ECONOMIC GROWTH AND OPTIMAL INCOME TAX EVASION

Oscar Mauricio Valencia[1]
Directorate of Economic Studies
National Planning Department
and
National University of Colombia
Department of Economics

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Abstract

We analysed the relationship between economic growth and income tax evasion. For this purpose we constructed a dynamic model with human capital in which income tax evasion was endogenized. The model captured the effect of income tax evasion on economic growth through three channels: 1) evasion alters the optimal path of consumption and savings 2) income tax evasion generates labour market distortions; 3) returns on assets are affected when tax evasion occurs.

The concept of optimal policy against evasion was introduced. Based in the Ramsey policy approach, we reformulated the Ramsey problem including income tax evasion. We found that optimal taxes and fines are determined by the public provision supply level, which lead to changes to optimal level of income tax evasion.

The model was calibrated for 2000 Colombian economy. Computable experiments show that different enforcement policies based on an increased probability of detection and punishment have a positive impact on welfare and growth. On the other hand, as income tax evasion increases so the capital cost goes up, the labor supply is reduced and economic growth and welfare decreases.

JEL classification code: H26, 041.

Keywords: Income Tax Evasion, Endogenous Growth and Optimal Tax Policies.

1 Introduction

The consecutive implementation of the income tax reforms have been the constant in the Colombian tax policy in the last three decades. The reforms have been characterized by increasing marginal rates, exemptions and changes in the taxable base [see Calderón and Gonzaléz 2002]. The implications of the above tax reforms have been traduced to lower income tax collections, and increasing the income tax evasion. Figure 1 depicts that fact[1]. This graph shows the relation between the growth of collections and the income tax evasion rate, and also points out a sharp negative relation between them. Moreover, in the periods when the reform has been implemented, the evasion rate increases and therefore the collection level falls.

In this line, when we calculate the tax productivity[2] for income tax rate, we found that tax productivity growth for the years 1998-1991, for firms was 57% while for households was 33%. This situation contrast with the performance of tax productivity in the last years when downturn to 32% and 23% for firms and households respectively. This behaviour of tax productivity is associated with economy activity and tax burden. An interesting stylized fact showed a relationship between economic growth and income tax evasion. Graph 2 illustrated this fact, we can see while in periods when income tax evasion increased the GDP growth decreased. Between 1998-1994 the GDP growth was in average 4.35% while the income tax evasion was 30%. Recently this tend is reversed, the income tax evasion growth in average 35% and the GDP growth was 1.5%.

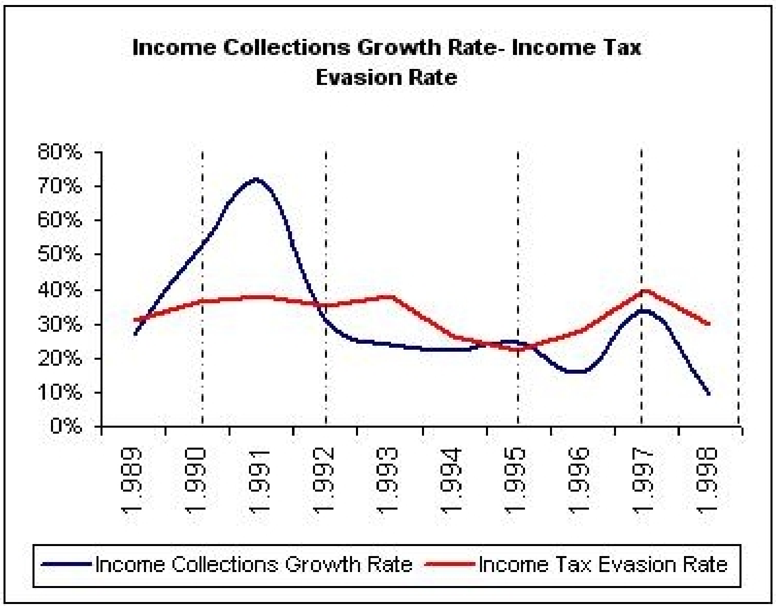
These stylized facts suggest the following questions: 1) What is the theoretical relationship between income tax evasion and economic growth ?. 2) How does implemented optimal tax policy against income tax evasion. 3) What are the quantitative impact of income evasion on welfare and economic growth?

