Real effects of nominal shocks: a 2-sector dynamic model with slow capital adjustment and money-in-the-utility*

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Abstract

This paper develops a two-sector model to study the effect and incidence of nominal shocks (fiscal or exchange rate policies) on sectors and factors of production. I adopt a classical two-sector model of a small open economy and enrich its structure with gradual investment and a preference for real money holdings. An expansive nominal shock (fiscal expansion or a nominal appreciation) leads to increased spending (due to the role of money), which pushes nontraded prices up (with gradual capital adjustment, the short-term transformation curve is nonlinear). This translates into changes in factor rewards, capital labor ratios and sector-level employment of capital and labor. Higher nontraded prices lead to extra domestic income, validating some of the initial excess spending. This propagation mechanism leads to a persistent real effect (on relative prices, factor rewards, capital accumulation) of nominal shocks, which disappears gradually through money outflow (trade deficit). I also draw parallels with the NATREX approach of equilibrium real exchange rates and the literature on exchange rate based stabilizations.

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1 Introduction

This paper has a dual objective. One is to develop a two-sector model *without price or wage rigidities* in which various nominal shocks (nominal appreciation, fiscal expansion, the choice of the euro conversion rate) still have a medium-term impact on relative prices, factor rewards,

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investment and sectorial reallocation. For example, the model produces an endogenous gradual passthrough of a nominal appreciation into wages and nontradable prices, even with a full and immediate passthrough into tradable prices. The model also has a "real equilibrium path", which is essentially a two-sector neoclassical open economy growth model with asymmetric exogenous productivity growth in the two sectors.

Besides its theoretical aspects, the model seems to be capable of capturing actual price and wage dynamics after nominal appreciations and fiscal expansions, particularly the recent development of the Hungarian economy. The latter situation can be characterized by (1) a massive increase in wages (without a matching rise in TFP); (2) a halt in investment with a marked sectorial asymmetry: increase in service sector investments, fall in manufacturing; (3) slow or even reversed FDI flows; (4) export sector production costs (wages) not adjusting to the fall in revenues; (5) an increase in the nontraded-traded relative price; (6) an overall consumption boom, accompanied with a deteriorating trade balance. The policy environment can be summarized as (1) an increase in minimum wage legislation, (2) followed by a large nominal appreciation (monetary restriction), (3) followed by a massive fiscal expansion, partly in the form of public sector wage increases. The exact timing of the fiscal expansion is somewhat unclear: the rise in public sector wages unambiguously came after the monetary contraction, but the fiscal stance before and after the monetary developments is subject to heated political debates in Hungary.

The picture strongly suggests that the relative price of capital to labor $\left(\frac{r}{w}\right)$ has fallen. If we do not attribute this entirely to changes in minimum wages and public sector wages, then the monetary restriction ("revaluation") and the overall fiscal expansion should also play a role. The model successfully produces the same economic developments with the latter two policies, pointing to their potential role in the process.

A similar moral applies to any exchange-rate based disinflation attempt, and its reverse conclusions are relevant to price and wage developments after large devaluations. Rebelo and Végh (1995) find the following main stylized facts of exchange rate based stabilization programs: (1) high economic growth, (2) which is dominantly fueled by consumption, (3) slow price adjustment, (4) deteriorating trade balance. Burstein et al (2002) analyze large devaluation episodes, and find that inflation (price level) anomalies can be traced to the behavior of nontraded prices and wages.

The main mechanism of the model is the following. Consider an appreciation of the nominal exchange rate. It changes the spending behavior of consumers, through influencing their intertemporal (savings) decisions. In particular, domestic (nominal) assets are revalued in terms of tradable goods. If there is a positive link between consumption expenditures and asset values (money holdings), then consumption increases. The money-in-the-utility framework does imply such a link. This is one "stickiness" in the model. Increased spending must lead to increased production of nontradables, while excess demand in tradables can be satisfied through imports as well. This shift in production leads to an increase in the relative price of nontradables as long as the short-term transformation curve is nonlinear. This is the second and last friction of the model, which can be attributed to gradual capital adjustment (q-theory), for example.

The virtue of having only these two dynamic frictions is that one can clearly see the intuitive developments behind all results. It is also evident that both rigidities are necessary for the mechanism to work: without the nominal effect, we could not consider nominal shocks, while without the real friction, excess spending would not alter relative prices (under flexible capital and labor, the transformation curve is linear, and the relative price is fully determined by the supply side). This is why a new-keynesian model with *mobile factors, money-in-the utility and flexible prices* cannot lead to a real effect of money (nominal shocks).

As the economy moves along its transformation curve, factor rewards must also change: if the nontraded sector is more labor-intensive, than r falls and w increases (Stolper-Samuelson theorem). There is a marked reallocation between the two sectors: both labor and capital migrate from tradables into nontradables. A lower $\frac{r}{w}$ increases capital intensity in both sectors. The decline in r initiates a fall in aggregate capital (slump in investment and FDI). Notice that this is compatible with an increase in sectorial capital intensities, since the expanding nontraded sector is less capital intensive than the contracting traded sector. Rising wages create extra income for consumers ("Dutch disease"), which makes the real effect persistent in the medium-term (i.e., more persistent than the trade deficit adjustment process with *exogenous* income): excess spending slowly returns to equilibrium, through a gradual outflow of domestic money (assets).

The paper is organized as follows. The next section explains the basic building blocks of the model. Section 3 develops the full details of the two-sector growth model with money-in-the-utility, which is then adopted for numerical solution in Section 4. Section 5 describes the main results (nominal and real growth paths), which are discussed in Section 6. The final section concludes with some empirical considerations, and the Appendix contains some skipped details.

2 The basics of a gradual income and capital adjustment model

2.1 General considerations

I consider a dynamic adjustment of a two-sector small open economy model (the "dependent economy" model[1]). One of the sectors is traded, the other is nontraded. The two sectors differ in

[1] See, for example, Dornbusch: Open Economy Macroeconomics, chapter 6.

pricing: traded prices are set by the law of one price (fixed international prices times the nominal exchange rate), while nontraded prices are determined through domestic market clearing. In traded goods, domestic supply and demand can temporarily deviate from each other, leading to a trade deficit or surplus. One could further distinguish between exportable and importable goods.

There are two dynamic factors in the model. The first one is the intertemporal aspect of consumer behavior: the gradual adjustment of expenditures to income. This can be also viewed as some sort of a nominal rigidity (illusion), which ensures that nominal shocks (nominal exchange rate movements, fiscal policy) have a temporary effect on spending. Such a behavior can be perfectly consistent with consumer optimization: as we shall see, this can be rationalized by an explicit intertemporal maximization of a utility function containing real money balances as well.

The nominal effect does not come from the rigidity or stickiness of prices or wages, but from the gradual response of consumption expenditures. This does not imply that real-world prices or wages were flexible, or there were no inflation persistence – all is meant to show that there are systematic effects of nominal shocks on relative prices even under price flexibility.

The other dynamic effect is the accumulation of capital. Due to adjustment costs, this is a gradual process, like in a regular Tobin's q model. For simplicity, I assume that capital is owned 100% by foreigners (in other words: capital owners consume only tradables, their opportunity cost of funds is the fixed world interest rate, which then makes the nationality of capital owners irrelevant). It implies that changes in capital income will not affect domestic nontraded demand. In reality, there should be such an effect, but one would expect the labor income effect to dominate.

This is already sufficient to produce real effects of a nominal shock: under a nominal appreciation, for example, the value of domestic money holdings (wealth) in terms of tradable goods increases. This leads to more consumption of tradables and nontradables. Since country-level capital is fixed in the short-run, and nontraded consumption must equal production, this implies a change in relative prices between the two sectors, and also influences wages and the rental rate. This latter implies a change in the capital accumulation process, while the former has an impact on consumer income, which may reinforce or counteract the initial excess consumption. If wages increase (which happens if the nontraded sector is more labor intensive than the traded sector), then consumer income increases, propagating the real effect of the nominal shock. My objective is to quantify these dynamic mechanisms.

For analytical and numerical tractability, I need to specify the production and consumption side. I work with a Cobb-Douglas assumption on both sides, and I also adopt certain simplifications on the dynamic equations of the model (neglect some second order effects and linearize around the balanced growth path). These simplifications do not alter the behavior of the model: Benczúr and Kónya (2003) considers a continuous time, full optimization version of the model, with qualitatively similar results.

The formulation and numerical solution of the model offers many interesting and important applications. One is a quantification of the price level impact of fiscal policy: we shall see that a fiscal expansion generates extra spending, and prices do not adjust immediately, due to the nonlinearity of the short-term transformation curve. This modifies all equilibrium prices (traded-nontraded relative prices, wages, rental rates), and then gradually disappears through income dynamics (and money outflow). Due to forward-looking investment behavior, this process counteracts with capital accumulation in a complex way, leading to rich dynamic consequences of a fiscal expansion.

A second application concerns the quantitative consequences of a *particular monetary restriction (nominal appreciation)*, which in fact will have similar effects than a *fiscal expansion*. Interpreting the monetary restriction as a revaluation of a fixed exchange rate, traded prices will fall (assuming immediate, and potentially full passthrough of the nominal exchange rate to tradable prices). This increases the value of domestic money holdings in terms of tradables, leading to a similar consumption boom and dynamic implications as a fiscal expansion. In particular, wages and nontraded prices will show an endogenous and gradual adjustment to the decreased tradable price level, which frequently puzzles central bankers.

A related but inherently fixed exchange rate situation is the choice of the EMU conversion rate. The model issues the warning that an overvaluation may imply a significant reduction in capital inflows, it may be persistent even with flexible prices and wages, and it has largely asymmetric effects on different sectors and different factors of production. Welfare implications are not clear-cut, since GDP growth may slow down, but consumers experience higher wages and consumption (financed by debt).

A fourth application comes from a surprising similarity between the dynamic equations of the model and different equilibrium concepts of the NATREX approach.[2] In this sense, the model can be viewed as an (almost) explicit optimization-based version of a NATREX model. The modifier "almost" applies only because I will have to adopt certain simplifications (approximations) of the full optimization model to ensure tractability (these are eliminated in Benczúr and Kónya (2003)). More precisely, the long-term equilibrium NATREX concept matches the steady state (balanced growth path) of my model (when both capital and money holdings are at their balanced growth path, which corresponds to the "traditional flexible"

[2] This framework was developed by Jeremy Stein, in Stein (1994) and various additional papers.

Balassa-Samuelson framework); while the medium-term equilibrium concept corresponds to the nonmonetary version of my model (when the adjustment of money holdings is much faster than that of capital, thus the income and expenditure of consumers are always equal to each other, and money does not influence any real variables). In other words, flows (the trade balance) are in equilibrium *given* the current level of stock variables.

