THE DEPENDENCY OF EXTREME RETURNS ON STOCK INDICES ACROSS BORDERS IN BULL AND BEAR PERIODS

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Abstract

The dependency of returns on stock indices in different countries manifests itself in a number of ways; for example, bull and bear periods may swap across borders, or non-zero correlation may be observed between weekly returns. The focus of our study is the dependency of (positive as well as negative) joint threshold exceedances of pairs of daily returns. In a first step, we fit a generalized Pareto distribution to the series of return threshold exceedances in each stock index considered. The second step is to analyze pairs of stock indices on the basis of a dependence function, which contains a parameter quantifying the degree of dependency, used for coupling pairs of threshold exceedance distributions. Our study analyzes bull and bear periods separately. Considering 190 international pairs of twenty stock indices, we obtain a picture of international dependencies. One of our findings is that the degree of dependency tends to increase during bear periods.

Key words: bivariate threshold exceedances; generalized Pareto distribution; logistic dependence function; daily stock index returns; bull and bear periods; dependence of stock markets

1 Introduction

National markets do not operate in isolation: They may affect each other in a multitude of ways, resulting in mean return and volatility spillovers between markets. To understand how characteristics of the return dynamics are transmitted among markets is of interest to economists, economic policy makers, and financial analysts.

There are several different approaches to capturing the international dependence of markets. The simplest way is to compute coefficients of correlation between pairs of weekly (say) returns, and correlation is indeed one of the basic ingredients of modern portfolio selection (Markowitz [6]). If a more general analysis of the structure of dependence between markets is intended, the shortcomings of the coefficient of correlation are twofold. First, it does not take the dynamic structure of the returns into account; the usual coefficient of correlation is always a measure of an average which pertains to an extended period of time, rather than an instantaneous measure of the degree of dependence. Second, the coefficient of correlation is not an adequate measure of the dependence which is present in the tails of distributions; it can therefore not model the dependence when returns jointly exceed a certain high threshold.

There are other approaches which can remedy these shortcomings, focussing on the goals of the analysis. If an analysis of volatility spillover between markets is intended, modeling a time series on the basis of a multivariate GARCH (MGARCH) process can be appropriate, since it is able to describe variances and covariances of the time series for each epoch conditional on what happened before the current epoch. Recent examples of research projects in this direction are Xu and Fung [12], who use a bivariate GARCH to investigate the pricing process of Chinese stocks listed at the stock exchanges of

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Hong Kong and New York; Higgs and Worthington [3] use an MGARCH to study the spillover between eight art markets; and Higgs and Worthington's [11] analysis of mean and volatility spillovers in Asian equity markets is based on nine-dimensional vector autoregressive and GARCH processes.

If, on the other hand, an assessment of extreme quantiles of the return distribution is the focal point of the analysis, it is more appropriate to fit a generalized Pareto distribution (GPD) to return excesses, that is, to the differences of returns and the 90% (say) quantile. This approach permits a reliable risk assessment and is therefore used in Value-at-Risk analysis; see Gençay and Selçuk [2] for a recent investigation of (one-dimensional) tail estimations of return distributions of stocks in emerging markets. Depending on how the tail of the distribution in question is specified, a generalized extreme value distribution (for extreme returns, in terms of a maximum or minimum, see e.g. Longin [4]) or a GDP (in the case of threshold excesses) will be more appropriate. Once two one-dimensional GDPs (or extreme value distributions) have been fitted to individual return series, it is possible to bring them together by estimating a dependence function (a copula, see Nelsen [8]) on the basis of joint extremes or joint threshold exceedances. In the context of returns on assets, this approach was used by Mendes and Moretti [7] (to merge extreme value distributions) and by Longin and Solnik [5] and Schmidbauer and Rösch [10] (to merge GPDs).

The goal of the present paper is to investigate the dependence of daily returns on stock indices on an international level, on the basis of returns on twenty stock indices. The assessment of dependence is in terms of pairs of return series, whose excesses are individually modeled as GPDs and then joined estimating a copula. In contrast to other studies (e.g. Mendes and Moretti [7]) we discriminate between bull and bear periods, and we do not identify a high gain (loss) with a bull (bear) period (see Longin and Solnik [5]), but bull and bear periods are defined such that a day is said to lie in a bull or bear period with respect to returns observed prior to that day. In particular, we are interested in finding whether the dependence among markets is higher during bull or bear periods. The methods used in the present paper are based on Schmidbauer and Rösch [10]; all computations were carried out in R [9].