[1] The point-marked line traces the periods of income tax reforms

[2] According with Steiner and Soto (1988), the tax productivity (TP) is measuring as follow:

$$TP = \frac{Income\ Collections}{GDP * Tax\ Rate}$$

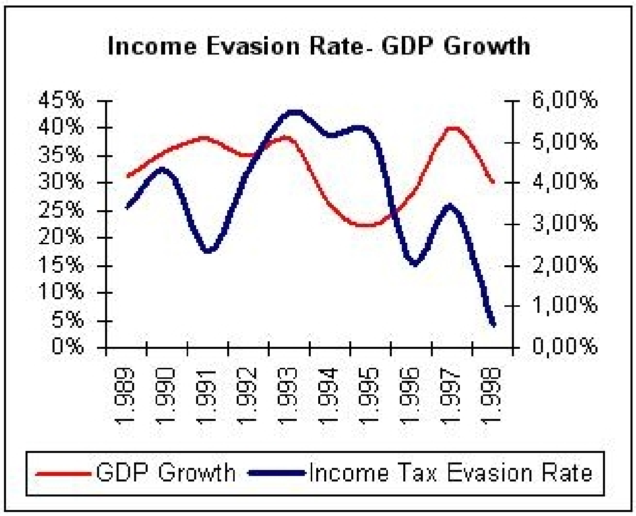
Graph 1



Source: DIAN, Steiner and Soto (1998), Avendano (2003).

To answer the above questions, we constructed an analytical-computable dynamic model with human capital for explaining the relationship between income tax evasion and economic growth. The model is constructed in such way as to characterized the channels of transmissions in which income tax evasion affects economic growth. Specifically, three channels are highlighted: *Evasion and Intertemporal consumption,* which shows that the income tax evasion today implies that taxes will increase in the future, which leads to a distortion in intertemporal savings decisions. The second channel is called *Evasion and labour decision*, which show us how income tax evasion alters labour and leisure decisions, and the third is *Evasion and accumulation process*, which studies the effects of income tax evasion on returns on physical and human capital. In this sense this model captures how income tax evasion leads through these channels to changes in the economic growth rate.

Graph 2



Source: DIAN, Steiner and Soto (1998), Avendano (2003).

For second question, the concept of optimal policy against evasion was introduced. Based in the Ramsey policy approach, we reformulated the Ramsey problem including income tax evasion. We found that optimal taxes and fines are determined by the public provision supply level, which lead to changes to optimal level of income tax evasion.

Theoretical model is calibrated for the 2000 Colombian economy for insights on quantitative impacts of income tax evasion and enforcement policies. The computable experiments showed that different enforcement policies based on increases in the probability of detection and punishment on being caught have a positive impact on welfare and growth. On the other hand, as income tax evasion increases, so the capital cost goes up and the labour supply goes down, resulting in a reduction in welfare and economic growth.

The outline of this paper is as follows. Firstly we review related literature, then in the second part we describe the model, and in the third we characterized the optimal tax policy against income tax evasion. In the fourth part we carry out computational experiments. The last section contains our concluding remarks

2 Related Literature

A large body of economic literature studies tax evasion as a risk decision. Allingham and Sandmo [1972] is a first contribution to the theory in this respect. This literature considers that the decision to evade taxes is based on uncertain behaviour. Agents may receive exogenous income and maximise the expected utility taken, given a probability of detection and fine. Optimal decisions by households are influenced by government policies. When the government changes the income tax rate, the probability of detection and the penalty rate, households alter the optimal consumption and investment decisions [Fullerton and Karayannis (1993)] . Srinivasan [1973] and Yitzhaki [1985] studies income tax evasion with risk neutrality. In these models there is no tax evasion if the expected penal tax rate is at least as high as the statutory tax rate. Furthermore, an increase in the probability of being caught or in the penal tax rate lowers tax evasion. Landskroner, Paroush, and Swary [1990] looked at tax evasion under uncertainty with risky assets. In these extensions, the same comparative static results, survive as long as there is increasing absolute risk. All these papers use static partial equilibrium models with exogenously given income.