2.2 Behavioral equations

Production

Traded sector: $Y_T = (A_T L_T)^{\alpha_1} K_T^{1-\alpha_1}$; $A_T(t) = A_T(0)(1+g)^t$. As we shall see, it is necessary to transform all variables into effective variables (as standard in growth theory). This means a normalization by some power of productivity growth (unlike in a one sector model, these powers are not necessarily the same across variables).

Nontraded sector: $Y_{NT} = (A_{NT} L_{NT})^{\alpha_2} K_{NT}^{1-\alpha_2}$. Let us keep A_{NT} constant for simplicity ($A_{NT} = 1$). One could also incorporate growth in nontraded productivity, or temporary innovations in productivity growth into the model.

In both sectors, firms maximize profits under perfect competition. This defines (for given prices) their demand for capital and labor. I assume the indifference of both factors *between the two sectors*, so $w_T = w_{NT} = w$, $r_T = r_{NT}$. This does not automatically imply full international mobility of capital: as we shall see, domestic rental rates can temporarily deviate from the fixed international rate.

I would not argue that the labor mobility assumption is fully realistic, or that the adjustment of labor is fast enough (compared to the adjustment of capital and nominal spending) to validate such an approximation. One could also set up a model with slow labor adjustment. This would, however, excessively complicate the model, while the other two adjustments are vital to my analysis (for a real effect of nominal shocks, we need to have slow adjustment of nominal spending; and slow capital adjustment is necessary to analyze investment behavior).

My other crucial assumption is that capital is indifferent between the *two domestic sectors*, but not necessarily between home and foreign. If the initial difference in sectorial returns of capital is not "too large", their equalization is feasible entirely through new investment. It is possible that a too large shock necessitates disinvestment in one of the sectors. Then one needs to assume that capital is mobile between sectors up to this degree. A further alternative would be to consider two separate q-theories in the two sectors. Benczúr et al (2003) contains such an attempt.

Demand

For a given money stock $H(t)$ (money, or wealth), consumption expenditure is proportional to money holdings: $E(t) = VH(t)$, where V is the (fixed) velocity of money. Qualitatively, the results would stay similar if consumption depended on current income as well.

For a given E, traded and nontraded consumption is driven by a Cobb-Douglas utility function: $p_T C_T = (1-\lambda) VH$, $p_{NT} C_{NT} = \lambda VH$. I could also assume a different degree of substitutability between the two goods – Section 6 discusses how my results would change with different substitutability in production or preferences.

Such a consumption-money behavior can follow from a precise intertemporal maximization framework: if $u(t) = \int v(\tau) e^{-\delta(\tau-t)} d\tau$, $v(t) = E(t)^{\alpha} H(t)^{1-\alpha}$ (money-in-the-utility specification of Sidrauski), then it is true for this special ("power Cobb-Douglas") case that E/H is constant along the saddle path (Dornbusch-Mussa (1975)). One needs to adjust the utility specification for the two good case:

$$v(t) = \left(C_T(t)^{1-\beta} C_{NT}(t)^{\beta} \right)^{\alpha} (H(t)/P(t))^{1-\alpha}.$$

The expenditure share λ corresponds to $1-\beta$ here. The price level variable P is the domestic price index ($P = P_T^{1-\lambda} P_{NT}^{\lambda}$). With some algebra, one can reestablish the property that $E = VH$ along the saddle path (the key observation is that for a given E, the per period problem implies fixed expenditure shares, so one ends up with an intertemporal objective function expressed in terms of H and E again). The Appendix contains this derivation.

If there is inflation (P changes), then this constant velocity in fact depends on inflation: $V = (\delta + \pi) \frac{\gamma}{1-\gamma}$, where π is the CPI-inflation (the change of P), and δ is the discount factor. My full model is one extra step more complicated, since inflation is not constant in the short-run (approaches the balanced growth path value from above or below). Then it is no longer true that E/H is constant along the saddle path.

The full optimization of consumers would substantially complicate my current framework. Benczúr and Kónya (2003) considers the continuous time version of the same model, with full optimization both on consumer and investor side. For a better comparability with new-keynesian open economy models, the utility specification is switched into logarithmic Cobb-Douglas (separable in money and consumption). In that case, $E = VH$ no longer holds even with zero or constant inflation, but there is no qualitative difference in the results. The main intuition also carries through: nominal shocks influence intertemporal consumption decisions, which moves the economy along a nonlinear short-term transformation curve. The Appendix sketches the proof of this argument.

Prices

In the traded sector $p_T = e p_T^* = e$, while p_{NT} comes from goods market clearing. In other words, there is an immediate and full passthrough of the nominal exchange rate into tradable prices, but not necessarily into nontradables. Wages and the rental rate are also determined through factor market clearing.

It is well-documented that the passthrough of exchange rate movements into tradable prices is far from full and immediate. My focus, however, is on the adjustment of the economy to a change in tradable prices. For this reason, similarly to most of the open economy macro literature, I will work with a perfect passthrough into tradable prices.

This completes the description of the per period equilibrium: for a fixed $K(t)$ and $H(t)$, the above considerations determine the per period values of r, w, p_{NT}/p_T, K_T, L_T, K_{NT}, L_{NT}, C_T and C_{NT}. The original dependent economy model solves such a per period model (though with fixed sectorial capital stocks, giving two different quasi-rental rates). To complete the model, I need to write down the laws of motion.

Money (H) dynamics

$$\begin{aligned} H(t+1) &= H(t) + eY_T + p_{NT}Y_{NT} - r(t)K(t) - eC_T - p_{NT}C_{NT} + DH(t) \\ &= H(t) + e(Y_T - C_T) - r(t)K(t) + DH(t). \end{aligned} \tag{1}$$

This is purely an accumulation equation (identity): money stock in the next period is equal to initial money holding, plus GNP minus expenditure, plus a potential exogenous term. GNP is the sum of traded and nontraded production (GDP) minus capital rents (that belongs to foreigners). Since the nontraded sector is in equilibrium, the value of nontraded production must equal the value of nontraded consumption. Change in money holdings thus equals the excess production of tradables, minus capital rents, plus the exogenous term DH.

This equation implies that the only asset of consumers is money (earning no interest). Under fixed exchange rates, any change in money holdings on top of DH enters as a money inflow or outflow. Since foreign and domestic interest rates on money are identical (zero), foreigners are willing to hold any level of domestic money under fixed exchange rates (their demand is perfectly elastic).Therefore, equation (1) also postulates the equilibrium of international money markets.

The exogenous term will play a dual role. One is related to growth: if there is a permanent productivity growth $g > 0$, consumption must be growing and hence H grows as well. If we do not want this increase to come only through a permanent money inflow, then the government must generate a fixed growth rate of domestic money. In principle, one should also worry about the distribution method of this money, but that would definitely complicate things. To cut it short, I revert to the classical "helicopter drop" method, which gives the fresh money lumpsum to consumers. This interprets monetary policy as choosing the level of the fixed exchange rate, its additional role is to increase money supply at a constant rate g.

The other role is to allow for a fiscal expansion (income shock). Formally, this is just a pure transfer of money to households – its essence is to give households extra income. This means an additional increase of money, on top of the constant rate. It is important to note that a zero present value fiscal shock might not have an effect in the full optimizing version of the model (Ricardian equivalence). In the approximate model, however, even temporary excess income leads to extra spending, consequently, it has a real effect. In my view, such a behavior is rather realistic: either due to some myopic consumer behavior, or a theoretical reason for the lack of Ricardian equivalence. Simon and Várpalotai (2001) contains an interesting case for the latter situation.

Under flexible exchange rates, the nominal appreciation should come from an increase in interest rates. This appreciation implies the same stimulus on consumption, through the revaluation of money holdings in terms of traded prices. High interest rates, on the other hand, may also depress consumption. This means that the effects in the flexible exchange rate model are smaller than with fixed exchange rates (they might as well be reversed). If we assume that, in line with actual experience, the net effect on consumption is stimulating, then the results cannot be reversed, only weakened.

Capital (K) accumulation

One of the cornerstones of the "standard", "long-run" Balassa-Samuelson model (the one advocated by chapter 4 of the Obstfeld-Rogoff textbook) is the full mobility of capital. It implies that the rental rate at home equals the international rental rate. However, this implies a very fast and also mechanical capital accumulation and adjustment process. If we add the standard labor flexibility assumption ($w_T = w_{NT}$), the real exchange rate (traded-nontraded relative price) is fully supply-determined. The transformation curve is linear, and nominal variables (or preferences) have no effect on relative prices, only on quantities.

Let us slow down this capital adjustment process: it means allowing a temporary deviation of domestic rental rates from world rental rates. For this, I adopt the framework of Tobin's q. Capital inflow does not immediately eliminate excess returns because that would imply too large adjustment costs. Gradual investment behavior reflects the balance between excess returns and adjustment costs, current and future.

This intertemporal maximization leads to a standard saddle path solution: the state variable is the capital stock, and the jump variable is q, which measures the difference between the internal and external value of a unit of capital. If the internal value is higher, then the firm invests, if the external, then disinvests (I or I/K equals $f(q)$, where f is increasing, and $f(1) = 0$). The interpretation of q is the extra profit implied by a marginal unit of extra capital, evaluated along the future optimal path. This is determined by two factors: one is the future marginal product, and the other is future saving on adjustment costs. Around steady-state (near constant K), the latter is negligible (second order), so q is approximately the present value of future per period returns, discounted by the world interest rate (r^*).

I will employ this approximation to my open economy model: investment depends on q, where q is the present value of future equilibrium rental rates ($r(\tau)$, $\tau \in [\tau, \infty]$), discounted by r^*. I shift the no investment point from $q = 1$ to $q = 0$, which means that my q is the present value of the excess yield, and not the yield itself. Measuring r^* in traded goods (foreign currency) and $r(t)$ in local currency, the equations become

$$\begin{aligned} K_{t+1} &= K_t + f(q_t) \\ q_t &= \frac{q_{t+1}}{1+r^*} + \frac{r(t)/e - r^*}{1+r^*} = \frac{r(t)/e - r^*}{1+r^*} + \frac{r(t+1)/e - r^*}{(1+r^*)^2} + \ldots \end{aligned}$$

In principle, f should be determined by the functional form of adjustment costs, but since it does not show up anywhere else, I can choose f directly (obeying $f(0) = 0$, $f' > 0$). This formulation corresponds to the adjustment cost being a function of I itself. An alternative is that it depends on relative investment (I/K), when the investment equation becomes

$$K_{t+1} = K_t (1 + f(q_t)).$$

The final form of the investment equation will be one step different, being written in terms of effective capital. As explained in Section 3, I will assume that adjustment cost apply only to changes in effective capital stocks.

3 Model details

To pin down the per period equilibrium, we need to determine all prices and quantities given a fixed level of $K(t)$ and $H(t)$ (the two state variables). Throughout these calculations, I will often drop time indices, and reintroduce them only at the summary of the per period solution.

