INGENUE v.2: a World OLG-CGE Model with Imperfect Financial Markets, Exchange Rates and Stochastic Lifetime

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1 Introduction

This paper presents the mathematical structure of a new version of INGENUE: a computable, general-equilibrium, multi-regional overlapping generations model. The aim of this research is to analyze the issues relating to wealth accumulation and the development of pension funds and other devices of saving for retirement in the context of global finance, hence to study the international capital flows that ought to be induced by the differences of aging and of technical progress growth in the various regions of the world.

The first version of the INGENUE model describes a multi-region, world model, in the spirit of those described by Obstfeld et Rogoff (1996), in which the structure of each regional economy is similar to that of other applied, OGGE models, such as Auerbach *et al.* (1983), Cazes *et al.* (1992, 1994) , except that labor supply is exogenous. Kotlikoff et al. (2003) and Börsch-Supan (2002) have developed two similar applied multi-regional OGGE models, but their analysis is restrained to developed countries. In INGENUE v1, the world was divided into six regions (three developed areas and three developing areas), each of which is made of three categories of economic agents: the households, the firms, and a PAYG retirement pension system. There was only one good, and only one financial asset, which is an ownership stake in the firms' productive capital ; both of them are freely traded on perfectly competitive world markets. There was no money and hence only two relative prices in each region: the (real) wage rate accruing to local, internationally immobile, workers; and the single (real) price of financial assets, both expressed in terms of goods, which may be chosen as *numéraire*. Hence, the various regions of the world were economically and financially perfectly integrated and there is only one world market for goods and one for financial assets. We have performed various pionner works (1999, 2001a, 2001b, 2002a, 2002b) with this first model[1] but this model could appear to be limited in some of its outcomes. So we developed this new version.

INGENUE v.2. vs INGENUE v.1. : What's new?

In this second version, we make the model more realistic by introducing a number of changes in the assumptions:

1. <u>Demographics:</u> the World is now divided in 10 regions. In order to make autonomous own demographic projections, we have built a population projection model based upon UN coefficient methods.

2. <u>Households:</u> We now assume uncertainty in lifetime expectancy at individual level. At the macroeconomic level there is still no uncertainties about it.

3. <u>International trade of commodities:</u> In order to deal with relative price movements of foreign and domestic goods we assume that the different countries produce, different imperfectly substitutable intermediate

[1]In a first technical report (1999), we present the rationale, theorical underpinnings, mathematical structure and background data of the model.

In the article (2001a), we propose a calibration and characterize the major features of the long-term, steady-state equilibrium of the model and we study the sensitivity of the results obtained for both the long-run equilibrium and the transition paths of the main endogenous macroeconomic variables of the model. In particular, we focus on the consequences of alternative assumptions concerning the rate of international technological convergence, an issue on which there is a little empirical consensus.

The paper (2002a) analyzes various effects of three simulated reforms of the European pay-as-you-go system. Reforms are presented around a baseline scenario where constant replacement ratio of public pension over net wage rate is assumed. Alternative reforms are: constant contribution rate, legal retirement age postponement and constant replacement ratio of public pension over gross wage rate. We then investigate the allocative, distributive and politic effects of these various policy.

In the paper (2001b), we systematically compare the domestic and international macroeconomic consequences of broad classes of pension reforms in Europe with those obtained in similar models where Europe is treated either in autarky or as a small, open economy. This comparison allows a better understanding of the role of financial openness and international capital flows in smoothing and spreading the effects of pension reforms in developed countries.

The paper (2002b) is a recapitulating work.

goods as in Backus et al. (1995). This will imply the existence of real effective exchange rates between the different regions. Here the main determinants of exchange rates are the relative productivity in the two productive sectors as in the standard view developed since Obstfeld and Rogoff works (i.e. the famous *Balassa-Samuelson effect* that is predominant in long run explanations of difference in real exchange rates).

4. Financial markets: We model region-specific interest rates to debtor that differ from the unique world interest rate to creditor by imposing an *ad hoc* convex function of the regional ownership ratio.

5. Calibration improvements: We introduce inheritances based upon a bequest motive, age-specific labor participation rates (exogenous), age-specific human capital (exogenous) and labor income of children in some parts of the world.