Other papers study the relationship between tax evasion and public spending. Cowell and Gordon [1988] analyse the relationship between public goods supply and tax evasion. This paper shows the effect of public goods on tax evasion. Falkinger [1991] studied how evasion affects the optimal supply of public goods. Slemord [1994] analysed the impact of income tax avoidance on the optimal taxation context. Borck [2003] studied a voting model on linear income tax which is redistributed as a lump-sum to taxpayers.

Following the same structure as Allingham and Sandmo, Penades and Caballe [1997] presented a dynamic model with evasion, studying the impact of the independent auditing process. He found that the growth effects of policy enforcement depend on public and private capital productivity. Therefore, when policy enforcement was based on high tax and penalty rates, this could lead to reduced economic growth.

Roubini and Sala-i Martin [1995] showed that in countries where tax evasion is substantial, the government chooses to increase seigniorage by repressing the financial sector and increasing the inflation rate. The consequences of repressing the financial accelerator translate into lower growth rates. Ho and Yang [2002] integrated a Persson- Tabellini model into an economic growth context. They constructed a model where the agents differed not only in income levels but also in terms of skills in concealing income from the tax authority. They found that a higher tax rate coupled to tax evasion led to a drop in the redistributive benefit and enhanced the distortionary cost of taxation in the margin; a higher tax rate therefore led to a lower redistribution level.

Chen [2003] constructed an endogenous economic growth model with public capital and tax evasion. He studied the relation between the impact of tax evasion on economic growth and public capital externality through income revenues. He found that an increase in the unit cost of both the tax evasion and punishment - fines- reduces tax evasion, while an increase in tax auditing reduced tax evasion only if the cost of enforcement was not too high. The empirical results showed that differing policy enforcement has a positive impact on reducing tax evasion, but in terms of economic growth the effects are ambiguous. A similar framework is used by Atolia [2003], who constructed a dynamic overlapping generation model with tax evasion, where the government revenue is used to provide public capital.

3 The Model

Following the Penades and Caballe [1997] and Chen [2002] approach, we constructed a dynamic general equilibrium model for explaining the relationship between economic growth and evasion. Evasion is introduced as part of the optimisation problem of economic agents which choose how much tax to evade, depending on the probability of being caught and punished, to maximize expected utility over time. On the other hand, the tax authorities choose income tax in such as way that revenues are maximized. For simplicity, the probability and the fine are taken as being exogenous to taxpayers and collectors.

3.1 Economic Environment

3.1.1 Preferences and Technologies:

The economy is populated by a large number of identical, infinitely-lived households. All households own the same stock of physical-capital claims in period 0, $k0 > 0$ that they can buy or sell in a free capital market. Each household derives utility from the consumption of a single final consumable good, over the infinite horizon. Households preferences are additively- separable function of consumption (c_t), leisure (l_t) and public goods (η_{ct}).

$$U = \sum_{t=0}^{\infty} \beta^t u \left(c_t, l_t, \eta_{ct}\right) \tag{1}$$

where $\beta^t \in (0, 1)$ represents the discount rate for intertemporal consumption, η_{ct} $= g_{ct}/c_t$ is the proportion of public goods that enters into utility function. The preferences are represented by the following CES-CRRA type function:

$$U(c_t, l_t, g_t) = \ln\left[c_t^{\theta}\left(1 - l_t\right)^{1-\theta}\right] + \lambda \ln\left(\eta_{ct}\right)$$

with $\lambda > 0$ and $\theta \in (0, 1]$.

3.1.2 Techology

Output is produced using only labour and capital, according to the Cobb-Douglas function with constant return to scale:

$$Y_t = k_t^{\alpha}\left(h_t n_t\right)^{1-\alpha} \tag{2}$$

where $0 < \alpha < 1$, h_t denotes the exogenous qualification level of the representative agent and n_t is a non- leisure time which could be used in the production of consumption goods and accumulation of the human capital.