3.1 Per period equilibrium

The first order conditions of profit maximization are (using $A_{NT} = 1$, $p_T = ep_T^* = e$):

$$w = p_{NT}\alpha_2 K_{NT}^{1-\alpha_2} L_{NT}^{\alpha_2 - 1} = \alpha_2 p_{NT} k_{NT}^{1-\alpha_2} \tag{2}$$

$$w = p_T A_T^{\alpha_1} \alpha_1 K_T^{1-\alpha_1} L_T^{\alpha_1 - 1} = \alpha_1 e A_T^{\alpha_1} k_T^{1-\alpha_1} \tag{3}$$

$$r = (1-\alpha_2) p_{NT} k_{NT}^{-\alpha_2} \tag{4}$$

$$r = (1-\alpha_1) e A_T^{\alpha_1} k_T^{-\alpha_1}. \tag{5}$$

This specification assumes that all prices are expressed in home currency.

Since we have assumed a permanent trend in A_T, we need to interpret the long-term equilibrium situation appropriately: instead of a steady state, we will have a balanced growth path. Just like in a one-sector Ramsey model with growth, we need to introduce effective variables, i.e., divide all variables by an appropriate power of productivity growth. In a one sector model, it means the same first power for all variables. In our asymmetrical two sector model (A_{NT} is constant), this will mean a different power for some variables (p_{NT}).[3]

Starting with (5), it is immediate that the transformation to effective labor gives the steady state:

$$r = (1-\alpha_1)\, e A_T^{\alpha_1} \left(\frac{K_T}{L_T}\right)^{-\alpha_1} = (1-\alpha_1)\, e \left(\frac{K_T}{A_T L_T}\right)^{-\alpha_1} = (1-\alpha_1)\, e \hat{k}_T^{-\alpha_1}.$$

The variable $\hat{k}_T$ is the amount of capital per effective worker in the traded sector. Let us introduce $\hat{p}_{NT} = \frac{p_{NT}}{A_T^{\alpha_2}}$, and get a fully homogenous system for the effective variables:

$$\hat{w} = \alpha_2 \hat{p}_{NT} \hat{k}_{NT}^{1-\alpha_2} \tag{6}$$

$$\hat{w} = \alpha_1 e \hat{k}_T^{1-\alpha_1} \tag{7}$$

$$r = (1-\alpha_2)\, \hat{p}_{NT} \hat{k}_{NT}^{-\alpha_2} \tag{8}$$

$$r = (1-\alpha_1)\, e \hat{k}_T^{-\alpha_1}. \tag{9}$$

This is a system of four equations with five unknowns. If we fix r for example, then we get back the "flexible Balassa-Samuelson" result, where supply completely determines the relative price of nontradables, wages and capital-labor ratios. The role of demand is reduced to the determination of the size of the sectors.

In the model, however, r is endogenous, and it is the total capital stock, $K_T + K_{NT}$, that is fixed (within a period). We also know the behavior of demand, since nominal spending is proportional to the other state variable, H. This enables a straightforward determination of r: for a given r, (6)-(9) defines $p_{NT}(r)$. Using H, we get demand (expenditure) for traded and nontraded goods. Nontraded demand must equal supply, so we have the value of nontraded production. With $\hat{k}_{NT}(r)$, we then also obtain $L_{NT}(r)$ and $K_{NT}(r)$. Labor market clearing defines $L_T(r) = L - L_{NT}(r)$. Combining this result with $\hat{k}_T(r)$ gives $K_T(r)$ as well. The last step is to write the market clearing condition for capital: $K = K_T(r) + K_{NT}(r)$, which defines

[3] Under $\alpha_1 \neq \alpha_2$, even ${}_T = {}_{NT}$ implies such an asymmetry, since productivity growth is labor augmenting, and the two sectors use labor with different intensity. If we assume a common growth rate of the TFP of the two sectors, we get back to full symmetry.

the per period equilibrium value of r.

The details are presented in Benczúr (2003), the final outcome of the calculations is

$$\hat{K} = \hat{k}_T + \lambda V \frac{\hat{H}}{e} \hat{k}_T^{\alpha_1} \left(\frac{1-\alpha_2}{1-\alpha_1} - \frac{\alpha_2}{\alpha_1} \right). \tag{10}$$

3.2 Money accumulation

Money accumulation follows the individual per period budget constraint (equation (1)):

$$H_{t+1} = H_t + e\left(Y_T(t) - C_T(t)\right) - r_t K_t + DH(t). \tag{11}$$

Apart from the exogenous money growth term, this expression is closely related to the balance of payments: $e\left(Y_T - C_T\right)$ is the trade balance, while $-rK$ is investment income paid to foreigners. This determines the dynamics of financial wealth ("net foreign financial assets"). It is important to make the qualification "financial", because K will have its own accumulation equation, and a full balance of payments would reflect capital flows as well.

Transform (11) into effective variables:

$$\hat{H}_{t+1} = \frac{1}{1+g} \left(\hat{H}_t + e\left(L_T^{\alpha_1} \hat{K}_T^{1-\alpha_1} - (1-\lambda) V \frac{\hat{H}_t}{e} \right) - r_t \hat{K}_t + \widehat{DH}(t) \right).$$

Along the balanced growth path, effective traded and nontraded production is constant, so effective expenditure ($e\hat{H}$) is also constant. The per period budget constraint still implies that $eL_T^{\alpha_1} \hat{K}_T^{1-\alpha_1} - (1-\lambda) V \hat{H}_t - r_t \hat{K}_t$ is zero, so

$$\hat{H} = \frac{\hat{H}}{1+g} + \frac{\widehat{DH}(t)}{(1+g)}$$

along the balanced growth path. This implies $\widehat{DH}(t) = g\hat{H}$. In order for the monetary model to reproduce the balanced growth path of the corresponding real model, there must be an exogenous growth of money at the rate of $(1+g)$. Productivity growth implies increasing consumption and money holdings. Unless consumers get the extra money exogenously, they would have a trade surplus every period to ensure growing money balances. This cannot coincide with the real equilibrium, because that has balanced trade every period. To reproduce this situation, one must assume that the domestic government prints gH extra money every period.

In summary, we must have $\widehat{DH}(t) = g\hat{H}_t$, which then yields

$$\hat{H}_{t+1} = \hat{H}_t + \frac{e}{1+g} \left(L_T^{\alpha_1} \hat{K}_T^{1-\alpha_1} - (1-\lambda) V \frac{\hat{H}_t}{e} \right) - \frac{r_t \hat{K}_t}{1+g}.$$

Substituting our previous results for $L_T^{\alpha_1} \hat{K}_T^{1-\alpha_1}$ and $r_t \hat{K}_t$, and then making use of (10), we get

$$\hat{H}_{t+1} - \hat{H}_t = \frac{e}{1+g} \alpha_1 \hat{k}_T \left(\hat{K}_t \right)^{1-\alpha_1} - \frac{V \hat{H}_t}{1+g}. \tag{12}$$

3.3 Interpretation of medium- and long-term equilibrium concepts

Equation (11) offers a direct reinterpretation of the dual concept of equilibrium exchange rates (traded-nontraded relative price), which is frequent in many approaches. The system has two state variables, $\hat{H}$ and $\hat{K}$. In the long-term equilibrium situation, both variables have reached their steady state levels (with growth, it applies to their effective versions). There is a corresponding relative price $\hat{p}_{NT}$, which implies a long-term path (a growth trend and a fixed "level") for the relative price. The medium-term equilibrium allows for $\hat{K} \neq \hat{K}^*$, but requires effective money holdings to be constant at every moment (the economy is always on the $\frac{d}{dt}\hat{H} = 0$ locus). In other words, consumers satisfy their budget constraint in every period, so traded consumption also equals traded production less capital income. According to (11), this is equivalent to the balance of payments "without capital flows" (the flow variable corresponding to money), which is the condition defining the medium-term equilibrium value (the standard definition of the medium-term NATREX value):

$$0 = e \left(Y_T - C_T \right) - rK.$$

Consequently, the medium-term equilibrium restricts the use of money, wealth or any other means of intertemporal consumption reallocation. Putting differently, money adjustment is so fast that the economy is practically always along the $\frac{d}{dt}\hat{H} = 0$ curve, and it converges towards $\frac{d}{dt}\hat{K} = 0$ along this curve. In "reality", money adjustment is slower, so the economy may be out of its medium-term equilibrium. In every moment (i.e., for every K_t), one can still define the corresponding medium-term equilibrium value of the real exchange rate $p_{NT} \left(H \left(K_t \right), K_t \right)$. The right measure of real exchange rate misalignment is thus $p \left(H \left(K_t' \right), K_t' \right) - p \left(H_t', K_t' \right)$.

More generally, the long-term equilibrium concept is the balanced growth path of a model with many state variables. The medium-term equilibrium corresponds to such a transition path where some of the state variables adjust immediately, the corresponding flow variables are in equilibrium, and only a subset of the laws of motion drives the dynamics. Disequilibrium (realized behavior of the economy) is then described by the full model, where all state variables adjust slowly, though at a different speed.

Within my model, the medium-term value of $\hat{H}\left(\hat{K}_T\right)$ can be obtained from (12):

$$\frac{\hat{H}}{e} = \frac{\alpha_1}{V}\hat{k}_T\left(\hat{K}_t\right)^{1-\alpha_1}.$$

Let us plug this into expression (10) for $\hat{k}_T$:

$$\hat{K}_t = \hat{k}_T\left(1 + \lambda\alpha_1\left(\frac{1-\alpha_2}{1-\alpha_1} - \frac{\alpha_2}{\alpha_1}\right)\right).$$

This reassures that the medium-term equilibrium (where H was eliminated) is independent from nominal variables (e). Moreover, we can see that $\hat{k}_T$ deviates from its steady state value in proportion to the deviation of $\hat{K}_t$. The convergence process is accompanied by a structural transformation. We also see that the rate of technology growth does not influence the equilibrium convergence process. As a consequence, the relative price movement implied by capital accumulation can be simply added to the balanced growth path trend behavior.

3.4 Capital accumulation

Now we can turn to capital accumulation:

$$\begin{aligned} K_{t+1} &= K_t + f(q_t) \\ q_t &= \frac{q_{t+1}}{1+r^*} + \frac{r_t/e - r^*}{1+r^*}. \end{aligned}$$

The essence of these expressions is that investment responds not only to current excess yields, but also to discounted future excess earnings. The investment function is increasing in q, and by our previous normalization, $f(0) = 0$. The simplest function satisfying these conditions is linear ($f(q) = cq$), which suffices for my purposes.

This formulation of q-theory is not compatible with balanced growth: we would like to see q as zero and $\hat{K}$ as constant. One way to ensure this property is to assume that the installation of new capital is also subject to the same productivity growth as the traded sector. Alternatively, if the adjustment cost depends on I/K, then q converges to $g = f(q)$.

