Dynamics of Loans in thePolish Banking System

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Abstract

At the macro level, the time-series of the amounts of loans granted (a flow) and repaid (a flow) to the banking system in each period are not available. The information on these flows is important in many analyses, such as, inter alia, the impact of the bank lending on the financing of investment outlays. However, one can get the information concerning the structure of loans (a stock variable) regarding their duration. In each period, the loans are granted for different terms: from overnight ones to those lasting several years. The preferences of the credit takers are reflected in the term distribution of the outstanding loans. In order to estimate these flows, a model aimed at linking the above-mentioned preferences, the levels of loans and flows has been developed. In the approach proposed, the amount of loans outstanding is a resultant of the rate of the new loans flow but also of the average duration of these loans, which in turn is the result of the term preferences. The evaluation of the flow of the loans granted in the Polish banking sector is presented.

1. **Introduction**

The investment of enterprises in a significant part is financed by bank loans. At the macroeconomic level, it is often of great interest to analyze the relationship between these two categories: the flow of the investment outlays, and the flow of loans granted by banks. The banking sector, however, and in consequence the central banking, is not keen to use the flows in the analysis. The methodology used is based on the balance sheet items. In effect, in order to make the above mentioned analysis possible, a trick has been employed, consisting in substituting flows by net flows, which are equal to the changes in the level of debt (loans outstanding). This approach is workable and simple, although cases where such an approach may be is inadequate may be easily indicated.

The traditional approach works best, when the term structure of the loans granted is relatively constant. In the periods of significant changes occurring in that structure, for example during recession, the share of the short-term loans increases while the share of the long term loans decreases. This is related to the shrunk investment demand and the increased demand for short-term money because of the increased liquidity problems. Moreover, the constant level of the outstanding loans can conceal the fact that the rate of flow of the loans granted has changed significantly. The decrease of the mean loan term of the granted loans indicates the growth of the rate of flow of the loans granted.

The above remarks are an argument for the flow-based analysis. However, this task is hard to perform, as the data are neither available nor direct. The data collected by most central banks from the commercial banks are the time-series concerning the aggregated outstanding loans with terms belonging to the defined time-ranges. In an attempt to perform the flow-based analysis, two options can be considered. One would require the central bank to collect data concerning the flows of the loans granted (and also the inflows and outflows of deposits), but in order to do that the central bank should be first persuaded that such an analysis serves a purpose. The other option consists in finding some way of analyzing the existing and available data. Being a substitute for the direct data, this solution could be used simply as a cheap alternative.

This paper proposes a method which makes it possible to evaluate the flows of the loans in the banking system. The paper consists of several parts. In Part 2, the basic available data are presented and some preliminary analysis is performed. It is shown that in the period analyzed, a significant change in the term structure of the loans granted occurred. Part 3 contains the general outline of the proposed method of analysis. It embraces modeling of the flow the loans repaid (and/or written off) as the distributed lag function of the loans granted. Such an approach makes it possible to treat the amount of the outstanding loans also as the distributed lag function of the flow of the granted loans. Moreover, the impact of the rate of growth of the flow of the loans granted on the structure of the outstanding loans is analyzed. Before any evaluation of the coefficients of the distributed lag is attempted, the preferences of the loan-takers have first to be determined. Part 4, in order to facilitate the evaluations, introduces certain simplifications. In Part 5 the monthly flows of the loans granted are evaluated.

The presented study is a continuation of research made in the Systems Research Institute of the Polish Academy of Sciences, Gadomski (2002). The time-series used in the analysis belong to the period December 1996 - November 2002. It should be noted that since 2003, the aggregation of data has been changed in order to adjust the Polish banking statistics to the classifications used in the European Union. This is the main reason why the data used have not been updated.

This study was also motivated by another factor – sheer curiosity. Since most heads of the central banks cannot directly answer the question what amounts of loans are granted and repaid in each period, finding an answer to that question is compelling.

2. **Data**

The data available in this research consist of the time-series concerning the total amount of the loans outstanding in the Polish banking system[1], Fig. 1, as well as the amounts of outstanding loans belonging to the aggregated term ranges. Each granted loan is included in one of the following aggregate range[2]: (1) up to 1 month, (2) from 1 month to 3 months, (3) from 3 months to 6 months, (4) from 6 months to 12 months, (5) from 12 months to 36 months, (6) from 36 months to 60 months, (7) more than 60 months.

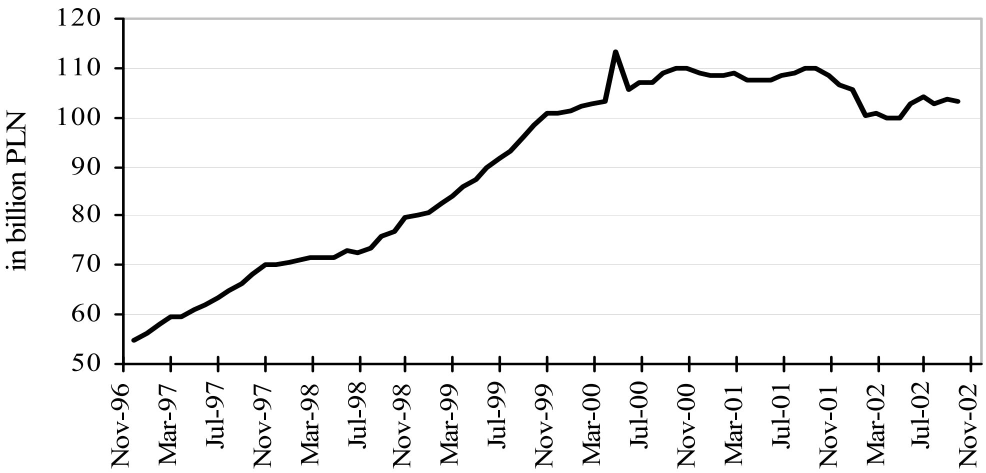

Fig.1. Total loans outstanding denominated in zloty (PLN).

The analysis of Fig. 1 leads to the conclusion that two sub-periods can be distinguished. The first one lasted from December 1996 to December 2000, while the second one lasted from January 2001 to November 2002. The first sub-period was characterized by a relatively steady growth of 20% per year; in the second sub-period outstanding loans tended to decrease at the rate of -3.2% per year.

At first glance, the shares of the outstanding loans belonging to the particular aggregated ranges in the total amount of the outstanding loans, Fig. 2, seem to be stable. A closer

[1] The research is limited to the analysis of Zloty denominated loans only.

