Goodwin « Growth-Cycle » and the NAIRU.

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The non-linear growth-cycle model is used to analyse the interactions between the growth cycle and the NAIRU. In particular it is shown that an increasing NAIRU will increase the length of the depression phase of the cycle and slow down the initial recovery phase.

1. Introduction..

This paper is a contribution to the debate about the labour demand – real wage nexus. It uses the non-linear growth-cycle model proposed by R.M. Goodwin in 1967[2].

With respect to the usual Cobb-Douglas growth model, it has the advantage of mixing explicitly both growth and cycle, enabling the model to deal with short-run variations in factor shares compatible with long-run stability of these shares. Furthermore, the model includes a wage formation model fully compatible with the NAIRU concept.

In other words, the model may be used to appreciate the impact on the growth cycle of changes in the level of the NAIRU.

2. Assumptions.

The model is highly stylised in order to remain tractable but is nonetheless able to emphasize the consequences of certain type of behaviour.

The basic assumptions are as follows

1. Steady technical progress, disembodied and Harrod-neutral
2. Steady growth in the labour force
3. Two factors of production, labour and capital both homogenous and non-specific
4. All quantities are measured in real and net terms.
5. All wages are consumed; all profits are saved and invested.
6. The capital-output ratio is assumed to be (roughly) constant over time
7. The real wage rate will rise in the neighbourhood of full employment.

Assumption 5 is only for convenience and can easily be relaxed without changing the nature of the results. It has also the side advantage of making the average saving rate cyclical rather than constant as in Solow's neo-classical growth model.

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[2] GOODWIN, R,M. " A Growth Cycle " in C.H. Feinstein (ed.) "Capitalism and Economic Growth", Cambridge University Press, 1967.

Assumption 6 should in fact be a result rather than an assumption but need to be only roughly true over time.

Assumption 7 finally should be expressed in nominal terms and incorporate inflation, but that would complicate considerably the model. As stated, it simply means that in the neighbourhood of full employment, nominal wages will grow faster than prices, which seems realistic enough.

3. The model.

The main variables are

Q = real output

K = capital stock

k = net investment (= dK/dt)

w = real wage rate

c = capital/output ratio (inverse of the average productivity of capital)

a = labour productivity

L = employment

n = labour supply

From assumption 1, we have $a=a_0e^{\alpha t}$ with α constant.

From assumption 2, we have $n= n_0e^{\beta t}$ with β constant

The workers' share of output is w/a, the profit share is (1-w/a)

From assumption 5, investment is k = (1-w/a).Q

Employment is given by L = Q/a

The profit rate is k/K = (1-w/a)/c

The average productivity of labour grows at rate α:

Defining q = dQ/dt and l = dL/dt one has d(Q/L) = q/Q-l/L = α so that

$$l/L = \frac{(1-w/a)}{c} - a$$

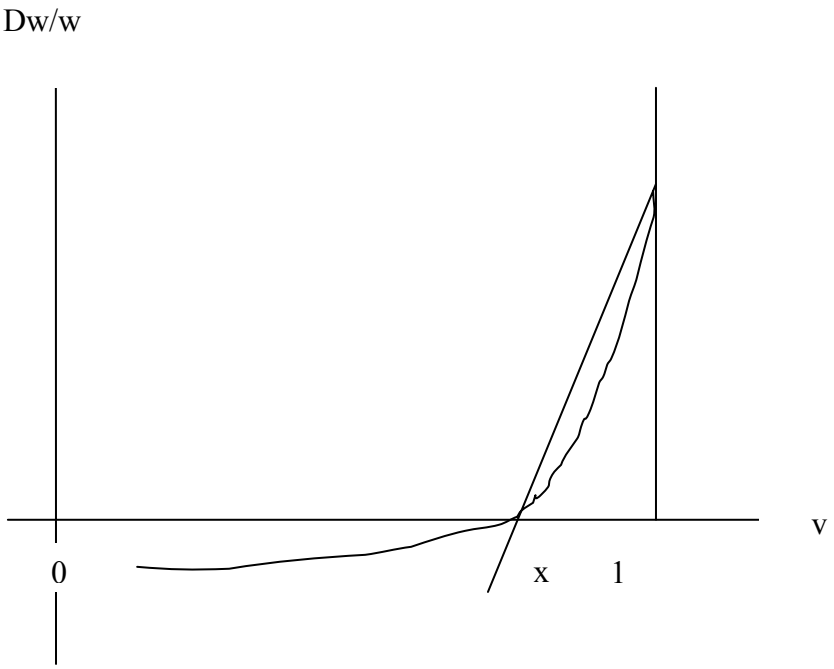
In order to shorten the notations we define