2 Demographics

2.1 Regions

In this new version of Ingenue, the World is now divided in 10 regions instead of 6 regions in the previous version, according to geographical criteria:

1. **"Western Europe":** 'Channel Islands', 'Denmark', 'Finland', 'Iceland', 'Ireland', 'Norway', 'Sweden', 'United Kingdom', 'Greece', 'Italy', 'Malta', 'Portugal', 'Spain', 'Austria', 'Belgium', 'France', 'Germany' (East + West), 'Luxembourg', 'Netherlands', 'Switzerland'.

2. **"Eastern Europe":** 'Estonia', 'Latvia', Lithuania', 'Bulgaria', 'Czech Republic', 'Hungary', 'Poland' 'Romania', 'Slovakia', 'Slovenia', 'Albania', 'Bosnia and Herzegovina', 'Croatia', 'TFYR Macedonia', 'Yugoslavia'.

3. **"North America":** 'Canada', 'United States of America', 'Australia', 'New Zealand', 'Melanesia', 'Fiji', 'New Caledonia', 'Papua New Guinea', Solomon Islands', 'Vanuatu', 'Micronesia', 'Guam', 'Polynesia', 'French Polynesia', 'Samoa'.

4. **"South America":** 'Argentina', 'Bolivia', 'Brazil', 'Chile', 'Colombia', 'Ecuador', 'French Guiana', 'Guyana', 'Paraguay', 'Peru', 'Suriname', 'Uruguay', 'Venezuela', 'Belize', 'Costa Rica', 'El Salvador',

'Guatemala', 'Honduras', 'Mexico', 'Nicaragua', 'Panama', 'Bahamas', 'Barbados', 'Cuba', 'Dominican Republic', 'Guadeloupe', 'Haiti', 'Jamaica', 'Martinique', 'Netherlands Antilles', 'Puerto Rico', 'Saint Lucia', 'Trinidad and Tobago'.

5. **Japan**

6. **"Mediterranean World":** 'Algeria', 'Egypt', 'Libyan Arab Jamahiriya', 'Morocco', , 'Tunisia', 'Western Sahara', 'Armenia', 'Azerbaijan', 'Bahrain', 'Cyprus', 'Georgia', 'Iraq', 'Iran', 'Israel', 'Jordan', 'Kuwait', 'Lebanon', 'Occupied Palestinian Territory', 'Oman', 'Qatar', 'Saudi Arabia', 'Syrian Arab Republic', 'Turkey', 'United Arab Emirates', 'Yemen'. 'Turkmenistan', 'Uzbekistan' 'Kyrgyzstan'

7. **"Chinese World":** 'China', 'Democratic People's Republic of Korea', 'Mongolia', 'Republic of Korea', 'Brunei Darussalam', 'Cambodia', 'East Timor', 'Lao People's Democratic Republic', 'Myanmar', 'Philippines', 'Singapore', 'Thailand', 'Viet Nam'.

8. **"Africa":** 'Burundi', 'Comoros', 'Djibouti', 'Eritrea', 'Ethiopia', 'Kenya', 'Madagascar', 'Malawi', 'Mauritius', 'Mozambique', 'Réunion', 'Rwanda', 'Somalia', 'Uganda', 'Tanzania', 'Zambia', 'Zimbabwe', 'Angola', 'Cameroon', 'Central African Republic', 'Chad', 'Congo', 'Democratic Republic of the Congo', 'Equatorial Guinea', 'Gabon', 'Botswana', 'Lesotho', 'Namibia', 'South Africa', 'Swaziland', 'Benin', 'Burkina Faso', 'Cape Verde', 'Côte d'Ivoire', 'Gambia', 'Ghana', 'Guinea', 'Guinea-Bissau', 'Liberia', 'Mali', 'Mauritania', 'Niger', 'Nigeria', 'Senegal', 'Sierra Leone', 'Togo'. 'Sudan'

9. **"Russian World":** 'Belarus', 'Russian Federation', 'Ukraine', 'Kazakhstan', 'Republic of Moldova',

10. **"Indian World":** 'India', 'Afghanistan', 'Bangladesh', 'Bhutan', 'Maldives', 'Nepal', 'Pakistan', 'Sri Lanka', 'Tajikistan', 'Indonesia', 'Malaysia'.