[2] From the beginning of 2003 this classification has been changed as a measure aimed at adjusting Polish statistics to those of EU.

inspection, however, reveals a more complex picture. The available data were used to analyze a variable $\mathbf{T_{Ut}}$ interpreted as the mean term of the loans in the total amount of the outstanding loans at period t:

$$\mathbf{T_{Zt}} = \sum_{l=1}^{7} \mathbf{i_l u_t^{(l)}}, \tag{1}$$

where:

$\mathbf{i_l}$ – average length of the terms belonging to the **l**-th aggregated terms range (mid-range term),
$\mathbf{u^{(l)}_t}$ – share of the loans outstanding $\mathbf{z_t^{(l)}}$ from the **l**-th aggregated range in the total outstanding loans $\mathbf{z_t}$ at period **t**:

$$\mathbf{u^{(l)}_t} = \frac{\mathbf{z_t^{(l)}}}{\mathbf{z_t}}$$

The values of the variable $\mathbf{T_{Zt}}$ (Fig.3) signal a significant change in the mean time of the loans being the part of the loans outstanding during the transition between the two sub-periods. In the first sub-period, the average value of $\mathbf{T_{Zt}}$ was about 48 months; in the second sub-period, this value decreased by 6 months. This change coincided with a drop in the investment outlays in the Polish economy in that period. Whatever the reason of the decreased investment, the demand for loans changed, and that change was not just of the quantitative but also of the qualitative nature. This very fact constitutes a sound premise for this research.

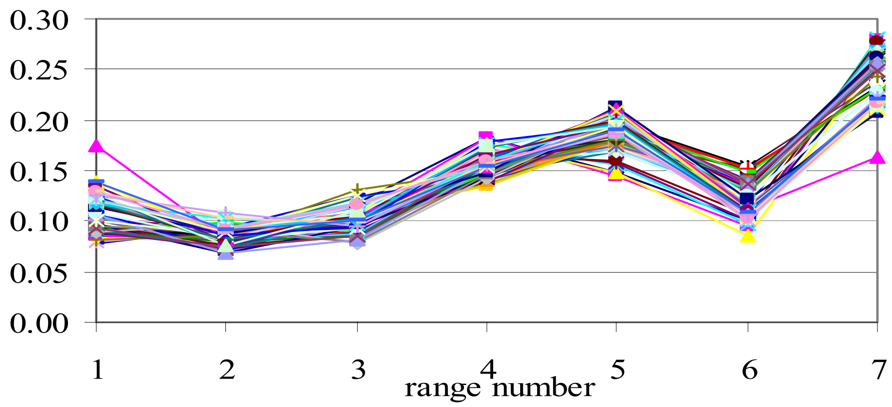

Fig.2 Structure of the outstanding loans in the Polish banking sector

3. Model of flows

In order to facilitate the presentation of the model, let us begin with its introductory version. At first it is assumed that the loans are granted for the periods of lengths expressed by natural numbers. This means that in the month **t**, loans are being granted for the immediate repayment, for one month, two months etc. (Later on a more realistic assumption will be adopted that the loans are being granted, for example, for periods expressed by real numbers.)

The amount $\mathbf{x}_t$ of all loans granted in the month **t** is thus distributed between loans with the different period lengths ranging from zero to a certain **n** which denotes the longest possible period of the loans:

$$\mathbf{x_t = \alpha_0\, x_t + \alpha_1\, x_t + ... + \alpha_n\, x_t;} \tag{2a}$$

or

$$\mathbf{x_t = x_{0t} + x_{1t} + ... + x_{nt}}\,, \tag{2b}$$

where $\boldsymbol{\alpha_i}$, $\mathbf{i = 0, 1,..,n}$; are coefficients which fulfill the conditions:

$$\boldsymbol{\alpha_i} \geq \mathbf{0,\ i = 0, 1,..,n;}\ \ \sum_{i=0}^{n} \alpha_i = \mathbf{1}\,, \tag{3}$$

and reflect the effective preferences of loanees (and to a certain extent of banks) as to the length of the repayment period. The values of these coefficients are the outcome of a negotiation process between the banks and their customers. The properties of the coefficients $\boldsymbol{\alpha_i}$, $\mathbf{i}$ = 0, 1,..,$\mathbf{n}$; make it possible to say that they form the preference distribution $\boldsymbol{A}$. In this model, the process of forming the values of $\boldsymbol{\alpha_i}$ is not analyzed, however, it is taken into account that the changes in the economic environment result in the changes in the preference distribution.

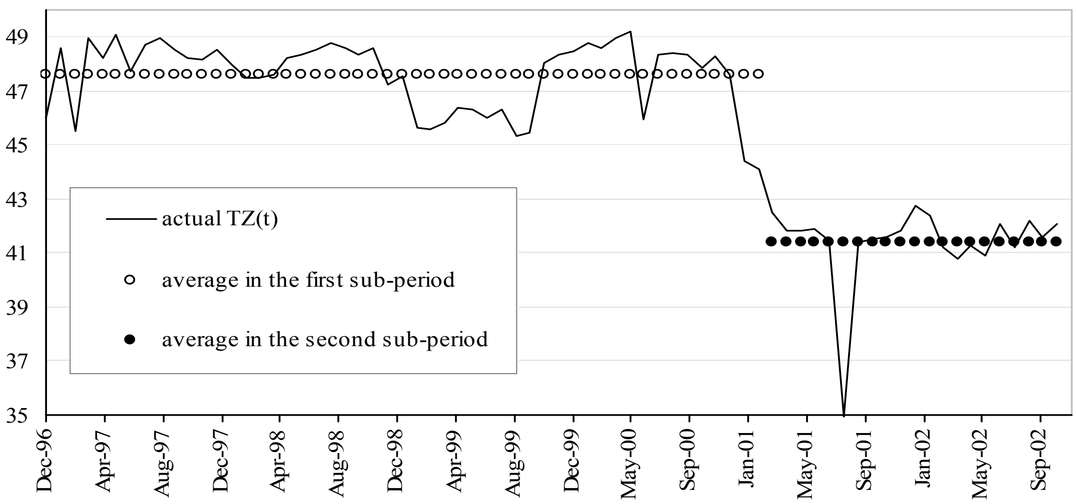

Fig. 3. Values of $\mathbf{T}_{U\,t}$ in the period December 1996 – November 2002.

We assume that the banking system is tight, which means that loans granted are always fully repaid. At first glance this assumption seems to be very strong; however the sum of the flows of the repaid as well as deteriorated loans (written off) provides the fulfillment of the balance condition: what enters the system will at some time leave it.