This still implies that the balanced growth path of the gradual investment model is different from that of the costless investment model. To eliminate this feature, one can assume that the "natural growth" of capital (at a rate of $1+g$) is costless. In other words, I write the same

q-theory formalism in terms of effective capital. The full dynamic system then becomes

$$\hat{H}_{t+1} - \hat{H}_t = \frac{e}{1+g}\alpha_1 \hat{k}_T \left(\hat{K}_t\right)^{1-\alpha_1} - \frac{V\hat{H}_t}{1+g}$$
$$\hat{K}_{t+1} = \hat{K}_t + cq_t$$
$$q_t = \frac{q_{t+1}}{1+r^*} + \frac{r_t/e - r^*}{1+r^*}$$
$$\text{transversality condition}: \; q_\infty = 0.$$

One can eliminate q from the system by incorporating a forward-looking term in the capital accumulation equation:

$$\hat{K}_{t+1} = \frac{\hat{K}_{t+2}}{2+r^*} + \frac{\hat{K}_t\left(1+r^*\right)}{2+r^*} + \frac{c}{2+r^*}\left(r_t\left(\hat{K}_t\right)/e - r^*\right) \tag{13}$$
$$\hat{H}_{t+1} - \hat{H}_t = \frac{e}{1+g}\alpha_1 \hat{k}_T \left(\hat{K}_t\right)^{1-\alpha_1} - \frac{V\hat{H}_t}{1+g}.$$

This is already a dynamic system without an explicit jump variable. It is three dimensional, but two initial conditions and asymptotic boundedness is sufficient for a unique solution (one of the eigenvalues is divergent, so a general stable solution is the linear combination of two eigenvectors).

4 Details of the numerical solution

4.1 The minimized dynamic system

After simplifying the per period equilibrium conditions, we are left with the following system, solvable by the software Winsolve (all hat variables correspond to effective variables, adjusted for TFP growth):

$$\hat{K}_t = \hat{k}_T\left(t\right) + \lambda V \frac{\hat{H}}{e} \hat{k}_T\left(t\right)^{\alpha_1}\left(\frac{1-\alpha_2}{1-\alpha_1} - \frac{\alpha_2}{\alpha_1}\right) \tag{14}$$
$$r_t = \left(1-\alpha_1\right) e \hat{k}_T\left(t\right)^{-\alpha_1}$$
$$\hat{K}_{t+1} = \frac{\hat{K}_{t+2}}{2+r^*} + \frac{\hat{K}_t\left(1+r^*\right)}{2+r^*} + \frac{c}{2+r^*}\left(r_t\left(\hat{K}_t\right)/e - r^*\right)$$
$$\hat{H}_{t+1} - \hat{H}_t = \frac{e}{1+g}\alpha_1 \hat{k}_T \left(\hat{K}_t\right)^{1-\alpha_1} - \frac{V\hat{H}_t}{1+g}.$$

For numerical tractability, one needs to linearize the dynamic equations. This means that one approximates the deviation of r_t and $\hat{k}_T\left(t\right)$ from their equilibrium values by the appropriate linear combination of the deviations of $\hat{H}$ and $\hat{K}$. Within-period equations can remain in their original, nonlinear form. The Appendix gives the details of the linearization process.

For given initial values of $\hat{H}$ and $\hat{K}$, we look at the implied paths of $\hat{p}_{NT}$, $\hat{K}$ etc. The model converges to the stationary point (it is a point in terms of effective variables, and a path in normal variables), which also corresponds to the long-term equilibrium relative price of tradables and nontradables. Along the transition path, however, it does not strictly follow the medium-term equilibrium path. This medium-term equilibrium path can be obtained by replacing the law of motion of $\hat{H}$ with

$$\frac{\hat{H}_t}{e} = \frac{\alpha_1}{V}\hat{k}_T\left(\hat{K}_t\right)^{1-\alpha_1}. \tag{15}$$

One can then check the deviations from the medium-term equilibrium path.

Notice that the medium-term equilibrium path (the real economy) does not depend on g, the rate of technology growth. It means that *the equilibrium real exchange rate can be decomposed into a TFP term ("standard Balassa-Samuelson" effect) and a capital accumulation term, and there is no interaction between the two.* This does not apply to the non-equilibrium path.

The basic shock to be considered is a nominal appreciation, but an exogenous increase in money (fiscal expansion, income shock) would lead to very similar results. One can see that the medium-term equilibrium path is unaffected, but the actual realization changes.

A nominal appreciation has an inflationary side effect: traded prices are down, but the resulting consumption boom pushes the nontraded-traded relative price up. This dies out only gradually, as a trade deficit restores the equilibrium level of $\hat{H}$ (measured in traded goods). An income shock has similar implications, but *without* the fall in tradable prices.

4.2 Calibration

This is just an illustrative calibration, trying to show that under reasonable parameter constellations, the model produces quantitatively relevant results. For actual policy simulations or implications, one would have to back up some of the parameters from actual data (elasticities, expenditure shares, capital and labor shares, etc.), or estimate approximate static or dynamic equations of the model. One likely pitfall of such an approach is the apparent nonhomothetic behavior of traded-nontraded relative prices and consumption, often found in many economies, including Hungary. One cannot simply incorporate nonhomothetic preferences into growth models, since they do not lead to well-defined steady states or balanced growth paths. Nonetheless, such a consumer behavior may have important implications for the evolution of relative prices, which is worth exploring in the future.

The required parameters are $\alpha_1, \alpha_2, \lambda, r^*, g, c, V$; and there are two initial conditions – H_0, K_0. One period is chosen to be approximately one *tenth* of a calendar year. It is not necessarily true that such a short period would be enough for all price adjustments to be over. One can recalibrate the model in a way that one period corresponds to one quarter. I have also run some simulations for this latter scenario, without any significant changes in the results.

- $\alpha_2 = 0.8$ – labor intensity of the nontraded sector.
- $\alpha_1 = 0.5$ – labor intensity of the traded sector. All this starting assumption does is to assume that $\alpha_2 > \alpha_1$, which is a standard choice, though it might not hold in certain countries (including Hungary). Another hint is the share of capital from GDP. With the current choices, it is 37.5%.
- $\lambda = 2/3$ NT expenditure share; this is not an unreasonable assumption, particularly if we take into account that traded prices also have large service components
- $r^* = 0.005$ – required real rate of return on capital (assuming that one year is ten periods, then it means 5% annually). With one period being one quarter, this value is 0.0125.
- g – I choose $g = 0.001$ (0.0025), which implies a growth of percentage point per year.
- c, V – one can choose one of them "freely" (say, c), by matching an a priori speed of adjustment. Based on this, the choice of $c = 3000$ is consistent with one year being 10 periods: the half-life of an innovation to the capital stock *in the equilibrium (real) model* is 20 periods (2 years). Then I select V in such a way that the speed of nominal adjustment be sufficiently faster than that of real adjustment. This led to $V = 0.1$ (a shock to H has a half life of around one year, *in the model with exogenous income*).
- $H_0/H^*, K_0/K^*$. The latter measures the ratio of current to steady state per capita capital stock. For illustration, I chose its value to be 100 and 90 percent. In various runs, I have chosen H_0 to be 100, 90 and 110 percent of H^*. Another possibility is to reproduce the initial trade deficit.

5 Results

5.1 The behavior of the real exchange rate during convergence

5.1.1 The medium-term equilibrium path

As discussed earlier, we get profiles for effective variables, which are to be added one in one to the growth part. For the non-monetary ("flexible exchange rate") version, this is independent from the speed of growth; but the same does not apply to the monetary version. Besides, there is no choice of the fixed exchange rate that is compatible with converging along the medium-term

NATREX path. We will see small deviations, which are partly related to the approximation that V is constant (and the negligence of the $\dot{V}$ term), and partly inherent to the nominal economy.[4]

Convergence implies an appreciating real exchange rate, if the nontraded sector is more labor-intensive. If convergence involves both TFP growth and capital accumulation, then what the model produces is the excess real appreciation relative to the standard Balassa-Samuelson situation. If labor intensities are equal across sectors, then capital accumulation has no impact on the equilibrium real appreciation, while if the nontraded sector is less labor-intensive, real appreciation should be smaller than the standard Balassa-Samuelson.

All these are fully consistent with international trade theory: as long as capital is scarce, it has a high factor price. In the flexible Balassa-Samuelson model, an increase in world interest rates increases the relative price of that sector which uses capital more intensively (inverse Stolper-Samuelson theorem). For high rental rates, if the nontraded sector is more labor-intensive, then the NT relative price starts from a low relative price, thus must increase. It means a positive but vanishing excess inflation (real appreciation) relative to the standard Balassa-Samuelson case.

The two figures (1 and 2) show the evolution of the nontraded relative price and price change (per period, so annual measures are tenfold larger). The steeper path (Run3) corresponds to $K_0 = 0.8K^*$, while the choice for the other is $K_0 = 0.9K^*$. We can see that there is a large difference between the two, but it disappears as capital approaches K^*. Under slower capital accumulation, the same cumulative difference (the relative price is determined by K/K^*, so both the initial and the terminal price level is independent from the speed of adjustment) is distributed along a longer time period, so it is more persistent, but also smaller.

5.1.2 Disequilibrium (nominal) paths

I show the results of two different scenarios, and compare them to the medium-term equilibrium convergence path. In both cases, I choose $K_0 = 0.9K^*$. The initial value of H is set such that $e = 1$ yields $p_{NT}(1) = p_{NT}^{eq}(1)$ In other words, if $e = 1$ (the first nominal scenario), then the nominal economy is initially on the (real) equilibrium convergence path. Numerically, it means that $H_0 = 0.949H^{st}$. In scenario 2, we have the same H_0, but the nominal exchange rate is 10% stronger ($e = 0.9$). The results are displayed on figures (3-7).

[4] The real economy moves along the $\frac{d}{dt}\hat{H}\left(\hat{K}\right) = 0$ curve. As $\hat{K}$ grows, this leads to an increase in $\hat{H}$ as well. The nominal economy cannot satisfy $\frac{d}{dt}\hat{H}\left(\hat{K}\right) = 0$ and still produce an increase in effective money holdings, unless there is an extra exogenous increase in $\hat{H}$.