u = w/a the ratio of real wage to labour productivity

v = L/N the employment ratio

Assumption 7 may be written as dw/w = f(v) as in figure 1

Figure 1



For convenience f(v) will be replaced by a linear approximation which holds for moderate movements of v near v=1 i.e. dw/w = -γ+ρ.v. It is clear from this graph that point x is the complement to one of the NAIRU i.e. the non-accelerating-inflation-rate of employment NAIRE v= γ / ρ

Since du/u = dw/w – α or

Du/u = -(α+γ)+ρ.v

The system then boils down to two differential equations

Dv = [{1/c-(α + β)}-u/c].v (1)

Du = {-(α+γ)+ρ.v}.u (2)

These equations are formally equivalent to the predator-prey model developed by Volterra in 1931[3] and, as describing the symbiosis between two populations, partly complementary but also partly hostile may describe adequately the structure of a capitalist economic system and, in particular the strategies of employers and trade unions.

Eliminating time and integrating we get

(1/c).u + ρ.v – [(1/c)-(α + ρ)] log u – (γ +α) log v = constant (3)

[3] VOLTERRA, V. Théorie mathématique de la lutte pour la vie. Gauthier-Villars, Paris, 1931

Defining, for simplification

$\Theta_1 = 1/c$ $\eta_1 = (1/c)-(\alpha + \rho)$

$\Theta_2 = c$ $\eta_2 = \gamma + \alpha$

We can transform equation (3) into

$$\Phi(u) = u^{\eta 1} e^{-\Theta 1 u} = Hv^{-\eta 2} e^{\Theta 2 v} = H.\psi(v) \qquad (4)$$

Where H is an arbitrary constant depending on initial conditions.

Since $(1/c) > (\alpha + \beta)$, all coefficients are positive and the slope of these two functions can be viewed in figure 2 by differentiating