2.2 Population structure and Projection Method

The period of the model is set to five years. In each region z, the economy is populated by 21 overlapping generations of one-sex agent who may no live longer than 105 years. For notation purpose cohorts will be indexed by $a \in [0, \dots, 20]$. The number of people of age a at time t is denoted by $L_a^z(t)$ (for any household variable, a subscript a denotes age and an argument t in parentheses denotes calendar time). At date t the number of "births" (individuals between 0 and 4 years old) is then denoted by $L_0^z(t)$ and total population alive at time t is $L^z(t) = \sum_{a=0}^{20} L_a^z(t)$.

Population evolution are exogenously calculated according to a standard population projection method on the basis of historical and prospective UN data. We have aggregated population structure, with the UN data from 1950 to 1995, over countries to build Ingenue's regions (see above). Then we can project fertility and mortality trends (for both sexes) at the region-aggregate level, this together with initial population structures in 1995, allow us to obtain population evolution in the future from 2000 until the ending date of the model. We implicitly assume that there is no migration flows in the future. With some usual population projection methods, we construct evolution of mortality and fertility tables on the only basis of life expectancy and global fertility rates evolutions in the future.

2.2.1 Mortality

People can die before 105 year ; let s_a the conditional probability of surviving between age a and age $a+1$, the number of age $a-1$ people then changes as:

$$L_a^z(t) = s_{a-1}^z \cdot (t-1) \cdot L_{a-1}^z(t-1) \quad \text{for all } a > 0 \tag{1}$$

$\prod_{i=0}^{a-1} s_i(t+i)$ is then the unconditional probability of being alive at age "a" when born at date t. For population projection we then need some process to describe evolution of $\{s_{a-1}^z(t-1)\}_{a>0}$ for $t = 2000, \ldots, T$ (for both sexes). For this we first have to precise beginning and ending mortality tables. Beginning table is given from UN data between years 1995 and 2000. Ending table are chosen among UN "typical" long run mortality tables (from Coale and Demeny, 1966). We then have to fix a date when the convergence to the long run table will be achieved and a process for convergence between initial date and this target date. According to UN methods we extrapolate future mortality tables on the basis of a expected trend for life expectancy. We adopt a linear process of convergence.

2.2.2 Fertility Process

At each time period the number of births will be equal to $L_0^z(t) = \sum_{a=3}^{9} f_a^z(t) L_a^z(t)$ (here L is only female population), where f_a^z are the average age-specific fertility rates, we implicitly assume following UN projections that women fertility occurs only between 15 and 50 years old.

To achieve stationarity of population at very long run fertility and survival rates have to satisfy the Lotka condition. Here this condition is satisfied by normalizing the components of matrix representing the low of motion of the deterministic population (fertility and survival rates) by the biggest eigenvalue of this matrix.

3 Macroeconomic framework

3.1 Households

Individuals are assumed to become adults when they turn 20 ($a_0 = 4$ in formula). During any period, the household sector is then made of 17 overlapping cohorts of "adults", of age between 20 and 105, and 4 cohorts of "young". Adults may not stay in the labor force after a legal maximal mandatory retirement age $\bar{r}^z$. They determine their optimal designs of consumption and saving with perfect foresight at the beginning of their adult life. Between 15 and 50 yrs. adults are supposed to give birth to children, according to the fertility calendar. Children are dependent until they turn 20, they consume with a cost per child that is supposed to be proportional to the parents consumption.

Labor supply is assumed to be exogenously given as the age-specific rate of participation to labor market: e_a^z. We use ILO data and projections to characterize activity from 1950 until 2015 and assume that after this date participation rates remain fixed at their 2015 level. According to this database people may work since the age of 10 so we will take into account children labor income to the budget constraint of their parents.

The intertemporal preferences of a new entrant on working-life are given by the following life-time utility function over uncertain streams of consumption c_a^z and leaving a voluntary bequest H^z to their children when they'll reach an age of T (if they survive until this age)[2]:

[2] Usually in these kind of model the age T is the biological limit to life (here 105 yrs.) but in order to imply a realistic pattern of inheritance among the children of deceased households, we will assume that T is equals to 80 yr old.

$$U^z_{a_0}(t) = \sum_{a=a_0}^{20} \rho^{a-a_0} \left[\prod_{i=a_0}^{a-1} s^z_i(t+i) \right] \frac{\eta}{\eta-1} c^z_a(t+a-a_0)^{\frac{\eta-1}{\eta}} \tag{2}$$
$$+ \rho^{T-a_0} \prod_{i=a_0}^{T} s^z_T(t+T-a_0) V\left(H^z(t+T-a_0)\right)$$

where ρ is the psychological discount factor[3], C_a is consumption at the age a ; η is the intertemporal substitution rate and $V(\cdot)$ is the instantaneous utility of bequest, so agent has some felicity from leaving a bequest but it is independent of the future stream of the consumption that the children draw from this bequest (*warm glow* altruism). This bequest behaviour is mainly adopted to calibration issues (empirically savings for life-cycle can only explain a part of saving motives).