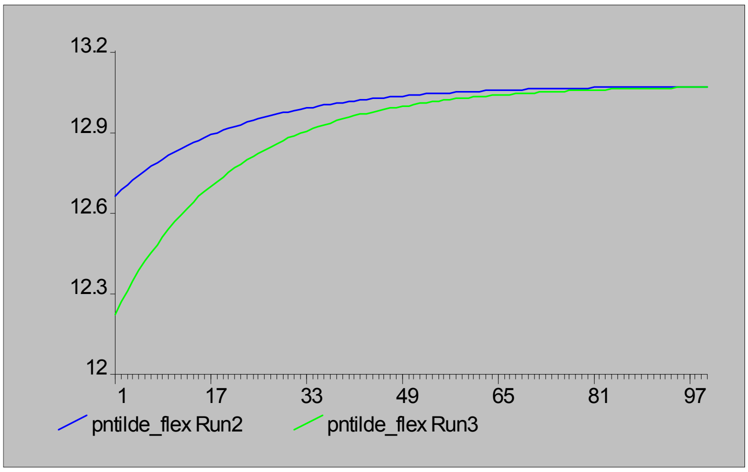

Figure 1: Convergence paths – the NT-T relative price

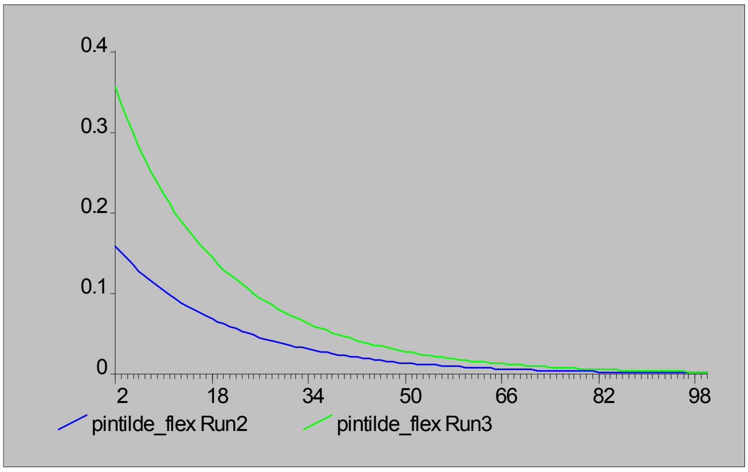

Figure 2: Convergence paths – excess nontraded inflation

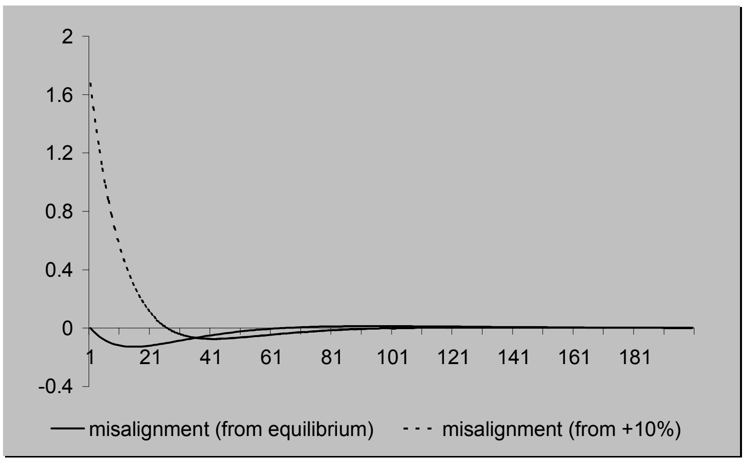

Figure 3: Percent deviation of the relative price from equilibrium

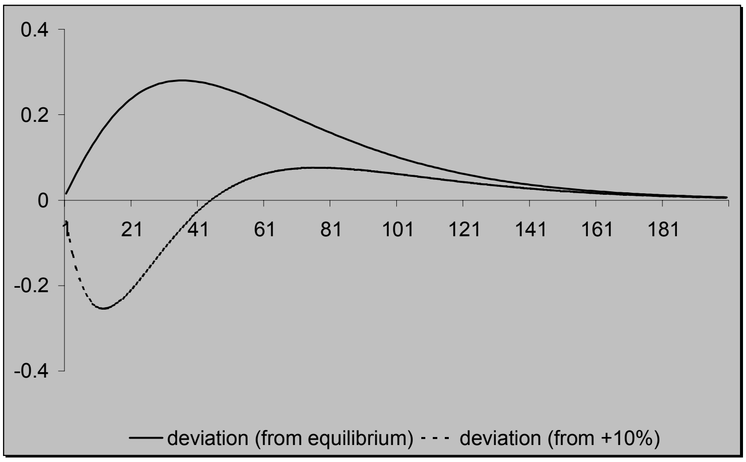

Figure 4: Percent deviation of the capital stock from equilibrium

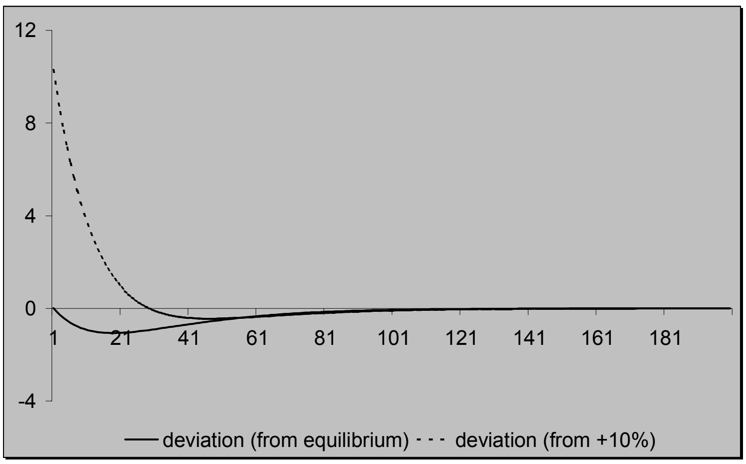

Figure 5: Percent deviation of the money stock from equilibrium

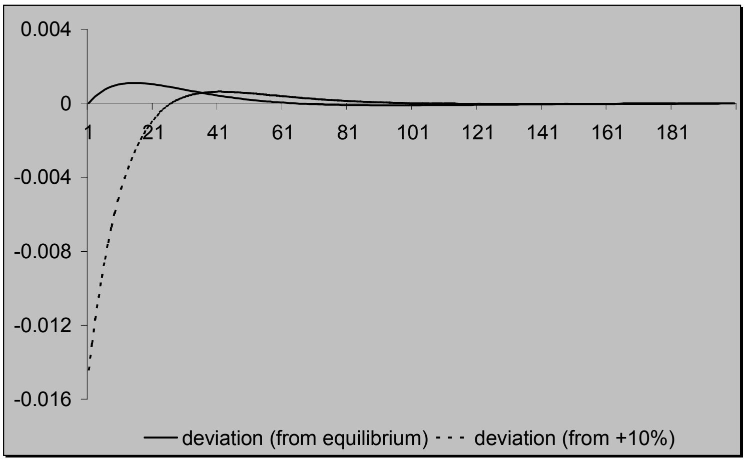

Figure 6: Percent deviation of the rental rate from equilibrium

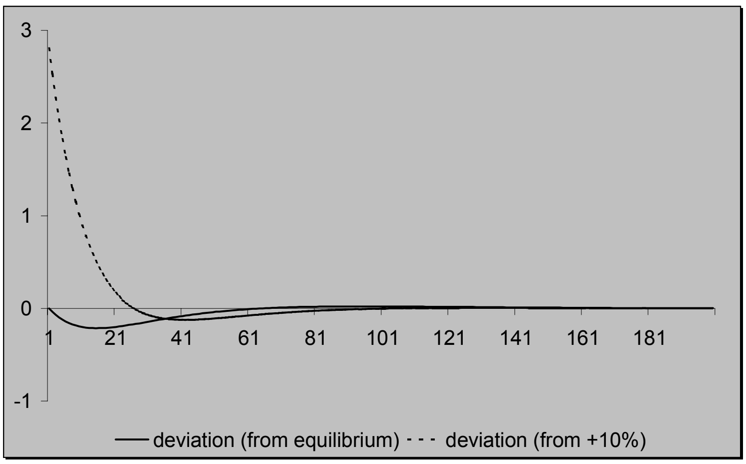

Figure 7: Percent deviation of wages from equilibrium

We can see that the nominal path starting from the real equilibrium departs from the medium-term equilibrium path, but the deviation is minor. The largest difference corresponds to capital, which depends on the discounted sum of future returns, so all deviations are added up in some sense. Overall, the exchange rate (relative price of nontradables) is undervalued during convergence. This is also shown by the difference between actual money stocks and the equilibrium values calculated according to equation (15): the disequilibrium path involves smaller money stocks, thus lower consumption. That benefits the price and accumulation of capital, while keeps wages low. Undervaluation has no systematic cause: the interaction between the two dynamic effects and the precision of my approximations jointly determine the deviation between the real and nominal paths.

A stronger initial exchange rate (by 10%) leads to overvaluation in the first 10 quarters of convergence (its impact is 170 basis point on the relative price), which then turns into an undervaluation. This undervaluation implies a boost to capital accumulation in two years. We can see a similar dual effect on most variables. In the next subsection, I will compare two such nominal paths in more detail (though with $H_0 = H^{st}$). Section 6 offers some interpretations, and relates all the results to international trade theory.

5.2 A shock to the nominal exchange rate

5.2.1 Base values

We start at the steady state capital and money stock, but there is a 10% revaluation at the beginning. This leads to a 170 basis points increase of nontraded-traded relative prices, and the effect disappears gradually in near 3 years. The stock of capital is reduced by 35 basis points for this time period (in fact, even longer), and there is an entire year with a loss of 45 basis points. A back of the envelope calculation:

$$
\begin{aligned}
rK &= 0.375 * GDP \\
K &= \frac{0.375}{0.05} * GDP = 7.5 * GDP \\
0.0035K &= 0.0265 * GDP,
\end{aligned}
$$

so the loss of capital is equivalent to 265 basis points of GDP. In the worst year, it is 337 basis points. Actual estimates of Hungary's net capital stock to GDP (Pula (2003), not necessarily consistent with a fixed share of capital income and a rental rate of 5%) are around 1.2-2.9, so the 35 basis points reduction in capital stocks is equivalent to 40-100 basis points of GDP.

The impact on the two sectors is much stronger: the nontraded capital stock increases by 15%, while its traded counterpart falls by 5%. The corresponding numbers of employment is

+7.5% and -10%, and 6% for capital-labor ratios in both sectors. The price of capital (measured in "euros") falls by 3%, while wages (also in euros) increase by 3%.

The volume of consumption increases in both sectors, which implies an equal increase in nontraded production. Traded production, on the other hand, falls, so part of the excess tradable consumption is imported, showing up in a trade deficit and a money outflow. This reflects partly increased spending, and partly decreased production of tradables.

One can also calculate the balance of GDP in fixed euro prices: we have the volumes of both sectors, and aggregate them using $p_T^* = 1$, $p_{NT}^* = p_{NT}^{st.st.}$ (which corresponds to steady state prices). This shows an initial loss of 5 basis points, which then accelerates, and the cumulative sum is 929 basis points. This is measured in per period GDP, so in terms of annual GDP, the loss is 93 basis points. One cannot really obtain a sacrifice ratio from this, since there was no disinflation, but just a correction of the price level.