Under taxation, the taxpayer declares income $\phi \in [0, 1]$ as a share of total income. Let τ be the income tax rate; therefore, the income reported by taxpayers to the tax collector is $\tau\phi Y$. Tax evasion involves a transaction cost (see Cowell (1990), for example lawyers' and accountants' fees. Generally, the transaction cost increases monotonically with tax evasion and income. In this sense, following Chen [2002], we assume that the transaction cost for tax evasion is $\omega_0 (1-\phi)^\gamma$with $\gamma > 1$. $0<\omega_0 < 1$ which reflects the level fixed cost.

When tax evasion occurs, the tax collector could be auditing and detecting evasion events. Let (p) be the probability that the taxpayer is discovered and caught by the tax authorities. The taxpayer who evades tax therefore stands a chance that his evasion will be a success, in which case the level of consumption increases, or alternatively, a chance of being caught and punished. In this case, the fine $(\chi > 1)$ is proportional to the income evaded. Hence, the disposable expected income when the taxpayer evades and is detected by tax collectors is $p\left((1-\phi_t\tau_t) - \omega_0 (1-\phi_t)^\gamma - \chi_t\tau_t (1-\phi_t)\right) Y_t$. Symmetrically, when the taxpayer evades and the tax authorities do not detect the evasion, disposable expected income is $(1-p)\left((1-\phi\tau_t) - \omega_0 (1-\phi)^\gamma\right) Y_t$. The expected income for the average taxpayer is:

$$Y_t^d = p\left((1-\phi_t\tau_t) - \omega_0 (1-\phi_t)^\gamma - \chi_t\tau_t (1-\phi_t)\right) Y_t + (1-p)\,(1-\phi_t\tau_t) - \omega_0 (1-\phi_t)^\gamma) Y_t \tag{3}$$

We define the income tax evasion rate as $e_t = 1-\phi :$ re-writing equation 1 in terms of the tax evasion rate, the expected income is:

$$Y_t^d = p\left((1-\tau(1-e_t)) - \omega_0 e_t^\gamma - \chi_t\tau_t e_t\right) Y_t + (1-p)\left((1-\tau_t(1-e_t)) - \omega_0 e_t^\gamma\right) Y_t \tag{4}$$

Simplifying expression 2, we can obtain:

$$Y_t^d = (1 - \tau_t\,(e_t(1-p\chi) + 1 - \omega_0 e_t^\gamma)\, Y_t \tag{5}$$

With $p\chi_t < 1$.

3.1.3 Endogenous Growth

We would like to consider the implication of income tax evasion on economic growth, for this we assume that source of growth is a non-convexity of Learning by Doing following the Arrow and Romer Tradition. The fundamental idea is based on the human capital generates spillovers effects on the qualification of labour economic activities. The positive effect of experience is captured every period and accumulated into knowledge capital. The human capital accumulation is determined by the following transition equation:

$$h_{t+1} = (1 - \delta_h) h_t + i_t^h$$

where $\left(h_t, \delta_h, i_t^h\right)$ are the human capital stock, depreciation, and investment level. The human capital growth rate is determined by the amount of work effort devoted to production:

$$\upsilon_t^h = \frac{h_{t+1}}{h_t} = (1 + \Delta_0 n_t)$$

3.2 The Firm Problem

A Representative firm solves a static problem, which is determined by choosing the optimal demand for labor and capital:

$$\max \Pi_t = Y_t - w_t (1 - l_t) h_t - r_t k_t \tag{6}$$

subject to

$$Y_t = k_t^{\alpha} \left((1 - l_t) h_t\right)^{1-\alpha} \tag{7}$$

The first order conditions are:

$$r_t = \alpha k_t^{\alpha-1} \left((1 - l_t) h_t\right)^{1-\alpha} \tag{8}$$

$$w_t = (1 - \alpha) \left(\frac{k_t}{(1 - l_t) h_t}\right)^{\alpha} \tag{9}$$

Factor prices are expressed in terms of the number of effective labour hours and capital purchased by the firm form market.