At any given period, the budget constraint is (with additional constraints $S_{a_0-1} = 0$ and $S_{20} \geq 0$) for all $a \in [a_0, \dots, 20]$:

$$\tau^z_a(t) p^z_f(t) C^z_a(t) + p^z_f(t) S^z_a(t) = Y^z_a(t) + p^z_f(t) S^z_{a-1}(t-1) \frac{R^z(t)}{s_{a-1}(t-1)}$$
$$+ p^z_f(t) h^z_a(t) - p^z_f(t) H^z(t) \Upsilon_T(t) \tag{3}$$
$$\text{with} Y^z_a(t) = \begin{cases} \zeta^z_a(t) + (1-\theta^z(t)) w(t)^z e_a(t) \vartheta_a & \text{for} \quad a < \underline{r}^a \\ (1-\theta^z(t)) w(t)^z e_a(t) \vartheta_a + (1-e_a(t)) P^z_a(t) & \text{for} \quad \underline{r}^a \leq a < \bar{r}^a \\ P^z_a(t) & \text{for} \quad a \geq \bar{r}^a \end{cases}$$

where S^z_a denotes the stock of assets held by the individual at the end of age a and time t, $R^z(t) \cdot S_a(-1)$ is financial income (domestic real return on assets holdings times wealth), τ_a is the age-specific equivalence scale that takes into account costs of child-rearing (see details hereafter), and Y_a is the non-assets net disposal income. $p^z_f(t)$ is the price of the domestic final good so $R^z(t)$ is the return to capital income expressed in units of this final good. Due to life uncertainty at the individual level one may think that there exists unintended bequests, instead of we assume here following (Yaari, 1965) that there exists perfect annuities markets that pool death risk within the same generation so that the return to capital is “corrected” by the instantaneous survival probability of the generation. Besides children received inherited assets $h^z_a(t-1)$ from the voluntary bequests of their parents. People will

[3] Notice that the effective discount rate is equal to $\prod_{i=a_0}^{a-1} s^z_i(t+i) \frac{\eta}{\eta-1} \rho^{a-a_0}$, meaning that agents only care of their future as long as they stay alive. In other words the expectation takes into account that the agent can die before 105 yrs. old.

leave bequest $H^z(t)$ to their heirs only at the age of T, so in budgetary equation $\Upsilon_T(t)$ is a dummy that will be equal to 1 if $a = T$ and zero in other case.

For full-time active years ($a \in [a_0, \underline{r}^a[$), Y_a is simply equal to the net labor income after social security taxes (at rate θ), where w is the real wage rate per efficient unit of labour at time t. When agent is partly retired ($a \in [\underline{r}^a, \bar{r}^a[$), she also receives a pension benefit P_a^z for the unworked hours. And when she is full-time retired ($a \in [\bar{r}^a, 20]$) she only receives the pension benefit. As a matter of fact, unless notice otherwise, pension benefit is assumed to be age independent.

The τ_a^z term is the age-specific equivalence scale, it takes account the direct and indirect private costs of child-rearing. In order to calculate this relative cost of child-rearing for each cohort we need first to know the age distribution of children for each parent (from their past fertility behaviour) and second we need the age "c" (for child) equivalence scale of children β^c, which will be assumed here to be constant:

$$\tau_a^z(t) = 1 + \sum_{c=\max(0,a-12)}^{\min(3,a-3)} \beta^c \cdot L_a^{c,z}(t) \qquad a = 4, \dots, 12 \tag{4}$$

where the average number L_a^c of children of age c raised by cohort of age a can be recovered from fertility evolution (given the fertility calendar and the early death of both children and parents). For simplicity, the children depending of parents younger that 20 years old are assumed to be "allocated" between the adults that have children of the same age (allocation with age-specific weights)[4]. $\zeta_a^z(t)$ is the labour income that children bring to their parents resources during their childhood (calculated in the same spirit that costs of children-rearing).