For each length of the repayment period $\mathbf{i}$, $\mathbf{i} = 0, 1,..,\mathbf{n}$; the flow $\mathbf{y_{it}}$ of repaid loans in the month $\mathbf{t}$ is the following function of the flow of the loans granted in the recent and preceding months[3]:

$$\mathbf{y_{it}} = \sum_{j=0}^{i} \mathbf{w_{ij} x_{it-j}} \,. \tag{4}$$

Equation (4) shows that in the month $\mathbf{t}$ the flow of repayment of the loans drawn for $\mathbf{i}$ months is composed of the repayments (or write-offs) of such loans drawn in the earlier months but not before the date $\mathbf{t-i}$. One can also notice that loans $\mathbf{x_{it}}$ granted in the month $\mathbf{t}$ for the period of $\mathbf{i}$ months are repaid in the following installments: $\mathbf{w_{i0}x_{it}}$ in the month $\mathbf{t}$, $\mathbf{w_{i1}x_{it}}$ in the month $\mathbf{t+1}$, $\mathbf{w_{i2}x_{it}}$ in month $\mathbf{t+2}$, etc. At this stage, however, no assumptions concerning the installments are necessary.

On the basis of (2a) and (2b) we have:

$$\mathbf{x_{it} = \alpha_i\, x_t,\ i = 0, 1,..,n};$$

thus the equation (4) can be rewritten in the following form:

$$\mathbf{y_{it}} = \alpha_i \sum_{j=0}^{i} \mathbf{w_{ij} x_{t-j}} \,. \tag{5}$$

For each $\mathbf{i}$, $\mathbf{i} = 0, 1,..,\mathbf{n}$; the coefficients $\mathbf{w_{ij}}$, $\mathbf{j} = 0, 1, 2,., \mathbf{i}$; in equation (5) constitute the lag distribution $\mathbf{W_i}$, so that:

$$\mathbf{w_{ij}} \geq 0 \text{ and } \sum_{j=0}^{i} \mathbf{w_{ij}} = \mathit{1} \text{ for all } \mathbf{i} = 0, 1,..,\mathbf{n}. \tag{6}$$

The rate of flow $\mathbf{y_t}$, which diminishes the amount of the loans outstanding, is the sum of loans granted for all period lengths:

$$\mathbf{y_t} = \sum_{i=0}^{n} \left(\alpha_i \sum_{j=0}^{i} \mathbf{w_{ij}} \right) \mathbf{x_{t-j}} \,. \tag{7}$$

Equation (7) can be rewritten in the following form:

$$\mathbf{y_t} = \sum_{i=0}^{n} \mathbf{w_i\, x_{t-i}} \,, \tag{8}$$

where :

$$\mathbf{w_i} = \sum_{j=0}^{n} \alpha_j \mathbf{w_{ji}} \,,\ \mathbf{i} = 0, 1, 2,., \mathbf{n};$$

On the basis of conditions (3), (6) and (7) it is easy to prove that coefficients $\mathbf{w_i}$, $\mathbf{i} = 0, 1,..,\mathbf{n}$; also form the lag distribution $\mathbf{W}$, because :

[3] The theoretical framework is based on Dhrymes(1977), Maddala(1981) and Gadomski (2003).

for all $\mathbf{i = 0, 1,..,n}$; $\mathbf{w_i} \geq 0$,

and

$$\sum_{i=0}^{n} \mathbf{w_i} = 1. \tag{9}$$

In order to prove (9) it is sufficient to recall the condition (3) and assume that for all $\mathbf{j}$ in equation (5) $\mathbf{x_{t-j}} = 1$.

Reassuming the above considerations one can state that the flow diminishing the amount of loans outstanding in the banking system is a distributed lag function of the past rates of the flow granted provided given preference distribution.

The main parameters characterizing distribution $\mathbf{W}_i$ are the expected value $\mathbf{T_{Wi}}$ and variance σ^2_{Wi}, which are defined by the following formulae:

$$\mathbf{T_{Wi}} = \sum_{j=0}^{i} \mathbf{j\, w_{ij}}\,, \tag{10a}$$

and

$$\sigma^2_{Wi} = \sum_{j=0}^{i} (\mathbf{j} - \mathbf{T_{w_i}})^2\, \mathbf{w_{ij}} \tag{10b}$$

Parameter $\mathbf{T_{Wi}}$ is interpreted as the mean time it takes an average monetary unit from the flow of the loans granted for **i** months to be repaid. However, one should note that such an interpretation is adequate only for the static steady state[4]. Generally, such mean time changes with the changes of **x**, and so a more adequate measure is presented by the following:

$$\mathbf{T_{Wi}(\,t\,)} = \frac{\sum_{j=0}^{i} \mathbf{j\, w_{ij}\, x_{t-j}}}{\mathbf{y_t}}\,. \tag{11}$$

In the case of the static steady state, when $\mathbf{y_t} = \mathbf{x_t} = \mathbf{x_{t-1}} = .. = \mathbf{x_{t-n}}$, the equality $\mathbf{T_{Wi}} = \mathbf{T_{Wi}(\,t\,)}$ always holds true. The introduction of $\mathbf{T_{Wi}(\,t\,)}$ is aimed at the analysis of the impact the changes of $\mathbf{x_t}$ have on the changes in $\mathbf{y_t}$.

Let us assume that **x** grew at a certain steady monthly rate **r**, so that $\mathbf{x_{t-i}} = \mathbf{x_t}\,(\,1 + r\,)^{-j}$. Under such an assumption, equation (11) can be rewritten in the following way:

$$\mathbf{T_{Wi}(\,t\,)} = \frac{\sum_{j=0}^{i} \mathbf{j\, w_{ij}\, x_t (1+r)^{-j}}}{\sum_{j=0}^{i} \mathbf{w_{ij}\, x_t (1+r)^{-j}}} = \frac{\sum_{j=0}^{i} \mathbf{j\, w_{ij} (1+r)^{-j}}}{\sum_{j=0}^{i} \mathbf{w_{ij} (1+r)^{-j}}} = \sum_{j=0}^{i} \mathbf{j} \frac{\mathbf{w_{ij}(1+r)^{-j}}}{\sum_{j=0}^{i} \mathbf{w_{ij}(1+r)^{-j}}} = \sum_{j=0}^{i} \mathbf{j\omega_{ij}(t)}, \tag{12}$$

where ω_{ij}, **i**=0, 1,2,..,**n**; are the coefficients:

[4] As defined in Solow (2000)

$$\omega_{ij} = w_{ij} \frac{(1+r)^{-j}}{\sum_{k=0}^{i} w_{ik} (1+r)^{-k}} , \tag{13}$$

which also form the lag distribution Ω_i, because for all **j**, **j**=0, 1, 2,..,**i**; $\omega_{ij} \geq 0$ and $\sum_{j=0}^{i} \omega_{ij} = 1.$