2 Measuring the dependence of threshold excesses

In the following we give a brief outline of the method applied. For a detailed description, see Schmidbauer and Rösch [10]. Our investigations discriminate between bull and bear periods. The first step is therefore to determine, for each time series of stock indices, which days belong to bull and which to bear periods. This is essentially achieved by smoothing the time series of returns using a one-sided moving average (length: 50 days) with decreasing weights (more distant days are given less weight), and a bull (bear) period is defined as a sequence of days for which this smoothed series is increasing (decreasing, respectively).

The next step is to fit a GPD to the series of return excesses. An exceedance is said to occur if a return exceeds a certain threshold, which is the 90% (or 10%, in the case of negative exceedances) quantile throughout our investigations. The distribution function of the GPD is defined as:

$$F(x;k,\sigma) = \begin{cases} 1 - \left(1 - k\dfrac{x}{\sigma}\right)^{1/k}, & k \neq 0 \\ 1 - \exp\left(-\dfrac{x}{\sigma}\right), & k = 0 \end{cases} \tag{1}$$

Here, $\sigma > 0$ is a scale parameter; it depends on the threshold and on the probability density of the returns. The shape parameter k is called the tail index, since it characterizes the tail of the density function: The case $k < 0$ corresponds to fat-tailed distributions; in this case, the GPD reduces to the Pareto distribution. The case $k = 0$ corresponds to thin-tailed distributions; the GPD then reduces to the exponential distribution with mean σ. The case $k > 0$ corresponds to distributions with no tail (i.e. distributions with bounded support). When $k = 1$, the GPD becomes a uniform distribution on the interval $[0, \sigma]$. The method developed by Castillo and Hadi [1] is adopted here to estimate the parameters k and σ. Negative return excesses are defined as the 10% quantile, minus the return.

In a second step, bivariate distributions are constructed, which have the estimated GDPs as marginal distributions. For each index pair, there are four joint distributions: positive and negative excesses, each combined with bull and bear periods. The joint distributions are modeled on the basis of a dependence function

$$D(y_1, y_2) = \left(y_1^{1/\alpha} + y_2^{1/\alpha}\right)^{\alpha}, \tag{2}$$

where $y_i = -\ln F(x_i; k_i, \sigma_i)$, and $x_i = r_i - q_i$ is the return excess, that is, the difference between observed return r_i and that index's 90% quantile, q_i, and F is the cdf of the GPD. The joint distribution function of return excesses is

$$G(x_1, x_2) = \exp[-D(y_1, y_2)] = \exp\left[-\left(y_1^{1/\alpha} + y_2^{1/\alpha}\right)^{\alpha}\right]. \tag{3}$$

The parameter $0 < \alpha \leq 1$ quantifies the degree of (positive) dependence between the return exceedances. Independence (i.e., $G(x_1, x_2) = \exp[-(y_1 + y_2)]$) corresponds to $\alpha = 1$, while complete dependence (i.e., $G(x_1, x_2) = \exp[-\max(y_1, y_2)]$) is obtained as $\alpha \to 0$. The model (2) is known as the symmetric logistic model. It is a special case of a so-called Archimedean copula (a copula is a multivariate cdf with uniform marginals), see Nelsen [8].

For each case (an index pair in bull/bear periods, positive/negative returns) the parameter α is estimated on the basis of joint daily return excesses. A joint exceedance is defined to occur if the distance between the individual threshold exceedances is not more than five days. It is thus not required that individual exceedances occur on the same day to be considered a joint exceedance; this reduces the noise in the data.

Finally, we have to explain what we mean by bull and bear periods in the case of a two-dimensional time series of returns. In any case, our index pairs are formed such that the index for which a longer time series is available is taken to be the first component, and a pair of returns (r_{1t}, r_{2t}) is said to lie in a bull (bear) period whenever day t belongs to a bull (bear) period for the first index. We found that this is a viable compromise between too small datasets if a more restrictive definition is used, and a simple identification of high returns with a bull period.