$$d\Phi/du = [-\Theta_1 + (\eta_1/u)]\,\Phi$$

$$d\psi/dv = [\Theta_2 - (\eta_2/v)]\,\psi$$

Figure 2.a

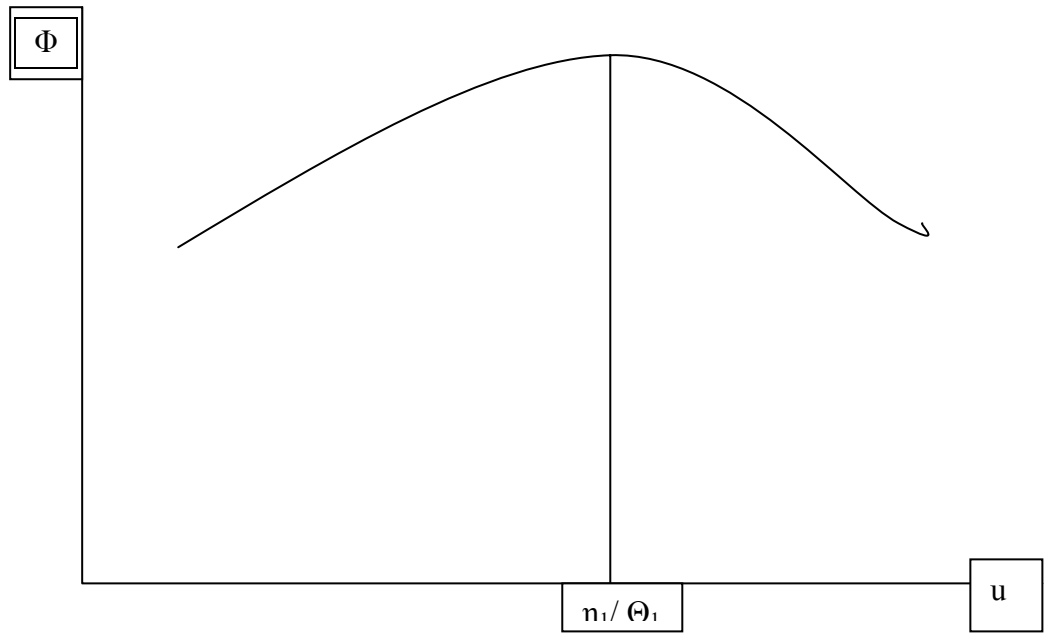
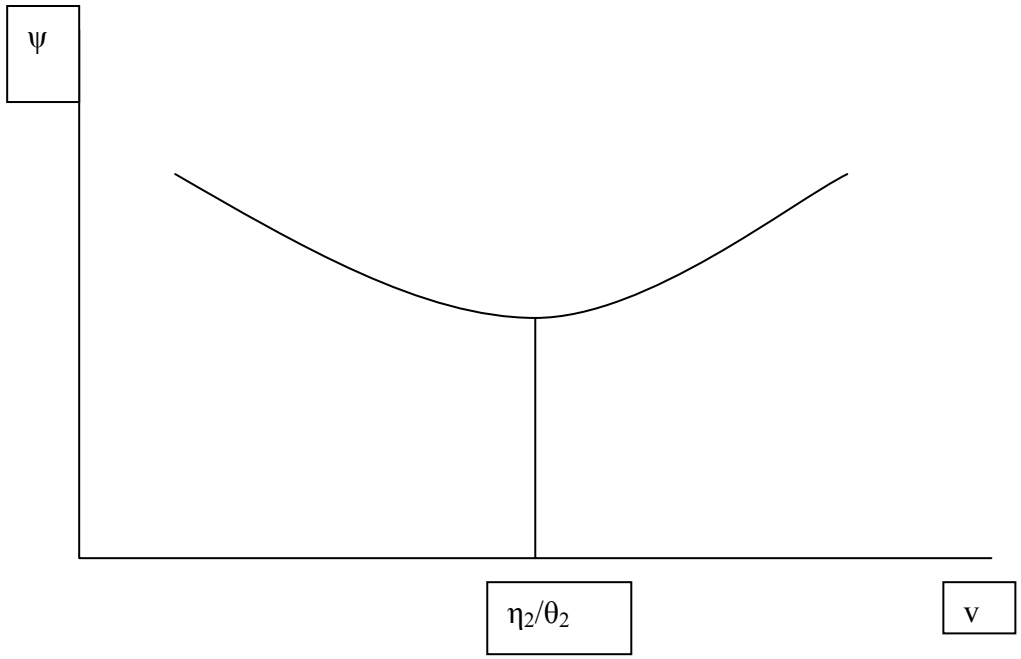


Figure 2b



These functions can be related into a four-quadrant diagram (figure 3) with the constant H being represented by a straight line trough the origin with slope φ/ψ =H (arbitrary since it depends on initial conditions. The two curves are then placed into symmetrical quadrants and equated via H. They give thus all pairs of u and v that are solutions of equation (3) and are plotted in the fourth quadrants where they take the form of a closed-looped oscillator, in the u, v space.

By using equations 1 and 2 one can determine in what direction one moves on B, as represented by the arrows in the graph.