A closer inspection of equations (12) and (13) reveals the fact that the growth of **x** affects the value of $T_{Wi}(t)$. One can notice that $\omega_{i0} \geq w_{i0}$ because the denominator in (13) is always smaller or equal to one. As, given **i**, with increasing **j** the ratio ω_{ij} / w_{ij} converges to zero, this shows that a faster growth of **x** increases the weight of the recent values of **x** while the earlier ones are of a smaller weight In result, for **r** > 0 we have:

$$T_{Wi}(t) \leq T_{Wi} \tag{14a}$$

and for **r** < 0

$$T_{Wi}(t) \geq . T_{Wi}. \tag{14b}$$

At this stage of the analysis it is reasonable to adopt a weak assumption that the mean times T_{w_i} , **i**=1,..,**n;** needed by the monetary unit to leave the stock of the outstanding loans granted for the period of **i** months are in the following relation:

$$T_{w_1} < T_{w_2} < ... < T_{w_n} .$$

The above assumption is justified by the fact that the repayment schemes usually assume evenly spread installments, and the longer the loan term, the larger the value of T_{Wi}.

When the distributed lags are employed in modeling the flows, the concept of the lagged stock is useful. The lagged stock consists of all units that entered the lag but did not flow out of it. In the case of loans, these lagged stocks are simply the outstanding loans granted for the period of **i** months, **i**= 0, 1,..,**n.** If we denote by z_t the level of the outstanding loans granted for **i** months at the end of month **t**, then we can write the following expression:

$$z_{i\,t} = z_{i\,t-1} + x_{i\,t} - y_{i\,t}, \tag{15}$$

which states that the level of the outstanding loans at the end of month **t** is the sum of the level at the beginning of month **t**, diminished by the outflow (consisting of the installments as well as the write-offs - deteriorated loans).

By substituting formula (4) for the value of $y_{i\,t}$ we have:

$$z_{i\,t} = z_{i\,t-1} + x_{i\,t} - \sum_{j=0}^{i} w_{ij} x_{it-j} = z_{i\,t-1} + (1 - w_{i\,0})\, x_{i\,t} - \sum_{j=1}^{i} w_{ij} x_{it-j} . \tag{16}$$

Using equation (15) to express the values of $z_{i\,t-1}$, $z_{i\,t-2}$,.., $z_{i\,t-i}$ one finally obtains:

$$\mathbf{z}_{i\,t} = \sum_{j=0}^{i}(\mathbf{1} - \sum_{k=0}^{j}\mathbf{w}_{ik})\mathbf{x}_{it-j} \ . \tag{17}$$

Equation (16) shows that at the end of month **t** the amount of the loans outstanding granted in months **t**, **t-1**,.,**t-i** , respectively $\mathbf{x}_{i\,t}, \mathbf{x}_{i\,t-1}, \mathbf{x}_{i\,t-2},.., \mathbf{x}_{i\,t-i}$, equals the sum of what left from the loans granted after these grants were repaid (or written off).

Denote by $\mathbf{s}_{ij}$ the part of the loans granted for **i** month $\mathbf{x}_{i\,t}$ residing in the stock of the loans outstanding $\mathbf{z}_{i\,t}$ at the end of month **t**:

$$\mathbf{s}_{ij} = (\mathbf{1} - \sum_{k=0}^{j}\mathbf{w}_{ik}),\ \mathbf{j}=0,\ 1,\ ..,\mathbf{i}. \tag{18}$$

All $\mathbf{s}_{ij}$ are non negative on the basis of the fact that: $\sum_{k=0}^{i}\mathbf{w}_{ik} = 1$.

Now expression (16) can be rewritten in the following way:

$$\mathbf{z}_{i\,t} = \sum_{j=0}^{i}\mathbf{s}_{ij}\mathbf{x}_{it-j} = \alpha_i\sum_{j=0}^{i}\mathbf{s}_{ij}\mathbf{x}_{t-j}\ , \tag{19}$$

because $\mathbf{x}_{i\,t-j} = \boldsymbol{\alpha}_i\ \mathbf{x}_{t-j}$.

Coefficients $\mathbf{s}_{i\,j}$, $\mathbf{j}= 0, 1,..,\mathbf{i}$; in expression (19) do not form the distributed lag because they do not sum to one. Moreover their sum can be much larger. If, for example $\mathbf{w}_{i\,0} = 0$, then $\mathbf{s}_{i\,0} = 1$. In this example the very first coefficient equals 1, so their sum must be much larger.

The latter sum is determined and is equal:

$$\sum_{j=0}^{i}\mathbf{s}_{ij} = \mathbf{T}_{\mathbf{W}i} \tag{20}$$

The proof of (20) is based on the equality:

$$(\mathbf{1} - \sum_{k=0}^{j}\mathbf{w}_{ik}) = \sum_{k=j+1}^{i}\mathbf{w}_{ik}\ .$$

Hence,

$$\sum_{j=0}^{i}\mathbf{s}_{ij} = \sum_{k=1}^{i}\mathbf{w}_{ik} + \sum_{k=2}^{i}\mathbf{w}_{ik} + ... + \sum_{k=i}^{i}\mathbf{w}_{ik} =$$

$$\begin{aligned}
&= \mathbf{w}_{i1} + \mathbf{w}_{i2} + \mathbf{w}_{i3} + \mathbf{w}_{i4} +..+ \mathbf{w}_{ii} + \\
&\qquad + \mathbf{w}_{i2} + \mathbf{w}_{i3} + \mathbf{w}_{i4} +..+ \mathbf{w}_{ii} + \\
&\qquad\qquad + \mathbf{w}_{i3} + \mathbf{w}_{i4} +..+ \mathbf{w}_{ii} + \\
&\qquad\qquad \ldots\ldots\ldots\ldots\ldots\ldots \\
&\qquad\qquad\qquad .+ \mathbf{w}_{ii} =
\end{aligned}$$

$$= \mathbf{w}_{i1} + 2\mathbf{w}_{i2} + 3\mathbf{w}_{i3} + \mathbf{w}_{i4} +..+ \mathbf{i}\ \mathbf{w}_{ii} = \sum_{j=0}^{i}\mathbf{j}\,\mathbf{w}_{ij}$$

Finally, on the basis of the definition of T_{Wi} , expression (16a), we have:

$$\sum_{j=0}^{i} s_{ij} = \sum_{j=0}^{i} j\, w_{ij} = T_{Wi}.$$

Now it is possible to express the relationship (18) in terms of the distributed lag. It can be achieved by the normalization of the coefficients s_{ij} **:**

$$z_{it} = T_{Wi} \sum_{j=0}^{i} \left(\frac{s_{ij}}{T_{W_i}} \right) x_{it-j} = T_{Wi} \sum_{j=0}^{i} v_{ij}\, x_{it-j} \quad , \tag{21}$$

where v_{ij}, i=0, 1, 2,..,n and j= 0, 1, 2,..,i; are coefficients forming lag distribution V_i, and:

$$v_{ij} = \frac{1 - \sum_{k=0}^{j} w_{ik}}{T_{W_i}} = \frac{\sum_{k=j+1}^{i} w_{ik}}{T_{W_i}} . \tag{22}$$

The above considerations lead to the important conclusion that there exists a strict correspondence between the relationships (4) and (22); namely, if we know the coefficients of the distribution W_i, then we can determine the coefficients of the distribution V_i. The opposite holds true as well: coefficients of the distribution W_i can be determined on the basis of the coefficients from the V_i distribution.