3 The stock indices used in the present study

Our analysis is based on pairs of twenty stock indices. This yields a total number of $\binom{20}{2} = 190$ pairs. The following table reports the indices, together with the day from which onward we use them (the last day is always in April 2004), the number N of days (observations), and the percentage of days in bull and bear periods.

name	symbol	country	first day	N	% bull	% bear
Dow-Jones	dji	USA	1950-01-04	13724	62.5	37.5
S&P 500	gspc	USA	1950-01-04	13645	63.3	36.7
DAX	gdaxi	Germany	1959-09-29	11114	56.6	43.4
Nikkei 225	n225	Japan	1984-01-05	4983	53.0	47.0
FTSE 100	ftse	UK	1984-04-03	5052	63.2	36.8
All Ordinaries	aord	Australia	1984-08-06	4963	61.7	38.3
Hang Seng	hsi	Hong Kong	1987-01-02	4268	63.4	36.6
ISE National 100	xu100	Turkey	1987-10-26	4085	61.3	38.7
Straits Times	sti	Singapore	1987-12-29	4049	52.3	47.7
CAC 40	fchi	France	1988-01-04	4078	64.1	35.9
Swiss Market	ssmi	Switzerland	1990-11-12	3335	65.4	34.6
BSE 30	bsesn	India	1991-01-03	3063	54.4	45.6
BEL-20	bfx	Belgium	1991-04-10	3252	59.6	40.4
IPC	mxx	Mexico	1991-11-11	3084	60.5	39.5
NZSE 40	nz40	New Zealand	1992-09-21	2887	57.5	42.5
AEX General	aex	Netherlands	1992-10-13	2910	63.8	36.2
ATX	atx	Austria	1992-11-12	2811	59.6	40.4
KFX	kfx	Denmark	1993-01-27	2794	64.7	35.3
Bovespa	bvsp	Brazil	1993-04-28	2700	63.7	36.3
MIBTel	mibtel	Italy	1993-07-20	2691	56.9	43.1

4 Empirical results concerning international dependency

For the sake of simplicity, we do not report the estimation results in terms of estimated parameter values here. Following Longin and Solnik [5], we compare the levels of dependence in terms of $1-\alpha^2$, where α is the parameter measuring the degree of dependence. It should be noticed that a high (low) value of $1-\alpha^2$ indicates a high (low) degree of dependence. As explained in Section 2, the parameter α was estimated separately for positive/negative excesses during bull/bear periods. In order to gain insight into the structure of dependence in the pairs and during bull and bear periods, the α estimates are plotted against overall correlation coefficients of pairs of weekly index returns (without discrimination between bull and bear periods) in the scatterplots in Figure 1. The x-coordinate (weekly correlation) is the same for corresponding points in all four scatterplots. Some points are labeled in order to exemplify the conclusions that can be drawn from our analysis.

Although it is not straightforward to detect a pattern in these scatterplots, it is clear at first sight (and will be more formally analyzed in the contingency table below) that the density of points in the upper part of the plot is highest in the lower left scatterplot: The degree of dependence grows particularly during bear periods. We could say: Positive news strengthens ties among stock markets most when it is least expected.

The weekly return correlation of an index pair does not necessarily give a hint as to its degree of dependence in threshold exceedances. A high level of weekly correlation may well go along with a small amount of dependence in exceedances, as in the case of the pair gspc_ssmi. On the other hand, lower weekly correlation can conceal a jump to high levels of daily threshold exceedance dependence, as in the case of xu100_aex.

With regard to general tendencies during bear periods, the levels of ftse_fchi and gdaxi_fchi, for instance, may be of interest. Excesses during bear periods appear to be more likely to swap over borders than during bull periods; an effect which is pronounced in the case of dji_fchi for positive exceedances only. The swap-over of negative exceedances seems rather contained for dji_fchi (in spite of the high overall correlation of weekly returns) — and this is in stark contrast to the interaction of dji_gdaxi.

We conclude this study with a comparison of how the index pairs behave during bull and bear periods in terms of a contingency table. Consider the two statements:

A: In the case of positive threshold exceedances, dependency increases in bear periods.
B: In the case of negative threshold exceedances, dependency increases in bear periods.

The following contingency table reports the numbers of cases in which A and/or B are true:

		statement A		
		true	false	
statement B	true	68	38	106
	false	49	35	84
		117	73	190

With these frequencies, the ratio of true/false counts is significantly different from 0.5 in the case of statement A (p-value: 0.0018), while it is not significantly different from 0.5 in the case of statement B (p-value: 0.1276). There is, however, symmetry in the behaviour of the stock index pairs with regard to positive and negative return exceedances in so far as a χ^2 test of independence does not reject the null hypothesis of independence in the contingency table (p-value: 0.59). This means: The behaviour of a pair of stock indices with respect to joint positive threshold exceedances does not generally give a clue as to its behaviour with respect to negative threshold exceedances.