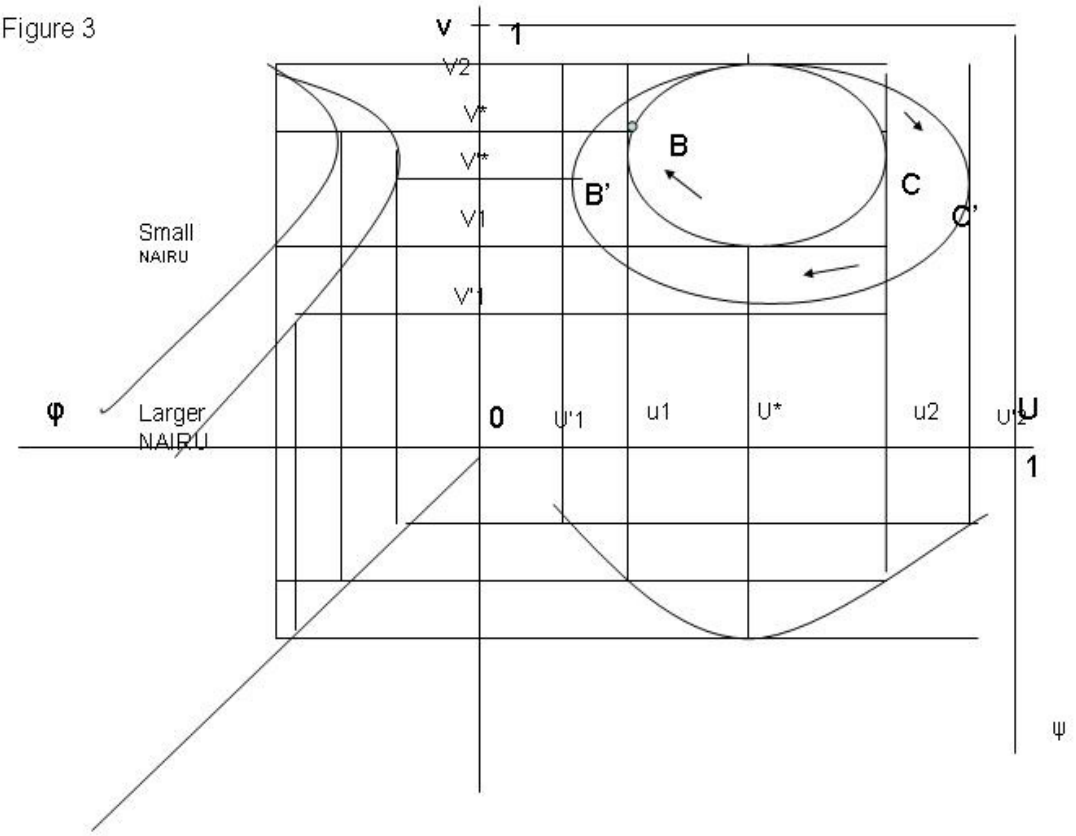
On the 0-1 u axis, any point indicates the distribution of real income with the wage share to the left and the profit share to the right.

The profit share is directly linkable to the profit rate and ultimately to output growth via the capital/output ratio.

When left undisturbed, the system has constant average values for u an v hence a constant long-run average distribution of real income and degree of employment.

When disturbed, the system will show alternative rate of growth for output and employment. Whether they actually decrease in level or merely grow more slowly will depend on the severity of the cycle generated by the disturbance.

Figure 3



How does the system work?

Starting at point B, the profit share is at its highest possible value (u1) with employment at its long-run average value (v*). This situation generates an investment boom pushing output, employment and wages net of productivity upward. Employment then reaches its maximum level at v2, but the profit rate then fall back to its average level u* and below it. At this stage, labour demand decreases and employment fall back to its equilibrium level but this time with the profit rate at its nadir (point C). At that stage, the low level of investment lead to a fall in the growth of output and employment well below full employment thus restoring profitability since wages are growing then at a lower rate than productivity. In turn, this increase in profitability brings about an output and employment recovery until the system is back at B and the cycle starts again.

The critical element is thus not the real wage growth but the fact that it grows faster or slower than productivity.

5. Linkage with the NAIRU.

An increase (for any reasons) of the NAIRU may be introduced in Goodwin's model though figure 2. Indeed, a rising NAIRU (i.e. a falling NAIRE) is equivalent to a displacement of the equilibrium value to a lower level. In other words in $dw/w = -\gamma+\rho.v$, both γ and ρ are taking smaller values but γ falls more than ρ, causing a fall in γ/ρ.

In the oscillator system the curve $\psi(u)$ is independent from γ and ρ whereas $\varphi(v)$ moves towards the central axe : the new v'* is reached for a lower value of v an of $\varphi(v)$. When put in

figure 3, this leads to a larger oscilator. Thus an increase of the NAIRU will therefore make the depression and initial recovery phases of the cycle longer and slower than before.The "inflationery" boom and initial slowdown are less affected.

Thus, all other things being equal, a rise in the NAIRU would cause a lower average rate of growth of output over a medium term horizon, together with a lower equilibrium level of employment.