Another important property of the relation between (4) and (22) is that whatever the shape of the distribution W_i (which is left unspecified), the coefficients of the V_i distribution, as illustrated by expression (22), are the non-increasing function of the index number j.

The parameters characterizing distribution V_i are the expected value T_{Vi} and variance σ^2_{Vi}, which are defined respectively by the following formulae:

$$T_{Vi} = \sum_{j=0}^{i} j\, v_{ij} \, , \tag{23a}$$

and

$$\sigma^2_{Vi} = \sum_{j=0}^{i} (j - T_{V_i})^2\, v_{ij} \tag{23b}$$

Parameter T_{Vi} , expression (23a), can be interpreted as the mean time a monetary unit spends in the stock of the outstanding loans which were formed by the loans granted for i months. Note that similarly to the case of the lag function (4), growth/decrease of the exogenous variable at a constant rate r results in decrease/increase of the effective mean time.

It will be shown now that the growth of the exogenous variable at constant rate r causes the growth of the endogenous variable also at that constant rate r.

On the basis of the above assumption we have:

$$x_{i\,t-j} = x_{i\,t}\,(1 + r)^{-j} \text{ and } x_{i\,t+1} = x_{i\,t}\,(1 + r)\,,$$

hence

$$y_{it+1} - y_{it} = \sum_{j=0}^{i} w_{ij} x_{it-j+1} - \sum_{j=0}^{i} w_{ij} x_{it-j} = x_t\,(1 + r)\sum_{j=0}^{i} w_{ij}(1+r)^{-j} - x_t \sum_{j=0}^{i} w_{ij}(1+r)^{-j} = r\,y_{it},$$

and finally:

$$y_{it+1} = (1 + r)\,y_{it}\,,$$

Which was to be proved.

Because the outstanding loans are the distributed lag function of the loans granted, it can be easily proved that the growth of the exogenous variable at a certain constant rate **r** causes the growth of the outstanding loans at the same growth rate **r**. This property will be used in the further analysis.

First, we will see that the mean time T_W , needed by the monetary to flow out from the stock of the loans outstanding, equals:

$$T_W = \sum_{i=1}^{n} \alpha_i T_{W_i}\,, \tag{24}$$

which means that T_W is the weighted average of the parameters T_{Wi} where the defined earlier preference coefficients α_i, **i**=1,..,**n**; play the role of the weight coefficients.

On the basis of the equation (8) we have:

$$w_i = \sum_{j=0}^{n} \alpha_j w_{ji}\;.$$

By determining T_W from the definition we have:

$$T_W = \sum_{i=1}^{n} i w_i = \sum_{i=1}^{n} i \sum_{j=0}^{n} \alpha_j w_{ji} = \sum_{j=1}^{n} \alpha_j \sum_{i=0}^{n} i\,w_{ji}\,,$$

which was obtained by a change in the summing order. As the last sum equals T_{Wi}, equation (10a), relationship (24) has been proved.

The following analysis will focus on some properties of the relationships between the level of the total outstanding loans and the total loans granted.

The total level of the loans outstanding at the end of the month **t** is the sum of the loans outstanding granted for periods ranging from **1** to **n** (in the introductory model the loans granted for 0 months do not participate in the outstanding loans):

$$z_t = z_{1\,t} + z_{2\,t} + .. + z_{n\,t} = \sum_{i=1}^{n} \alpha_i T_{w_i} \sum_{j=0}^{i} v_{ij} x_{t-j} = \sum_{j=1}^{n}\left(\sum_{i=0}^{i} \alpha_i T_{w_i} v_{ij}\right) x_{t-j}\;. \tag{25}$$

Now we will turn to the analysis of the shares $\mathbf{u_{k\,t}}$, $\mathbf{k}$=1, 2,..,$\mathbf{n}$; of the outstanding loans $\mathbf{z_{it}}$ granted for $\mathbf{i}$ months in the total stock of the outstanding loans $\mathbf{z_t}$:

$$\mathbf{u_{k\,t}} = \frac{\mathbf{z_{kt}}}{\mathbf{z_t}} = \frac{\alpha_k \mathbf{T_{W_k}} \sum_{j=0}^{k} \mathbf{v_{kj}\, x_{t-j}}}{\sum_{i=1}^{n} \alpha_i \mathbf{T_{w_i}} \sum_{j=0}^{i} \mathbf{v_{ij}\, x_{t-j}}}, \ \mathbf{k}= 1, 2,..,\mathbf{n}, \tag{26}$$

As all coefficients $\mathbf{u_{kt}}$ are non-negative and their sum equals one, for all $\mathbf{t}$ they constitute the probability distribution $\mathbf{U_t}$ and the appropriate methods of analysis can be employed. In the static steady state the formula (26) is reduced to the simpler form:

$$\mathbf{u_k} = \frac{\alpha_k \mathbf{T_{W_k}}}{\sum_{i=1}^{n} \alpha_i \mathbf{T_{w_i}}} = \frac{\alpha_k \mathbf{T_{W_k}}}{\mathbf{T_w}}, \ \mathbf{k}= 1, 2,..,\mathbf{n}, \tag{27}$$

because $\mathbf{T_W} = \sum_{i=1}^{n} \alpha_i \mathbf{T_{W_i}}$ and for all $\mathbf{i}$, $\sum_{j=0}^{i} \mathbf{v_{ij}\, x_{t-j}} = \mathbf{1}$.

As it was assumed earlier, it can also be noted that the values of $\mathbf{T_{w_i}}$ are the growing functions of the term length. In the case of the loans granted for $\mathbf{i}$ months, the share of those loans outstanding in the total stock of the loans outstanding is the function of the number $\mathbf{i}$ and the preference coefficient $\alpha_{\mathbf{i}}$.