3.3 Household Problem

Representative household solves the following problem:

$$\max_{\{c_t,e_t,n_t,k_{t+1},h_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \left\{ \ln \left[c_t^{\theta} \left(1-l_t\right)^{1-\theta} \right] + \lambda \ln \left(\eta_t\right) \right\} \tag{10}$$

subject to:

$$\begin{aligned}
c_t + i_t^k + i_t^h &= \left(1-\tau_t \left(e_t(1-p\chi)+1-\omega_0 e_t^{\gamma}\right)\right)\left(w_t(1-l_t)h_t + r_t k_t\right) && (11)\\
k_{t+1} &= (1-\delta)k_t + i_t^k && (12)\\
h_{t+1} &= \left(1-\delta_h\right) h_t + i_t^h && (13)\\
n_t + l_t &= 1 && (14)\\
& h_{0,}k_0 \text{ are given} && (15)
\end{aligned}$$

This problem could be seen as a dynamic mathematical programming problem where the state variables are (k_t, h_t) and the control variables are given by $(c_t, e_t, n_t, k_{t+1}, h_{t+1})$. The Bellman equation for this problem is:

$$V(k,h) = \left\{ \begin{array}{c} \max_{k',h',n,e} \ln \left[\begin{array}{c} (1-\tau\left(e(1-p\chi)+1-\omega_0 e^{\gamma}\right)\left(w(n)h_t + rk\right) + \\ (1-\delta)k + \left(1-\delta_h\right)h - k' - h' \end{array} \right]^{\theta} (1-h)^{1-\theta} \\ + \ln\lambda\left(\eta\right) + \beta V(k',h') \end{array} \right\}$$

Where (x') denote the variables in the next period. Using the first order conditions and the Envelope Theorem yields:

$$\frac{1}{c_t} = \beta \left\{ \frac{1}{c_{t+1}} \left[\left(1-\delta_k\right) + \left(1-\tau_t\left(e_t(1-p\chi)+1-\omega_0 e_t^{\gamma}\right) r_t\right] \right\} \tag{16}$$

$$\frac{\theta}{c_t} w_t \left(1-\tau_t\left(e_t(1-p\chi)+1-\omega_0 e_t^{\gamma}\right) = \left(\frac{1-\theta}{1-n_t}\right) \tag{17}$$

$$\left(1-\tau_t\left(e_t(1-p\chi)+1-\omega_0 e_t^{\gamma}\right)\left[w_t\left(1-l_t\right) - r_t\right] = \delta_h - \delta_k \tag{18}$$

$$\left(\tau_t\left(1-p\chi_t\right)-\gamma\omega_0 e_t^{\gamma-1}\right)=0 \tag{19}$$

$$\lim_{t\to\infty}\prod_{i=0}^{T-1}\left(\frac{1}{r_i}\right)k_{T+1}=0 \tag{20}$$

$$\lim_{t\to\infty}\prod_{i=0}^{T-1}\left(\frac{1}{r_i}\right)h_{T+1}=0 \tag{21}$$

Equation 16 is the Ramsey-Keynes rule which describes a necessary condition that has to be satisfied in the optimal path. The household equals the marginal cost of quitting one unit of consumption today to convert it into tomorrow's capital ($u'(c_t)$) with the benefit of this plan $\beta\left[u'(c_{t+1})\left(\frac{\partial(y_t^d)}{\partial k_t}+(1-\delta^k)\right)\right]$. Note that evasion reduces expected future consumption given that it reduces the marginal productivity of capital. These effects capture the fact that evasion alters the allocation of resources at intertemporal level.

Equation 17 shows the optimal decision between labour and leisure by the representative agent. This equation equates the marginal benefit of one additional unit of employment (n_t) to the marginal cost of supplying that additional unit of employment (n_t). Equation 17 shows the distortion generated by income tax evasion on labour allocations.When income tax evasion increases, real wages decrease hence leisure as a normal good increases. An rise in the income tax rate therefore, reduces the labour supply but increases the level of evaded income.

Equation 18 shows the non-arbitrage condition between fiscal capital and human capital, note that the expression is adjusted by the amount of income evaded and depreciation for each capital. Intuitively, this equation analyses the optimal decision between two assets by a representative household. Therefore, in equilibrium the return on each type of capital is equal. This implies that wages are equal to the return on capital in effective terms.