We can see that the model succeeds in matching the recent Hungarian experience (as detailed in the introduction): (1) a massive increase in effective wages; (2) a halt in aggregate investment, with an increase in service sector investments, and a fall in manufacturing; (3) reversed FDI flows (in the model, it is equivalent to an investment bust); (4) slow wage adjustment; (5) an increase in the nontraded-traded relative price; (6) an overall consumption boom, accompanied with a deteriorating trade balance. Though the Forint's appreciation was not literally a revaluation within a fixed exchange rate regime, but rather the outcome of a monetary restriction, the size and persistence of the appreciation (near 10%, for around 1.5 years) makes the fixed exchange rate results readily applicable to the situation. We also see the consumption boom, persistent trade deficit, and slow price adjustment feature of Rebelo and Végh (1995), and the increase in wages and nontraded prices as suggested by Burnstei et al (2002). Detailed results are displayed on Figures 8 – 16: each figure shows the percentage deviation from baseline, with the exception of the trade balance, which is the percentage point deviation from baseline.

5.2.2 Alternative scenarios

I have experimented with the adjustment speed of capital and nominal spending, and also the initial conditions for capital and money. The results have not changed in any substantial way. Benczúr (2003) gives some extra details on these scenarios.

5.3 Government spending

This has similar implications than a nominal appreciation, though its effect on the CPI is different, since there is no decline in traded prices. Qualitatively, we would get the same answers as with a revaluation. I have not run any specific scenarios.

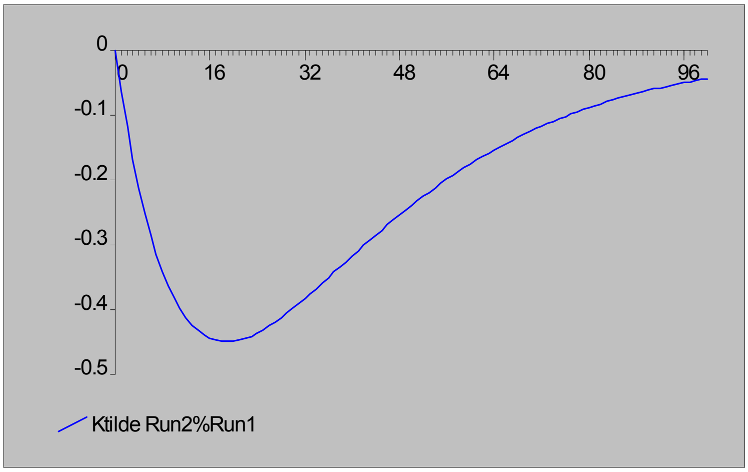

Figure 8: Shock response: capital

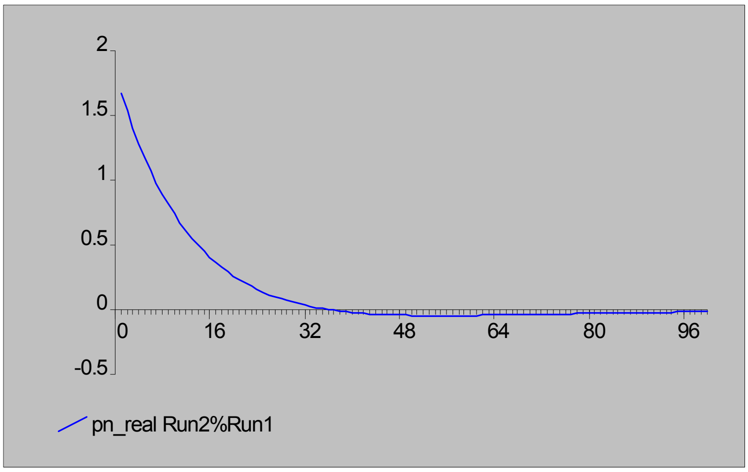

Figure 9: Shock response: NT-T relative prices

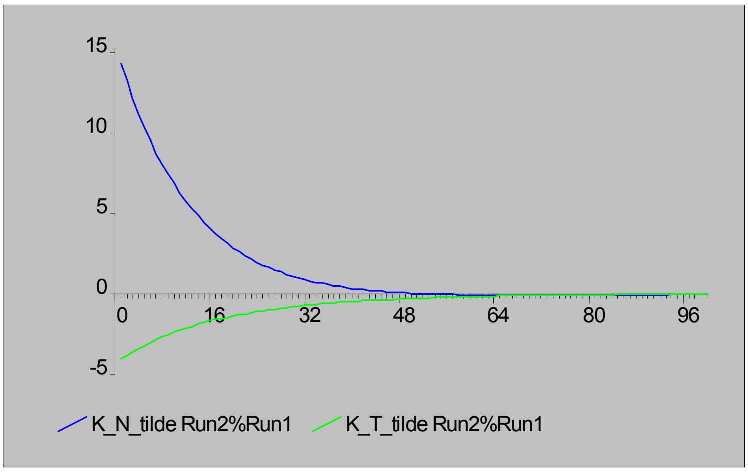

Figure 10: Shock response: sectorial capital employment

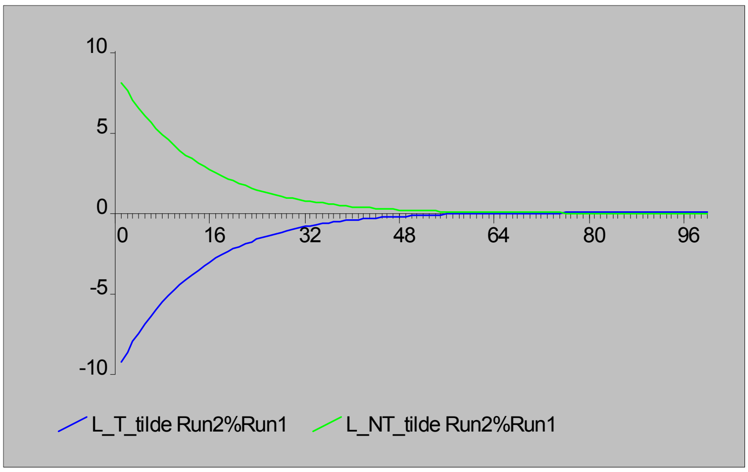

Figure 11: Shock response: sectorial labor employment

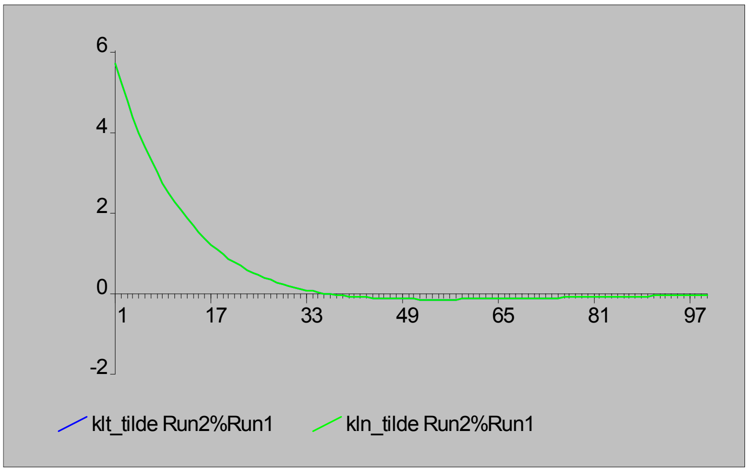

Figure 12: Shock response: sectorial capital-labor ratios (identical)

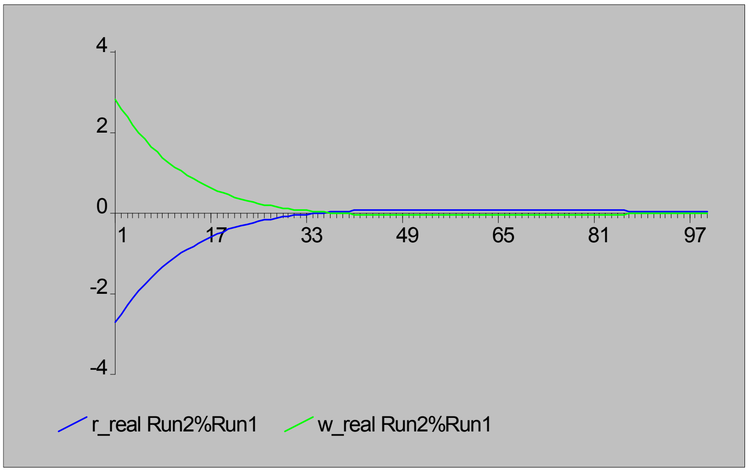

Figure 13: Shock response: factor prices in euros

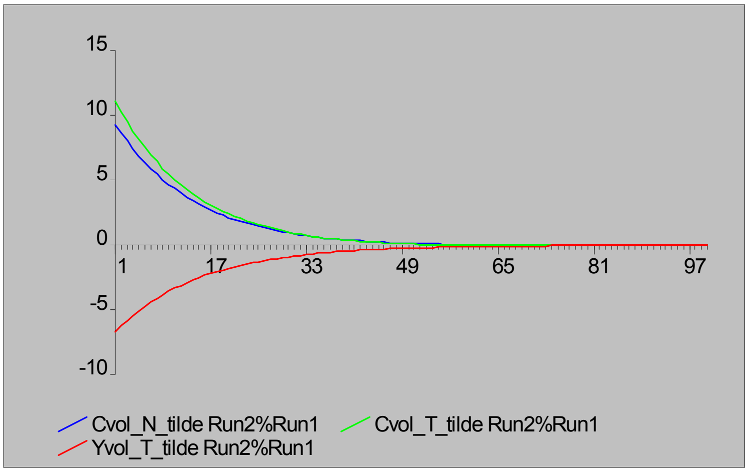

Figure 14: Shock response: volume of T and NT production and consumption

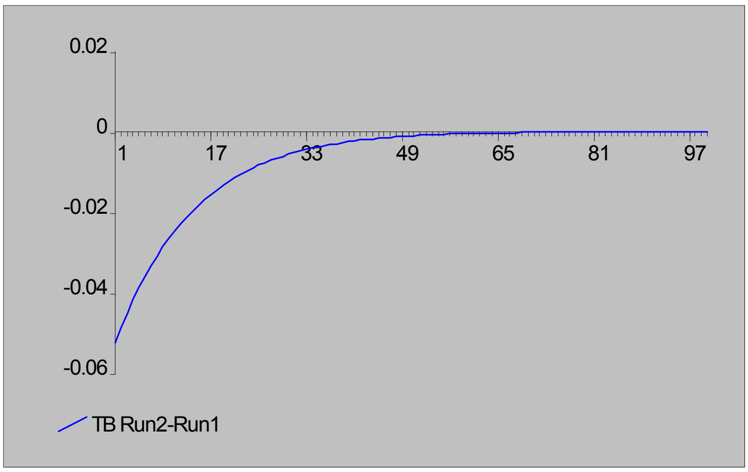

Figure 15: Shock response: trade deficit relative to fixed-price GDP

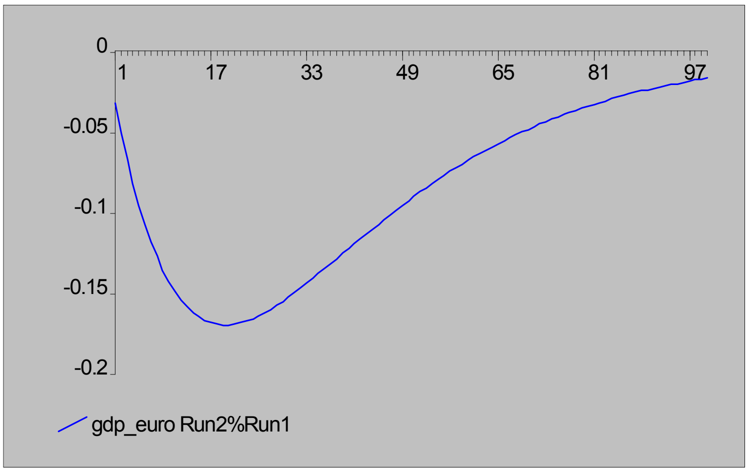

Figure 16: Shock response: evolution of fixed price GDP

6 Discussion

The signs of the previous results are easy to interpret intuitively. Both a fiscal expansion and a revaluation increases the value of H in terms of traded goods. This leads to an increase in spending on nontraded goods, which increases nontraded production. Given the short-run nonlinearity of the transformation curve, nontraded prices must increase. This is the dominant shock to the economy, all of the other results can be traced to this through the Stolper-Samuelson theorem: if the price of a sector increases, it leads to a more than proportional increase in the price of the factor which is used more intensively by the windfall sector. The price of the other factor of production decreases.