An agent's earning ability is assumed to be an exogenous function of its age. These skill differences by age are captured by the efficiency parameter ϑ_a which changes with age in a hump-shape way to reflect the evolution of human capital. For simplicity, we assume that this age-efficiency profile is time-invariant and is the same in all region. In the baseline case we adopt

[4]Being more precise will need to conserve the distribution of child with respects to their grand-parents and will complicate in an useless way the number of state variables in the system.

Miles (1999) human capital profile's estimation (for UK) and normalize ϑ_a such that $\vartheta_{a_0} = 1$.

New cohorts choose optimal plan for current and future consumption as well as bequest in order to maximize their lifetime utility under their intertemporal budget constraint taking prices, social contribution and benefits as given. At the equilibrium, the first order conditions for this program will yield (together with budget constraint and the transversality condition $S_{20} = 0$ assumed to be satisfied), $\forall z$:

$$C^z_{a+1}(t+1) = C^z_a(t) \cdot \left[\rho R^z(t+1)\frac{\tau^z_a(t)}{\tau^z_{a+1}(t+1)}\right]^\eta \qquad a \in [a_0, 20[\tag{5}$$

$$C^z_T(t)^{-\frac{1}{\eta}} = V'\left(H^z(t)\right) \tag{6}$$

Voluntary bequests are distributed to children according to the fertility calendar of their deceased parents. Taking Blinder (1975) functional form for V, we obtain a simple linear relation: $C^z_T = \Psi^z H^z$, where Ψ indicates the degree of altruism. At the equilibrium the sum of voluntary bequest will be equal to the inheritance received by children:

$$L^z_T(t)H^z(t) = \sum_{a=T-3}^{T-9} L_a(t)h_a(t) \tag{7}$$

Notice that people of age T have only children between age $T-9$ and $T-3$ so we have $h_a(t) = 0$ for all $a \notin [T-9, \ldots, T-3]$. For $a \in [T-9, \ldots, T-3]$, we assume that bequests are distributed equally to all children then h_a is proportional to the size of the children born from cohort of age T (according to her past fertility calendar).

In our international context, households can choose the region where they want to invest their wealth. So we need another equilibrium equation that characterizes the trade off between distinct assets:

$$R^z(t) = R^\star(t)\frac{p^z_f(t-1)}{p^z_f(t)} \qquad \text{for all t} > 0 \tag{8}$$

where $R^\star(t)$ is the unique world interest factor (in terms of the world *numéraire*), the condition of trade-off means that if a region z'household save one unit in is domestic asset (capital) it will yield $R^z(t)$ in real terms at the next period, if he chooses to invest on international market he will receive in real terms $R^{\star}(t)\frac{p_f^z(t-1)}{p_f^z(t)}$. Positive assets holdings of both kind of savings will imply that the two returns will equalize at the equilibrium.

3.2 The public sector

The public sector is reduced to a social security department; it is a pay-as-you-go (PAYG) public pension scheme, that is supposed to exist in all regions of the world. It is financed by a payroll tax on all labor incomes and pays pensions to retired households. The regional PAYG systems operate according to a defined-benefit rule: pensions π paid to individual retired are a fraction - or replacement rate $(\kappa(t))$ – of the current average (net of tax) wage. We assume a time-to-time balanced-budget rule:

$$\frac{\theta^z(t)}{1-\theta^z(t)} = \kappa(t) \cdot \frac{\sum_{a \geq \underline{r}^a}(1-e_a(t))L_a(t)}{\sum_{a \leq \bar{r}^a} e_a(t)L_a(t)} \tag{9}$$

In the baseline case, the regional age $\underline{r}^a$ of minimum legal retirement age as well as the maximum age $\bar{r}^a$ and the ratio $\kappa(t)$ are fixed (at least after year 2000), then payroll tax rates $\theta(t)$ are endogenously determined by this rule.