Similarly, the Euler equation for tax evasion (see equation 19) implies:

$$e_t^*=\left[\frac{\tau_t(1-p\chi)}{\gamma\omega_0}\right]^{1/\gamma-1}\geq 0 \tag{22}$$

Equation 19 shows the optimal tax evasion chosen by households. Optimal tax evasion is positive with respect to income tax, and negative with respect to the probability of detection and fines.

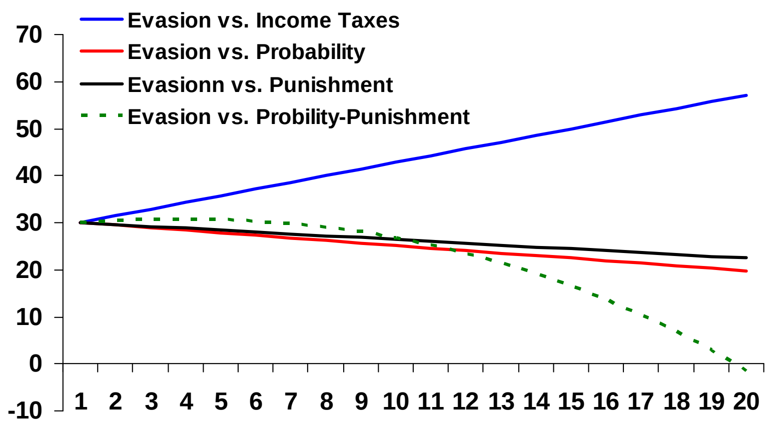
$$\frac{\partial e}{\partial p} = \frac{-\tau\chi}{(\gamma-1)\,\gamma\omega_0}\left[\frac{\tau(1-p\chi)}{\gamma\omega_0}\right]^{2-\gamma/\gamma-1} < 0$$

$$\frac{\partial e}{\partial \chi} = \frac{-\tau p}{(\gamma-1)\,\gamma\omega_0}\left[\frac{\tau(1-p\chi)}{\gamma\omega_0}\right]^{2-\gamma/\gamma-1} < 0$$

$$\frac{\partial e}{\partial \tau} = \frac{(1-p\chi)}{(\gamma-1)\,\gamma^2\omega_0^2}\left[\frac{\tau(1-p\chi)}{\gamma\omega_0}\right]^{2-\gamma/\gamma-1} > 0$$

On the other hand, when evasion has a zero value, the fine is the inverse of probabilities, i.e $\left(\chi = \frac{1}{p}\right)$. This implies that when the optimal value for evasion equals zero, the fine decreases as probabilities rise. As depicted in graph 3, evasion increases when income taxes rise. On the other hand, evasion falls when the probability of getting caught and being fined increases. This result contradicts the Allignam-Sandmo Paradox, in which the impact on evasion is ambiguous when income taxes increase [see Miles 1995].

Graph 3



3.4 Government and Tax Collector

We assume that there no exist a decentralised tax authority. The government spends a fixed quantity for own consumption, which is financed only by source income taxes. For collection activities, the government uses different enforcement instruments, penalities and taxes, the probabilities of detection are exogenous. Let $\pi = \{\tau_t, \chi_t\}_{t=0}^{\infty}$ be a tax policy consisting of an infinite sequence of income taxes, fines.

We assume that government owns a collection technology, as suggested by Roubini and Sala-i- Martin [1992]. In their paper the collection technology relates reported income to actual income. With tax evasion, the marginal change in the actual income is less than the marginal change in reported income. The functional form of collection technology is:

$$\phi = \frac{\nu_0}{(\tau_t)^{1-\varepsilon}} \tag{23}$$

where $\nu_0 \in [0,1]$ and $\varepsilon \in [0,1]$ represents parameters that relate to the extent that taxes are avoided.. ν_0 is a parameter that reflects of the efficiency of tax authority. ε is the elasticity of reported with respect to actual income. Under high income tax evasion, the government has poor technologies for collecting taxes, which is illustrated in low values of ν_0 and ε , while high values reflect efficiency in tax authority activities. Note when $\nu_0 = 1$ and $\varepsilon = 1$ we have all income reported; on other hand, when the income reported government revenues are:

$$R_t = \tau_t \phi Y_t + p\chi_t \tau_t (Y_t - \phi Y_t) \tag{24}$$

where R_t is the level of government, revenues which is equal to the revenues from reported income plus the income from fines of the taxpayer who evades. Replace equation 20 into 21, we obtain:

$$R_t = \tau_t Y_t \left[\nu_0 \tau_t^{\varepsilon-1} (1 - p\chi_t) + p\chi_t\right] \tag{25}$$

Note when $\varepsilon \to 1$ and $\nu_0 \to 1$ the revenues are $R_t = \tau_t Y_t$. Symmetrically when $\varepsilon \to 0$ and $\nu_0 \to 1$, the revenues proportional to the level of punishment and collected income taxes. We assume that the government sets the level of public consumption as a share of consumption. According to this, the government constraint is:

$$g_t = \eta_{ct} c_t \leq \tau_t Y_t \left[\nu_0 \tau_t^{\varepsilon - 1} (1 - p\chi_t) + p\chi_t \right] \tag{26}$$

The last expression shows the rule for public goods provision. In this case, choosing a tax policy is equivalent to choosing an entire fiscal policy.

3.5 Competitive Equilibrium

Definition 1 *Given a tax policy π and a sequence of government expenditure $\{g_t\}_{t=0}^{\infty}$, a competitive equilibrium in this economy is a sequence of individual allocations $\{c_t, n_t, k_{t+1}, h_{t+1}, e_t\}_{t=0}^{\infty}$, production plans $\{k_t, h_t\}_{t=0}^{\infty}$ and relative prices $\{r_t, w_t\}_{t=0}^{\infty}$, such that:*

- *Given $\{r_t, w_t\}_{t=0}^{\infty}$ and π the firms problem is solved, i.e (6-7) are satisfied*
- *Given $\{r_t, w_t\}_{t=0}^{\infty}$ and π the household problem is solved, i.e (10-15) are satisfied*
- *Factor markets clear:*

  $n_t = N_t$

  $k_t = K_t$

  $g_t = G_t$
- *The government budget constraint (26) is satisfied.*
- *Feasibility condition $C_t + K_{t+1} - (1 - \delta) K_t + G_t \leq Y_t^d$ is satisfied for all t.*

Definition 2 *Steady- state for the economy is defined as competitive equilibrium such that for all t, $\frac{c_{t+1}}{c_t} = c^*$, $\frac{n_{t+1}}{n_t} = n^*$, $\frac{k_{t+1}}{k_t} = k^*$, $\frac{e_{t+1}}{e_t} = e^*$, $\frac{h_{t+1}}{h_t} = h^*$ grow at a constant rate. Let υ^h be this rate*

$$\upsilon^h = c^* = n^* = k^* = e^* = h^* = (1 + \Delta_0 n_t)$$

Proposition 3 *If* $\frac{1-\tau_t}{\tau_t} > e_t(p\chi_t - 1) - \omega_0 e_t^\gamma$ *then there is a unique competitive equilibrium with tax evasion.*

Proof. *See technical appendix.* ■

The above proposition shows that the optimal path of capital accumulation is choosing if $\frac{1-\tau_t}{\tau_t} > e_t(1 - p\chi_t) - \omega_0 e_t^\gamma$ is achieved. This expression relates the benefits derived from income tax evasion (right side) to the ratio between the share of income not subject to tax burden (left side). Note that the benefits are expressed as the cost for households of being caught and punished and the transaction cost for tax evasion, which that is : $e_t p\chi_t + \omega_0 e_t^\gamma$. The share of income which is evaded is e_t therefore, the benefits derived of income evasion are: $e_t - (e_t p\chi_t + \omega_0 e_t^\gamma) = \Xi_e$.