In particular we are interested in the expected value $\mathbf{T_Z(t)}$, which can be interpreted as the mean term of the loans outstanding:

$$\mathbf{T_Z(t)} = \sum_{i=1}^{n} \mathbf{i\, u_{it}}\ . \tag{28}$$

If the exogenous variable grows at the steady rate $\mathbf{r}$, then equation (26) can be expressed in the following form:

$$\mathbf{u_{k\,t}} = \frac{\mathbf{z_{kt}}}{\mathbf{z_t}} = \frac{\alpha_k \mathbf{T_{W_k}} \sum_{j=0}^{k} \mathbf{v_{kj}\,(1+r)^{-j}}}{\sum_{i=1}^{n} \alpha_i \mathbf{T_{w_i}} \sum_{j=0}^{i} \mathbf{v_{ij}\,(1+r)^{-j}}}, \ \mathbf{k}= 1, 2,..,\mathbf{n}. \tag{29}$$

Expression (29) shows that the share of the $\mathbf{k}$-months loans in the total stock of the outstanding loans does not depend on the value of $\mathbf{x_t}$, but on the rate of growth, preferences and particular lag distributions $\mathbf{W_i}$. One can expect that the greater $\mathbf{r}$, the greater the share of the shorter term loans in the total amount of the outstanding loans and the smaller the shares of the longer term loans. This should affect the mean term $\mathbf{T_Z(t)}$.

Assume now that the exogenous variable grows at the steady rate **r**. If the preference distribution A is known, as well as the lag distributions $\mathbf{W_i}$, **i**=0, 1,.,**n**; it is possible to determine the share of the **k**-months loans in the total loans outstanding.

In this analysis, however, we are interested in another, more difficult problem: how to determine the unknown preference coefficients on the basis of the known shares of loans outstanding of the particular term lengths in the total stock of the loans outstanding.

4. Some adjustments

In order to proceed further, some assumptions are necessary for twofold reasons.

First, some simplifications are justified on the ground of reality. The family of the lag distribution belongs to that category. There is no need to assume sophisticated distribution where banking practice is simple. From now on we will assume that the installments are paid monthly and are evenly distributed. Moreover, one ought to take into account that the flows occur not in the discrete time but happen in almost continuous time. Hence in reality, the term lengths, which vary from zero to **n** months, are not necessarily represented by natural numbers. It is also necessary to account for the fact that some amount of loans granted in the given month for, say, a half-month period is carried over to the next month (if granted in the second half of proceeding month).

Second, as we are interested in the flows which are unknown, a way to determine them is to assume some sort of relationship between the stocks of the loans outstanding (which are known) and the flow of the loans granted and/or the flow of the loans repaid and written off.

In specifying the lag distribution, two lag distributions are introduced. The first one is used for explaining the repayment $\mathbf{y_{0t}}$ of the granted loans with terms not longer than 1 month. This repayment consists in one half of the loans granted in the period **t** for the term shorter than one month, and in the other half of the loans granted in the period **t-1** for the term shorter than one month:

$$\mathbf{y_{0t} = \tfrac{1}{2}\,\alpha_0\,(\,x_t + x_{t-1}\,)} \qquad (30)$$

The second lag distribution is used for explaining loans granted for the terms longer than one month. The elements of the following sum represent the installments of the loans granted for the term equal **i** and repaid in period **t**:

$$\mathbf{\alpha_i\ x_{t-1}/i + \alpha_i\,x_{t-2}/i + .. + \alpha_i\,x_{t-i}/i}$$

On the other extreme within the same range, there are the installments of loans granted for the duration of **i+1** months:

$$\alpha_i\ x_{t-1}/(i+1) + \alpha_i x_{t-2}/(i+1) + .. + \alpha_i x_{t-i}/(i+1) + \alpha_i x_t/(i+1)$$

It is assumed that the flow y_{it} is a simple mean of the above expressions:

$$y_{it} = \tfrac{1}{2}\alpha_i\ [\ (x_{t-1} + x_{t-2} + .. + x_{t-i})\ i^{-1} + (x_{t-1} + x_{t-2} + .. + x_{t-i} + x_{t-(i+1)})\ (i+1)^{-1}\ ]$$

$$= a_i\left[\ \sum_{j=1}^{i}\left(\frac{1}{2}\frac{2i+1}{i(i+1)}\right)x_{t-j} + \left(\frac{1}{2}\frac{1}{i+1}\right)x_{t-(i+1)}\right],\ i = 1, 2,.,n.$$

Note that the lag coefficients are all equal except the last one. Note also that these lag coefficients can be explicitly expressed in the following way:

$$w_{ij} = \begin{cases} \frac{1}{2}\frac{2i+1}{i(i+1)}, \text{if } j\le i \\ \frac{1}{2}\frac{1}{i+1}, \text{if } j=i \end{cases} \qquad i = 0, 1, .. ,n. \tag{31}$$

Knowing these coefficients for all monthly ranges of the loan terms, it is not difficult to determine the coefficients of the lag distributions V_i, $i = 0, 1,.., n$; equation (22). It is now possible to determine the amount of the loans outstanding $z_t^{(l)}$ belonging to the l-th aggregated range:

$$z_t^{(l)} = \sum_{i=i_l}^{i_u} z_{it}\ , \tag{32}$$

where i_l and i_u denote the shortest and the longest terms respectively, belonging to the l-th aggregated range.

On the basis of equation (32) the formula for shares $u^{(l)}_t$ can be easily developed:

$$u^{(l)}_t = \frac{z_t^{(l)}}{z_t},\ l = 1,..,7.$$

We know, on the basis of the considerations related to the expression (29), that during the period of the steady growth of the loans granted, the shares $u_{k\,t}$ do not depend on the absolute value of the loans granted but on the growth rate r of the loans granted, the coefficients of the lag distributions and the preference distribution.

If one looks at the equation (29), it can be noticed that the unknown variables are the preference coefficients α_i , $i = 0, 1, .. , n$. Now an attempt will be made to determine those coefficients by minimizing the mean squared error:

$$\min_{\alpha_i}\sum_{l=1}^{7}\left(u_t^{'(l)} - u_t^{(l)}\right)^2 ,$$

where:

$u'^{(l)}_t$ - loans outstanding $z_t^{(l)}$ belonging to the l-th aggregated range, $l = 1,..,n$; empirical data

$\mathbf{u^{(l)}_t}$ - loans outstanding $\mathbf{z_t^{(l)}}$ belonging to the **l**-th aggregated range, **l** = 1,..,**n**; value from the model.