In our case, the price of the nontraded sector has increased, and it is more labor-intensive. This leads to a rise in wages and a fall in rental rates. Production becomes more capital-intensive, and the fall in rental rates decreases capital inflows (q falls).

What makes this situation persistent? The explanation is closely related to the phenomenon of "Dutch disease": a country receiving a transfer also sees its relative wages (terms of trade) improving. The extra consumption enabled by the transfer falls partly on nontradables, which pushes domestic wages up. In our two factor model, we need some extra conditions for the transfer effect: if the only source of income of domestic consumers is their labor earnings, then the nontraded sector must be more labor intensive than the traded sector. The price of capital falls, but that does not influence domestic spending.

This is the underlying propagation mechanism: the initial shock to consumption increases domestic income, so the excess money stock will flow out only slowly. If some of the capital is domestic, and its income is used for consumption expenditures, then excess spending still creates some of its excess income, but to a smaller degree. In this case, we can get persistence even without the labor intensity assumption.

One can give a similar interpretation to the nominal convergence path starting from the medium-term (real) equilibrium position: since the money accumulation process governed by consumer optimization is not the same as the medium-term equilibrium path, period one money stocks differ. This changes all prices in equilibrium, through the "demand effect". For a smaller than equilibrium H, wages also become smaller, which then reinforces the initial undervaluation and makes it persistent. Undervaluation thus even increases initially. This is balanced by the effect on capital accumulation: due to a higher rental rate, there is more capital inflow, which leads to an increase of wages eventually. Undervaluation starts to disappear. With my choice of parameters (and approximations), the system converges to the equilibrium path (medium- and long-term) through undervaluation. In general, it is possible to have a shift to overvaluation during this process.

It is clear that the parameter V plays an important role in determining the speed of adjustment through the trade balance: excess spending is proportional to VH, so a small V leads to a slow outflow of the extra money. Looking one step behind, V is $(\delta+\pi)\frac{\gamma}{1-\gamma}$, the sum of the discount factor and inflation times the substitutability between consumption and money holdings. Consequently, V represents the degree of intertemporal consumption smoothing – how fast consumers deplete their excess money stocks. Another important determinant of persistence is the weight of nontradables in consumption expenditures, since the larger it is, the more valid the Keynesian thesis that "excess demand creates its supply".

It is important to note that the sectorial labor intensity (or the sectorial factor mobility) assumption is not relevant for the increase of the price of nontradables. Its role is to make the price of capital fall and wages increase (through the Stolper-Samuelson theorem). The "wealth effect" of a revaluation hurts or benefits capital (investment), depending on relative factor intensities. I have explored a scenario with the traded sector being more labor intensive. Nontraded prices increased, wages fell, the rental rate increased, and capital accumulation accelerated.

If we do not attribute the current Hungarian experience entirely to changes in minimum wages and public sector wages, then the monetary restriction ("revaluation") and the overall fiscal expansion offers an explanation. Note that this requires the sectorial factor intensity assumption, which seems to be somewhat problematic with Hungarian data. The observed behavior of the economy, however, does support this assumption indirectly.

The degree of substitutability between the two goods (by consumers) and the factors of production (by producers) also influences the quantitative behavior of the economy. Starting with the preference side, let us assume that consumption utility is

$$\left((1-\lambda)^{\frac{1}{\theta}}C_T^{\frac{\theta-1}{\theta}}+\lambda^{\frac{1}{\theta}}C_{NT}^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta}{\theta-1}}.$$

The choice of $\theta=1$ corresponds to my Cobb-Douglas specification. Suppose that $\theta>1$. An increase in p then implies a larger substitution towards traded goods, so an increase in consumption expenditure must lead to a smaller increase in C_{NT} and p. Keeping the same transformation curve between traded and nontraded goods, a smaller price increase leads to a smaller wage increase and a smaller decrease of the rental rate. This muted impact effect also weakens the endogenous persistence of the shock, since a smaller wage increase leads to a faster outflow of excess money. In summary, a higher degree of substitutability between traded and nontraded goods increases both the impact effect and the persistence of nominal shocks on the real economy. Conversely, $\theta<1$ increases both the impact effect and the persistence. One can even find parameters such that a nominal appreciation initially *improves* the trade balance (wages increase more than one in one relative to the nominal exchange rate). Later on, the corresponding decline in r and K leads to a fall in w, and excess money flows out in the long-run.

Intuitively, one would expect the opposite impact of substitutability between factors of production: if it is easy to substitute labor with capital, then the same price increase leads to a smaller wage increase. Consequently, the same increase in nontraded expenditure (pC_{NT}) leads to a smaller increase in p and w, thus a smaller impact effect and smaller persistence. The combination of nonunit substitutability both in preferences and technology would produce very complicated general equilibrium cross-effects.

One comment is in line here, about the large sectorial reallocations showed by the results. These are the consequences of the assumption that only the cross-border adjustment of capital is slow. The price of domestic labor and installed domestic capital is equalized between the two sectors, so there is free sectorial mobility.

In reality, all of these adjustments should be gradual, leading to sectorial wage and rental rate differences. This is likely to increase the impact effect of the nominal shock (nontraded prices must increase even more, since near fixed capital and labor imply a larger increase in the cost of production), but its persistence should decrease – by the time labor starts to switch sectors, the price and wage differentials have been nearly eliminated. My results still issue a warning about the direction of asymmetries between sectors and factors, and indicate the direction of reallocation.

One can reinterpret the "money effect" as a "wealth effect", or even as a portfolio resizing and rebalancing effect. The common feature is that a nominal shock will influence the value or the returns of nominal wealth/assets, leading to a change in the intertemporal behavior of consumers. Such a change will translate into a change in consumption expenditures, which is the necessary starting point of my model.

In the wealth effect interpretation, it is implausible to assume that expenditures are determined entirely by wealth. In terms of a consumption function

$$E = \mu V H + (1 - \mu) Y,$$

I have assumed so far that μ is one (here H is total wealth, and not just money, though it does not yet earn any returns). This implies that V controls both the steady state wealth-income ratio and the persistence of the nominal shock response. The model readily extends to $\mu \neq 1$, but I have not solved any specific scenarios.

7 Some concluding empirical considerations

There is vast literature on exchange rate based stabilizations (for example, Rebelo and Végh (1995)), their stylized facts, the sources of success or failure. Darvas (2003) identifies two main groups of countries who experienced large (nominal and real) appreciations at the start of disinflations: fixed exchange rate, mostly Latin American countries with a failure in their stabilization programs, and floating countries, mostly industrial, with a success. The question is then whether these differences in country experiences can be related to differences in the strength of the mechanisms of my model. Naturally, there are many additional issues and considerations determining the overall success or failure (credibility, fiscal side, just to name a few), but this "transfer effect" might also be an important contributor.

According to my results, a nominal appreciation has a negative side effect, through nontraded prices: even if there is immediate and full passthrough of the nominal appreciation into tradable prices, the "equilibrium" (flexible price, but gradual adjustment of money balances/wealth) relative price of nontradables increases. Under certain factor intensity assumptions (the nontraded sector is more labor intensive than the traded sector), it is also true that wages increase (relative to tradable prices, which is the same as relative to wages in the rest of the world). Assume that these *qualitative* effects are equally present in successful and failed disinflation episodes.

What can determine the difference in success? If nontraded inflation or wage inflation remains relatively high, then expected inflation may adjust only little. If we manage to introduce inflation persistence (expectations) into the model, then a slow adjustment of wages and nontraded prices should increase inflation persistence (expectations), thus making the success of the disinflation less probable. This interpretation would trace the difference in success or failure into differences in the impact and persistence of the wealth (money) effect of an appreciation.

These differences can come from the role of money/wealth directly (like parameters V and μ), or from different production functions, factor intensities, preferences, intertemporal and intratemporal substitutability of goods. The distance from steady state (industrial country or emerging market), or the role of capital income within GNP may also matter. Thinking in terms of an external asset position, it may also matter whether the country is a net lender or borrower, since a revaluation decreases the value of external assets, so it decreases the value of outstanding foreign debt. Besides this direct wealth effect, there can be a portfolio rebalancing between foreign and domestic assets as well.

These are all differences which can be measured for both groups of countries. For most of them, one would expect that the failure countries are subject to larger wealth (nominal) effects. It is central to have data on wages and nontraded-traded relative prices. Burstein et al (2002) document a systematic behavior of prices after large *devaluations*: once one accounts for various conceptual and measurement issues in tradable prices, the only remaining mystery is the surprisingly low increase in nontradable prices. That effect can also be related to the negative wealth or money effect of a devaluation, cutting domestic spending, thus decreasing the relative price of nontradables.

Apart from data availability issues, a major problem of implementing this empirical cross-country comparison is that success and failure countries also differed in their fiscal policies. Overall, successful countries did not tend to "match" the appreciation with a fiscal expansion, which is much less true for failure countries. As argued earlier, a fiscal expansion has qualitatively similar implications for relative prices and factor rewards as an appreciation. In order to relate success or failure to a different money or wealth effect, one needs to filter out the effect of fiscal policy. It is in fact possible that fiscal policy gets all the blame of a failure, but it may also happen that disinflation would have failed even without a fiscal expansion. Unfortunately, this filtering might require extensive data on government behavior, and its result might be sensitive to parameters and model specification.

The last piece of speculation concerns the reason for certain countries switching to a fiscal expansion around the appreciation. Is there a potential mechanism that leads to an automatic fiscal expansion, depending on the equilibrium implications and price incidence of the nominal appreciation? For example, a government closely allied with exporters or domestic capital owners might be tempted to compensate domestic firms for the strong exchange rate. A government concerned with slowing investment may also adopt a fiscal expansion. The common theme is that an appreciation hurts exporters and capital, and a fiscal policy aimed at offsetting these effects ends up giving substantial extra income to domestic consumers as a byproduct.