5 Summary and conclusions

We investigated the degree of pairwise dependence, during bull and bear periods, between joint threshold exceedances of returns on stock indices. The empirical basis of the study consists of twenty stock indices, from which 190 pairs are formed. For each pair, the degree of dependence is measured by estimating the parameter of the dependence function. It was found that the amount of weekly return correlation does not permit a clear assessment of the degree of dependence in times of threshold exceedance. Furthermore, it was found that the dependence is generally increasing during bear periods, this effect being more pronounced for positive returns: Positive news seems to have its strongest impact when nobody expects it.

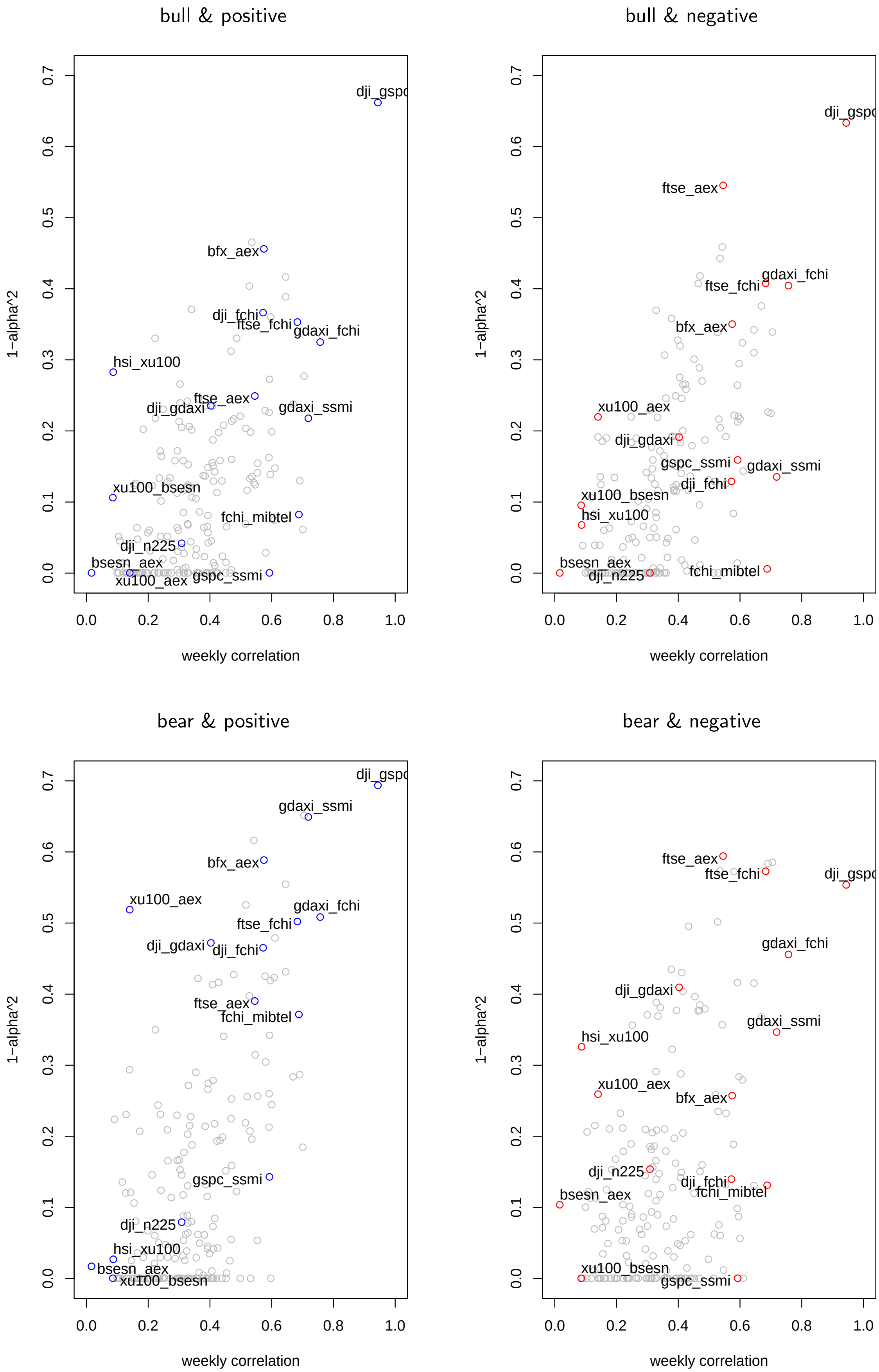

Figure 1: Scatterplots of levels of dependence vs. correlation of weekly returns

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