3.3 Production side

3.3.1 Intermediate good sector

Each zone z specializes in the production YI^z of a single intermediate good labelled, where subscript z indicates that the specific nature of this good lies in its region of origin. Production in period t takes place with a constant return to scale-Cobb Douglas production function using capital stock $K^z(t-1)$ installed at the beginning of the period t in the country z and the full domestic labor force $N^z(t)$, $\forall z$:

$$YI^z(t) = AI^z(t)\left(K^z(t-1)\right)^{\alpha}\left(N^z(t)\right)^{1-\alpha} \qquad 0<\alpha<1 \tag{10}$$

With this formulation YI^z also denotes GDP in the country z in terms of the local intermediate good. The current cash-flow of the representative domestic firm of the intermediate sector (in terms of the world *numéraire*):

$$\begin{aligned} CF^z(t) &= p_I^z(t)YI^z(t) - w^z(t)N^z(t) - p_f^z I^z(t) && \text{for t} > 0 \\ CF^z(t) &= p_I^z(t)YI^z(t) - w^z(t)N^z(t) - p_f^z(1-\delta^z(t))K^z(t-1) && \text{for t} = 0 \end{aligned}$$

where p_I^z is the price of the domestic intermediate good ; $I^z(t) = K^z - K^z(t-1)(1-\delta^z(t))$ are the gross investment expenses in domestic final good and $\delta^z(t)$ is the rate of economic depreciation. In time 0 the present value of the firm is equal to:

$$\Pi^z(0) = CF^z(0) + \sum_{t=1}^{\infty} \frac{CF^z(t)}{\prod_{s=0}^{t} R^\star(t)} \tag{11}$$

where $R^\star(t)$ is the factor of interest on the world financial market. Let us denote $k^z(t-1) = K^z(t-1)/N^z(t)$ as the capital-labor ratio, the maximization of the firm value will imply at the equilibrium ($\forall t$):

$$R^\star(t+1)\frac{p_f^z(t)}{p_f^z(t+1)} + \delta^z(t+1) - 1 = \frac{p_I^z(t+1)}{p_f^z(t+1)}\alpha A I^z(t+1)\left(k^z(t)\right)^{\alpha-1} \tag{12}$$

$$w^z(t) = p_I^z(t)(1-\alpha)AI^z(t)\left(k^z(t-1)\right)^{\alpha} \tag{13}$$

3.3.2 A "trick" to model real imperfections on world financial market

For a world model to be realistic the world asset capital market have to be imperfect. Because sources of imperfection and asymmetries in financial markets are various and uneasy to model with rigourous micro-foundations in such a large scale model as Ingenue we adopt the following *ad hoc* formulation for δ^z the region-specific rate of economic depreciation, with $\varepsilon > 0$:

$$\delta^z(t) = \bar{\delta}^z + (1-\bar{\delta}^z)\Delta^z \cdot \text{Max}\left(1 - \frac{S^z(t-1)}{K^z(t-1)}; 0\right)^{\varepsilon} \quad \text{for all z} \tag{14}$$

where $S^z(t) = \sum_{a=a_0}^{19} L_a(t)S_a(t)$ is the aggregate wealth across overall cohorts in region z. Aggregate financial wealth is equal to the sum of the region capital stock and the net assets on the rest of the world. This equation then indicates that capital invested in a region z depreciates more rapidly than the average when the region is a net debtor of the rest of the world. In other world the net-of-depreciation return from capital invested in indebted regions are, other things being equal, lower than in creditor regions. With this formulation the domestic cost of capital for the intermediate good sector may be greater than the world cost but never be lower.

3.3.3 Final good production sector

In the spirit of Backus et Al. (1995), we assume that the domestic, composite final good of region z (consumption and investment) YF^z is produced thanks to a combination of two intermediate goods : a "domestic" intermediate good in proportion B^z and a "World" intermediate goods in proportion M^z, according to the following CES technology, where $\sigma \geq 0$ denotes the elasticity of substitution, $\forall z$:

$$YF^z(t) = A \cdot F^z(t) \cdot \left[(\omega^z)^{\frac{1}{\sigma^z}} \left(B^z(t)\right)^{\frac{\sigma^z-1}{\sigma^z}} + (1-\omega^z)^{\frac{1}{\sigma^z}} \left(M^z(t)\right)^{\frac{\sigma^z-1}{\sigma^z}} \right]^{\frac{\sigma^z}{\sigma^z-1}} \tag{15}$$

with $\omega^z \in [0,1]$. This CES combination of external and internal good to produce domestic final good is a *reminiscence* of *Armington (1969) aggregator*. Taking prices as given and competitive behaviour the producer determines B^z and M^z that minimizes current profit: $p^z_f(t)YF^z(t) - p^z_I(t)B^z(t) - p^\star(t) \cdot M^z(t)$ subject to (15), where p^z_I is the price of the home-specific intermediate good and $p^\star$ is the price of the world intermediate good, both expressed in terms of the specific final good. The static maximization problem of a representative competitive firm gives at the equilibrium the following first order conditions:

$$B^z(t) = \omega^z \left(\frac{p^z_I(t)}{p^z_f(t)}\right)^{-\sigma^z} \frac{YF^z(t)}{\left(AF^z(t)\right)^{1-\sigma^z}}, \tag{16}$$

$$M^z(t) = (1-\omega^z) \left(\frac{p^\star(t)}{p^z_f(t)}\right)^{-\sigma} \frac{YF^z(t)}{\left(AF^z(t)\right)^{1-\sigma^z}} \tag{17}$$

where p^z_f can be shown to be equals to the following aggregate price index formulation that at the equilibrium:

$$p^z_f(t) = \frac{\left[\omega^z_z p^z_I(t)^{1-\sigma^z} + (1-\omega^z)\, p^\star(t)^{1-\sigma^z}\right]^{\frac{1}{1-\sigma^z}}}{AF^z(t)} \tag{18}$$

3.3.4 The fiction of a world producer of an homogenous world intermediate good

In order to simplify the exchanges of intermediate goods between regions of the world we assume that there exist a fictive world producer that uses region-specific intermediate goods in quantities $X^{\star,z}$ in order to produce a specific world intermediate good $Y^\star$ according to the following CES function:

$$Y^\star(t) = A^\star(t)\left[\sum_z \gamma^z(t)^{\frac{1}{\mu}} X^z(t)^{\frac{\mu-1}{\mu}}\right]^{\frac{\mu}{\mu-1}} \tag{19}$$

where $\gamma^z(t)$ is a weighted coefficient that represents the importance of the region z specific intermediate good in the world trade market (for instance think of oil producer regions, with respect to traditional theoretical trade determinants their share in world trade would be relatively small so one way to reproduce their actual huge trade balance is to allow an important size to this parameter).

This agent is assumed to act competitively, taking prices as given, he chooses $\{X^z(t)\}_z$, at each period, to maximize its static profit : $p^\star(t)Y^\star(t) - \sum_z p_I^z(t) \cdot X^z(t)$, subject to (19). This yields at the equilibrium the the following factor demand functions:

$$X^z(t) = \gamma^z(t)\left(e^z(t)\right)^{-\mu} Y^\star(t)A^\star(t)^{\mu-1} \qquad \text{for all z} \tag{20}$$

where for convenience $e^z(t) = \frac{p_I^z(t)}{p^\star(t)}$ is defined as the real exchange rate. It can be shown that at the equilibrium $p^\star$ equals to:

$$p^\star = \frac{\left[\sum_z \gamma_z p_I^z(t)^{1-\mu}\right]^{\frac{1}{1-\mu}}}{A^\star(t)}. \tag{21}$$

4 The world general equilibrium

4.1 The competitive world equilibrium

Definition :

Given the initial stock of capital installed in each zone $\{K^z(0)\}_{z=1,..,10}$*; initial distributions* $\{S_a^z(0)\}_{z=1,..,10;a=a_0,..,19}$ *of savings across age groups a and zone z; initial prices* $\{p_f^z(0)\}_{z=1,..,10}$ *and exogenous population prospects* $\{L_a^z(t)\}_{t\geq1;z=1,..,10;a=0,..,20}$ *with* $\{L_a^{c,z}(t)\}_{t\geq1;z=1,..,10;a=0,..,20;c=0,..,9}$ *the children distributions ; the technical progress process* $\{AI^z(t), AF^z(t)\}_{t\geq1;z=1,..,10}$ *and social security policies* $\{\bar{r}^a, \underline{r}^a, \kappa^z(t), \theta^z(t)\}_{t\geq1;z=1,..,10}$ *that satisfy (9), a competitive world-equilibrium with social security is a set of sequences for prices and* *social security transfers* $\{w^z(t), R(t)^z, p_I^z(t), p_f^z(t)\}_{t\geq 1; z=1,..,10}$, $\{R^\star(t), p^\star(t)\}_{t\geq 1}$, $\{P_a^z(t)\}_{t\geq 1; z=1,..,10; a\geq \underline{r}^a}$, *and an allocation of quantities* $\{H_T^z(t)\}_{t\geq 1; z=1,..,10}$,

$\{h_a^z(t)\}_{t\geq 1; a=T-9,..,T-3; z=1,..,10}$, $\{S_a^z(t), C_a^z(t), \}_{t\geq 1; a=a_0,..,20; z=1,..,10}$,