In effect of the trial and error analysis it was assumed that the preference coefficients are generated by the linear combination of three preference distributions: the short-term, mid-term and long-term ones:

$$\alpha_i = \mathbf{d_1}\, \alpha_i^{(1)} + \mathbf{d_2}\, \alpha_i^{(2)} + \mathbf{d_3}\, \alpha_i^{(3)}\,,\ \mathbf{i}= 0, 1, .. , \mathbf{n}.$$

where: $\mathbf{d_1, d_2, d_3}$ - the weight coefficients fulfilling the following relations

$$\mathbf{d_1, d_2, d_3 \geq 0,\quad d_1 + d_2 + d_3 = 1; \qquad d_3 = 1\text{-}(d_1 + d_2).}$$

Parameters $\alpha_i^{(1)}$, $\alpha_i^{(2)}$ and $\alpha_i^{(3)}$ are respectively the coefficients of the preference distributions generated with the use of the Pascal distribution:

$$\boldsymbol{\alpha}^{(j)}{}_{\mathbf{i}} = \binom{\mathbf{r_j + i - 1}}{\mathbf{i}} (\lambda_j)^{\mathbf{i}} (1-\lambda_j)^{\mathbf{r_j}}.$$

with parameters:

$\mathbf{r}_j$- the order of the Pascal distribution

λ_j – parameter associated with the average lag $\mathbf{T^{(i)}}$ by the relation:

$$\lambda_j = \mathbf{r}_j / (\mathbf{r}_j + \mathbf{T}_j).$$

5. Evaluation of the preference parameters and the flow of granted loans

The examination of Fig.1 and Fig.2 provided the evidence in support of the hypothesis that in the analyzed period, two sub-periods can be distinguished. These sub-periods differ both in the dynamics of the outstanding loans, and the value of the mean term of the outstanding loans. The first period lasted from November 1996 till December 2000, while the second one lasted from January 2001 till November 2002. The differences mentioned are the reason for the separate evaluation of the preference distributions in both sub-periods.

The best results were obtained with these parameters generated by the Pascal distribution. The values of these parameters are shown in Table 1.

Table 1. The estimated values of parameters $\mathbf{T^{(i)}}$, $\boldsymbol{r}_j$ and $\boldsymbol{u}_j$.

	sub-period Dec-96 – Dec-00			sub-period Jan- 01 – Nov-02		
	$\mathbf{T^{(j)}}$(months)	$\boldsymbol{r}_j$	$\boldsymbol{u}_j$	$\mathbf{T^{(j)}}$(months)	$\boldsymbol{r}_j$	$\boldsymbol{u}_j$
j = 1	**0.325**	**1**	**0.715**	**0.25**	**1**	**0.759**
j = 2	**8**	**3**	**0.2325**	**8**	**3**	**0.21**
j = 3	**60**	**6**	**0.0525**	**58**	**6**	**0.031**

The actual and fitted values of shares are shown in Fig.4 and Fig.5.

Having determined the values of the preference distribution $\boldsymbol{A}$, one can determine the lag distribution **W**, the relation between the flow of the granted loans and the flow of the loans repaid (and written off), equation (7), as well as the lag distribution **V**.

The coefficients of the estimated distributions $\boldsymbol{A}$ and W are presented in Fig. 6.

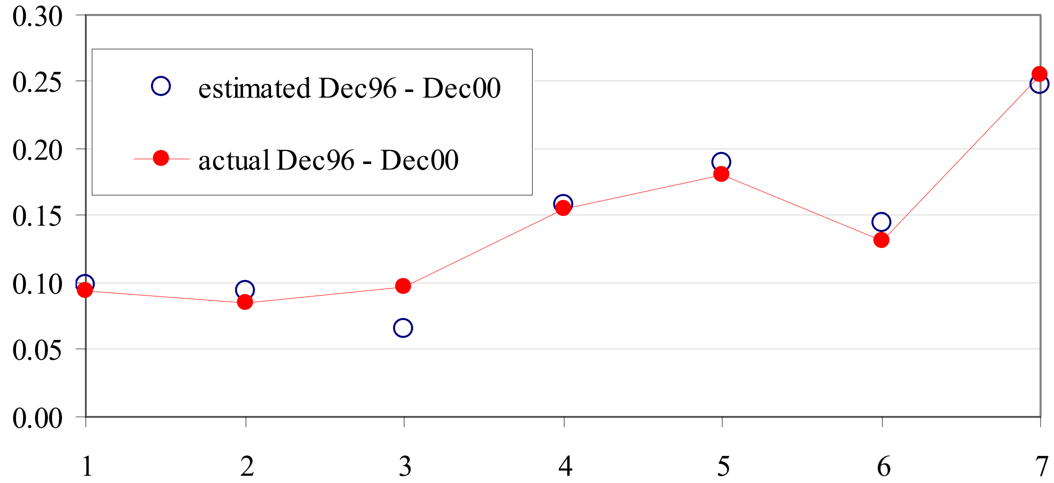

Fig. 4. The estimated and actual shares of the loans outstanding belonging to the particular term ranges in the first sub-period.

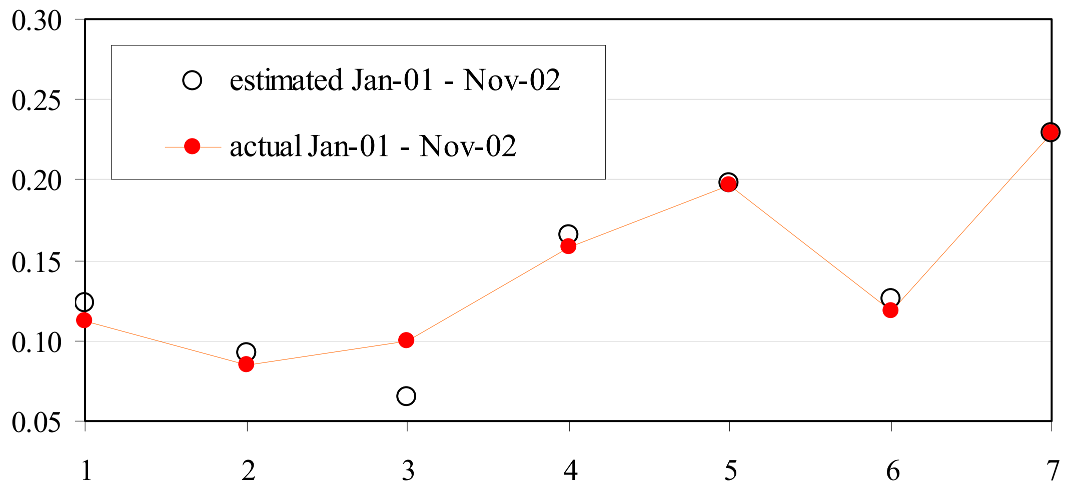

Fig. 5. The estimated and actual shares of the loans outstanding belonging to the particular term ranges in the second sub-period.