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Appendix

Derivation of $E = VH$

The per period utility function is

$$v(t) = C_T^{\beta\alpha} C_{NT}^{(1-\beta)\alpha} (H/P)^{1-\alpha},$$

where P is the ideal price index corresponding to the Cobb-Douglas preference specification. Denoting consumption expenditures by E, we know that $E = \lambda E/P_T$, $C_{NT} = (1-\lambda) E/P_{NT}$. Thus

$$v(t) = \lambda^{\beta\alpha} E^{\beta\alpha} P_T^{-\beta\alpha} (1-\lambda)^{(1-\beta)\alpha} E^{(1-\beta)\alpha} P_{NT}^{-(1-\beta)\alpha} H^{1-\alpha} P_T^{(1-\beta)(\alpha-1)} = const \cdot E^{\alpha} H^{1-\alpha} P_T^{-\beta} P_{NT}^{\beta-1}.$$

The consumer therefore maximizes

$$\int_0^{\infty} (E/P)^{\alpha} (H/P)^{1-\alpha} \exp(-\delta t)\, dt,$$

under the constraint

$$\frac{d}{dt} H = YP - E.$$

Here Y is contemporaneous (real) income, which is exogenous for our representative consumer. Switch from nominal to real money and define π as CPI-inflation:

$$\frac{d}{dt}\frac{H}{P} = \frac{d/dtH}{P} - \frac{H}{P}\frac{d/dtP}{P} = Y - \frac{E}{P} - \frac{H}{P}\pi.$$

Let h denote real money holdings and ε denote real expenditures, then the problem becomes

$$\max \int \varepsilon^{1-\alpha} h^{\alpha} \exp(-\delta t)\, dt, \text{ s.t. } \frac{d}{dt} h = Y - \varepsilon - h\pi.$$

Define the Hamiltonian:

$$\mathfrak{H} = \varepsilon^{1-\alpha} h^{\alpha} \exp(-\delta t) + \lambda_t (Y - \varepsilon - h\pi).$$

The first order conditions are

$$\begin{aligned} 0 &= \frac{d}{d\varepsilon}\mathfrak{H} = (1-\alpha)\left(\frac{h}{\varepsilon}\right)^{\alpha} \exp(-\delta t) - \lambda_t \\ -\frac{d}{dt}\lambda_t &= \frac{d}{dh}\mathfrak{H} = \alpha \left(\frac{h}{\varepsilon}\right)^{\alpha-1} \exp(-\delta t) - \lambda_t \pi. \end{aligned}$$

The first implies that

$$\lambda_t = (1-\alpha)\exp(-\delta t)\left(\frac{h}{\varepsilon}\right)^{\alpha}.$$

Differentiating this expression with respect to time:

$$-\frac{d}{dt}\lambda = \delta(1-\alpha)\exp(-\delta t)\left(\frac{h}{\varepsilon}\right)^{\alpha} - (1-\alpha)\exp(-\delta t)\,\alpha\left(\frac{h}{\varepsilon}\right)^{\alpha-1}\frac{\dot{h}\varepsilon - \dot{\varepsilon}h}{\varepsilon^2},$$

sothe second derivative condition becomes

$$\frac{\alpha}{1-\alpha}\left(\frac{h}{\varepsilon}\right)^{\alpha-1} - \left(\frac{h}{\varepsilon}\right)^{\alpha}\pi = \delta\left(\frac{h}{\varepsilon}\right)^{\alpha} - \alpha\left(\frac{h}{\varepsilon}\right)^{\alpha-1}\left(\frac{\dot{h}}{h} - \frac{\dot{\varepsilon}}{\varepsilon}\right)\frac{h}{\varepsilon}.$$

This simplifies to

$$\frac{\alpha}{1-\alpha}+\alpha\frac{d}{dt}\left(\frac{h}{\varepsilon}\right)=(\delta+\pi)\frac{h}{\varepsilon}.$$

One can easily check that $\varepsilon=(\delta+\pi)\frac{1-\alpha}{\alpha}h$ satisfies this optimality condition, and remains bounded, so the transversality condition also holds. This means that this is the saddle point solution, so indeed $E=VH$ holds along the optimal path.

In the full optimization model, however, π changes, which kills the fixed velocity property. For this reason, Benczúr and Kónya (2003) considers the full consumer and investor optimization setup. The per period utility function (power Cobb-Douglas) is replaced by the log version of Cobb-Douglas ($\log E+\gamma\log(H/P)$). This coincides with the standard choice of newkeynesian open economy models. Its most important property is that the marginal utility of consumption is independent from money holdings. A calculation similar to the one above shows that the solution does not satisfy $E=VH$: the optimality condition becomes

$$\alpha\frac{\varepsilon}{h}-(1-\alpha)\frac{\dot{\varepsilon}}{\varepsilon}=(\delta+\pi)(1-\alpha),$$

which is inconsistent with a constant ε/h and a changing ε.

It remains true, however, that an increase in H (or H/e, its euro value) increases consumption expenditures (with H/e, their euro value). Due to the production side (nonlinear short-term transformation curve), it leads to an increase in nontraded prices, and then feeds into factor prices as well. This is the same impact effect, and under the appropriate capital intensity assumption, we also get the same propagation mechanism.

The key of this intuition is thus the property that consumption expenditures (the jump variable corresponding to money) are increasing in nominal money (a state variable). This can be verified by filling in the necessary steps of the following reasoning.

The dynamic system consists of two state variables, capital (K) and nominal money (H). There are two jump variables, q belongs to K, and nominal expenditure E belongs to H. The dynamic system can be written as

$$\begin{pmatrix}\dot{K}\\ \dot{H}\\ \dot{q}\\ \dot{E}\end{pmatrix}=\mathbf{f}(K,H,q,E).$$

Loglinearizing around the steady state point ($\tilde{K}$ etc. denotes log deviation from steady state):

$$\begin{pmatrix} \tilde{K} \\ \tilde{H} \\ \tilde{q} \\ \tilde{E} \end{pmatrix} = \mathbf{A} \begin{pmatrix} K - K^* \\ H - H^* \\ q - q^* \\ E - E^* \end{pmatrix}.$$

Matrix $\mathbf{A}$ must have two convergent and two divergent eigenvalues, since the system is pinned down by two initial conditions (for capital and money) and two terminal conditions (coming from the transversality conditions of consumer and investor optimization). Denote the two eigenvectors corresponding to the convergent roots by $\mathbf{v}_1$ and $\mathbf{v}_2$. Then

$$\left(\tilde{K}, \tilde{H}, \tilde{q}, \tilde{E}\right)_0 = a\mathbf{v}_1 + b\mathbf{v}_2.$$

Coefficients a and b are set by the two initial conditions, so they can be expressed as linear combinations of $\tilde{K}$ and $\tilde{H}$. Then $\tilde{q}$ and $\tilde{E}$ are also linear combinations, so

$$\tilde{E}_0 = c\tilde{H}_0 + d\tilde{K}_0,$$

where c and d are (known) functions of the two eigenvectors. We need to show that c is positive, which follows easily if one uses that $\mathbf{A}\mathbf{v}_i = \lambda_i \mathbf{v}_i$.

Linearization of the laws of motion

Start from equation (10):

$$\hat{K} = \hat{k}_T + \lambda V \frac{\hat{H}}{e} \hat{k}_T^{\alpha_1} \left(\frac{1-\alpha_2}{1-\alpha_1} - \frac{\alpha_2}{\alpha_1} \right).$$

Here all variables are effective, but to simplify notations, I will drop the hats from here on. Rewrite K as $K^* + dK$, $k_T = k_T^* + dk_T$ etc. Then (10) becomes

$$K^* + dK = k_T^* + dk_T + \frac{\lambda V \left(H^* + dH\right)}{e} \left(k_T^* + dk_T\right)^{\alpha_1} \left(\frac{1-\alpha_2}{1-\alpha_1} - \frac{\alpha_2}{\alpha_1} \right).$$

Replace all nonlinear functions by their first order approximations:

$$K^* + dK = k_T^* + dk_T + \frac{\lambda V \left(H^* + dH\right)}{e} \left(k_T^* + \alpha_1 k_T^{*\alpha_1 - 1} dk_T\right)^{\alpha_1} \left(\frac{1-\alpha_2}{1-\alpha_1} - \frac{\alpha_2}{\alpha_1} \right),$$

neglect the second order (product) terms and eliminate the steady state form of (10):

$$\begin{aligned} dK &= dk_T + \lambda V \left(\frac{1-\alpha_2}{1-\alpha_1} - \frac{\alpha_2}{\alpha_1}\right)\left(\frac{H^*}{e}\alpha_1 k_T^{*\alpha_1 - 1} dk_T + k^* \frac{dH}{e}\right)^{\alpha_1} \\ dk_T &= \frac{1}{1 + \lambda V \left(\frac{1-\alpha_2}{1-\alpha_1} - \frac{\alpha_2}{\alpha_1}\right)\frac{H^*}{e}\alpha_1 k^{*\alpha_1 - 1}} \left(dK + \lambda V \left(\frac{1-\alpha_2}{1-\alpha_1} - \frac{\alpha_2}{\alpha_1}\right)\frac{dH}{e}\right). \end{aligned}$$

Using $r = (1-\alpha_1)\, e k_T^{-\alpha_1}$:

$$dr = -\left(1-\alpha_1\right) e \alpha_1 \left(k_T^*\right)^{-\alpha_1 - 1} dk_T.$$

Plugging these into the capital accumulation equation:

$$\begin{aligned} K_{t+1} &= \frac{K_{t+2}}{2+r^*} + \frac{K_t\left(1+r^*\right)}{2+r^*} + \frac{c}{2+r^*}\left(r_t\left(K_t\right)/e - r^*\right) \\ dK_{t+1} &= \frac{dK_{t+2}}{2+r^*} + \frac{dK_t\left(1+r^*\right)}{2+r^*} - \frac{c}{2+r^*}\left(\left(1-\alpha_1\right) e\alpha_1 \left(k_T^*\right)^{-\alpha_1 - 1}\right) dk_T. \end{aligned} \tag{16}$$

Since dk_T is also a linear combination of dH and dK, (16) gives a linear law of motion for dK in terms of state variables (current and future).

One can repeat the same procedure for

$$H_{t+1} - H_t = \frac{e}{1+g}\alpha_1 k_T \left(K_t\right)^{1-\alpha_1} - \frac{VH_t}{1+g}$$

as well:

$$dH_{t+1} - dH_t = \frac{e}{1+g}\alpha_1\left(1-\alpha_1\right)\left(k_T^*\right)^{-\alpha_1} dk_T - \frac{VdH_t}{1+g}.$$

This is again linear in state variables.