$\{K^z(t), \delta^z(t), YI^z(t), YF^z(t), X^z(t), B^z(t), M^z(t)\}_{t\geq 1; z=1,..,10}$ *such that the following equations are satisfied for each period* $t \geq 1$*:*

(i) Households maximization behaviour gives (3) – (5) – (6),

(ii) profit maximization of firms in intermediate sector gives (10) – (12) – (13), and (14) holds, for all z,

(iii) profit maximization in final good sector gives (15) – (16) – (17), for all z,

(iv) world producer profit maximization gives (19) and (20),

(v) markets are cleared at each date, ie the followings equations hold:

$$C^z(t) + I^z(t) = YF^z(t) \qquad \forall z \tag{22}$$

$$N^z(t) = \sum_{a=2}^{\bar{r}_a} \mathrm{e}_a^z(t) L_a^z(t) h_a^z \qquad \forall z \tag{23}$$

$$X^z(t) + B^z(t) = YI^z(t) \qquad \forall z \tag{24}$$

$$\sum_z M^z(t) = Y^\star(t) \tag{25}$$

where $C^z = \sum_{a\geq a_0} \tau_a^z L_a^z c_a^z$ is the aggregate consumption in region z. (22) is the equilibrium condition on the final good domestic markets in all regions, (23) is the intermediate goods market in all regions, (24) is the labor market in all regions and (25) is the equilibrium condition in the market for the world intermediate good.

4.2 Walras'law and additional accounting identities

It can be easily shown that these equations (22)–(25) together to the previous equilibrium equations are sufficient to describe the real equilibrium of the world economy. As a matter of fact the equilibrium of the world capital market does not appear in the previous definition because it is redundant here (i.e. *Walras' Law*). But because only relative prices are relevant to obtain equilibrium allocation one can also drop (or fix) one absolute price in the model. For calibration purpose we will choose that at each time the price of the intermediate good in the region "North America" will be set to one ($p_I^{US}(t) \equiv 1$), so all the value can be expressed in dollars in our model.

Notice that implicitly one can recover standard aggregate budget constraint of national accounts or equilibrium on world capital market from previous equilibrium equation. One can verify that the sum of budget constraint over all individuals (3) give an expression of C^z that can be substituted in (22) in order to obtain (together with PAYG constraint (9)):

$$p_f^z YF^z = w^z N^z + p_f^z[K^z + (1-\delta)K^z(-1)] + p_f^z[R^z S^z(-1) - S^z] \tag{26}$$

YI^z is linearly homogeneous then using the *Euler rule* and substituting prices for quantities in the equation, with (12) and (13), gives:

$$p_f^z YF^z - p_I^z YI^z = p_f^z[K^z - R^z K^z(-1)] - p_f^z[S^z - R^z S^z(-1)] \tag{27}$$

From (23) and given $p_f^z YF^z = p^\star M^z - p_I^z B^z$ (resulting from YF^z being linearly homogenous) we also have:

$$p_f^z YF^z - p_I^z YI^z = p^\star M^z - p_I^z X^z \tag{28}$$

one can then recovers the following accounting identities (for all z):

$$PIB^z(t) = p_I^z(t) YI^z(t) = p_f^z(t) YF^z(t) + p_I^z(t) X^z(t) - p^\star(t) M^z(t) \tag{29}$$

where : $p_I^z(t)X^z(t) - p^\star(t)M^z(t)$ is the trade balance of region z expressed in units of the domestic final good.

Equilibrium of the world financial market :

Let now show that at the equilibrium the world capital market is equilibrated. Competitive behaviour of the World producer and $Y^\star$ being linearly homogenous gives to us $Y^\star(t) = \sum_z \frac{p_I^z(t)}{p^\star(t)} \cdot X^z(t)$. This together with the world market equilibrium for the world intermediate good (25) implies that the sum of trade balance over the world is equal to zero:

$$\sum_z [p_I^z(t) X^z(t) - p^\star M^z(t)] = 0 \tag{30}$$

Then summing identities (27) over the regions implies (knowing from (28) that this is equivalent to (30): $\sum_z p_f^z[K^z - R^z K^z(-1)] = \sum_z p_f^z[S^z - R^z S^z(-1)]$. Then once the world market is cleared at time $t-1$ this implies that it also clears at time t:

$$\sum_z p_f^z K^z = \sum_z p_f^z S^z \tag{31}$$

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