The evaluation of the flow of the loans granted uses the property of Koyck's model which makes it possible to express the value of the outflow $\mathbf{y_t}$ (repayment and write off) as the following function of the outstanding loans $\mathbf{z_t}$:

$$\mathbf{y_t = \lambda\, x_t + (1 - \lambda)\, z_t.}$$

Because:

$$\mathbf{z_t = z_{t-1} + x_t - y_t},$$

it is easy to arrive at the expression:

$$\mathbf{x_t = \frac{\lambda\, z_t - z_{t-1}}{1-\lambda}},$$

which is used for determining the values of $\mathbf{x_t}$.

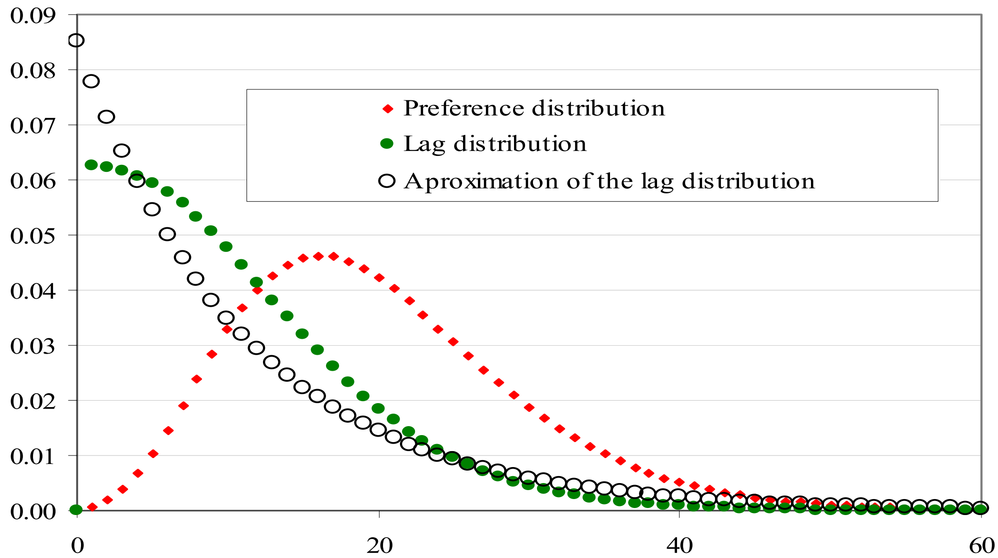

Fig.6. Estimated distributions ***A***, **W** and the approximated distribution.

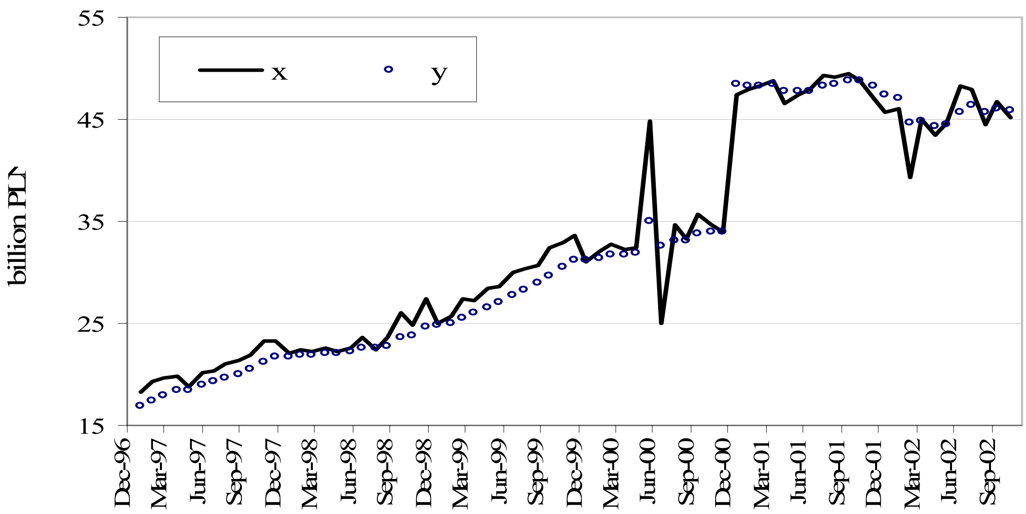

Fig.7. The monthly flow of the granted loans in the Polish banking sector.

The monthly flows of the loans granted are depicted in Fig. 7.

Some details in Fig.7 require comment. First of all, the sharp increase and then the equally sharp decrease of the flow in July and August 2000 are the effect of short-term (less than one month) privatization loans.

The sharp increase in the rate of the considered flow at the beginning of 2001 is, on the one hand, a result of a change in the preferences of the loanees (and, to a significant extent, of banks). On the other hand, so sharp an increase is also the result of the fact that the lag distribution **W** has been approximated with Koyck's distribution, which weights the most recent value of the exogenous variable, so the impact of the change is overvalued.

6. **Conclusions**

The above-presented considerations show that although the relationship between the flow of the loans granted and the amount of the loans outstanding is complex, it can be expressed by a conceptually simple distributed lag model. When drawing a loan, the loanee makes two main decisions: on the required amount of the loan, and the time it would take to fully repay the capital. The factors influencing this relationship are as follows: loan term preferences, scheme of the repayment of the principal, and the deteriorated loans. The changes in the economic environment can affect the demand for the investment loans and have a significant impact on the term structure of the loans drawn.

The structure of the loans outstanding is influenced by the term preferences and by the very dynamics of the loans granted. The bigger the rate of growth given constant term preferences, the smaller the share of long-term loans and the smaller the mean time a monetary unit spends in the loans outstanding. Also valid is the opposite: a decrease in the flow of the loans granted results in the increase of the share of the long-term loans, thus increasing the mean time a monetary unit spends in the loans outstanding. On the other hand, a change of the term preferences, ceteris paribus, may result in significant changes of the loans outstanding.

This analysis leads to the conclusion that the dynamics matters: the amount of the loans outstanding can grow either with the growth or with the decrease of the loans granted. Hence, in the formulation of the monetary policy the dynamics of the loans granted and changes in the term preferences should be taken into account.

It was proven that the flow of the loans granted could be evaluated on the basis of the available data. In the Introduction, two solutions were considered. First, that the central bank becomes convinced to the idea of evaluating the rates of flows of loans and collects the relevant data, and the second, that a method can be found to use the existing and available data for that purpose. This study, in the opinion of the author, presents such a method. Moreover, as these solutions should not be contradictory, it would be interesting to compare actual data with the results obtained using the proposed method.

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