Note that this condition is very important because it guarantees positive allocations of consumption and physical and human capital. Intuitively, if the tax burden is very high, the rental price of capital increases and if the rental price of capital is more than depreciation, the intertemporal allocation of consumption is positive.

It is important to highlight the fact that the condition for there to be equilibrium is simply a feasibility condition. Note that income tax payments without evasion plus income tax payments with evasion are less than income, that is $Y_t \leq Y_t\tau_t\Xi_e + Y_t\tau_t$.

4 ¿How Does Income Tax Evasion Affect Economic Growth?

To understand the effects of income tax evasion on economic growth we distinguished different channels of transmission. Specifically, we described the following channels:

- *Income Tax Evasion affects the optimal path of consumption and accumulation.*

  According to the Ramsey- Keynes rule (see equation 16) income tax evasion produces an increase in the income tax rate in the future. This is because the budget constraint on the government has been achieved in the long term.When tax evasion occurs, disposable income rises, which implies that present consumption could be higher than without tax evasion, but in the future this implies that the rental capital price increases because the future tax burden rises. In fact, the marginal product of capital goes up, which reduces the capital stock in equilibrium. Therefore we have two effects: income and substitution effects. The first is because income tax evasion produces an increase in disposable income. On the other hand, if current income tax evasion occurs this is equivalent to reducing future consumption, which decreases the optimal saving.

- *Income Tax Evasion affects the labour supply and therefore the optimal labour-leisure choice.*

  A with the first channel, the effects of income tax evasion could be described as the substitution effect, namely the optimal decision leisure and labour. Equation (17) describes this choice; note that when the income tax rate rises, the representative households substitute labour by leisure, and that income tax evasion amplified this effect, and hence the labor supply is reduced. Intuitively, households analyse other occupational choices when these offer greater opportunities for evasion than others that achieve the same intertemporal utility level.

  The income effect is when income tax rate rises the disposable income is reduced. For that representative household achieved the same level of utility, can it should offer more hours' work or evade more income.

- *Income Tax Evasion produces changes in the human accumulation process.*

  In our economy, the total income is generated by labour and physical and human capital. The labour income is determined by the level of human capital accumulation. The income tax rate is applied to two types of income. As remarked in equation (18), as income tax rate increases, income tax evasion rises, which leads to cost of physical and human capital are increasing and therefore the investment in equilibrium for each type of capital falling.

5 Ramsey Policies and Optimal Policy Enforcement.

The government faces an optimization namely choosing the tax-enforcement policy that will maximise welfare and revenues. This problem is similar to the Ramsey taxation approach [1927] [3]. The equations in that show that competitive equilibrium is a function of tax policy represented by a sequence of income taxes and punish of caught. This means that under a tax- enforcement policy, allocations of goods and factors that reflect the optimal behaviour of economic agents with respect to tax-enforcement policy should be feasible. In this sense, as presented in Stokey and Lucas[1983], Lucas [1990], and Sargent [2001] , the government problem is choosing the tax policy and allocations such that will maximise the present value of representative agents, subject to the government budget constraint and competitive equilibrium conditions. This implies, as remarked by Manuelli and Rosi [1993], that given a path of allocations, "the prices and tax policy can be reconstructed using the conditions describing competitive equilibrium"[Manuelli, Jones and Rossi page 489].

Definition 4 *Given relative prices* $\{r_t, w_t\}_{t=0}^{\infty}$*, a Ramsey Policy is a tax policy* $\pi \equiv \{\tau_t, \chi_t\}_{t=0}^{\infty}$ *such that given probabilities of detection* $\overline{p}$*, government solves the the following problem:*

$$\max_{\pi} V = \sum_{t=0}^{\infty} \beta^t u\left(c_t, l_t, \eta_{ct}\right)$$

subject to:

$$\frac{1}{c_t} = \beta\left[\frac{1}{c_{t+1}}\left((1-\delta) + (1-\tau_t\left(e_t(1-p\chi) + 1 - \omega_0 e_t^{\gamma}\right) r_t\right)\right]$$

[3] The original problem posed by Ramsey was:

"The problem I propose to tackle is this: a given revenue is to be raised by proportionate taxes on some of or all uses of income, the taxes on different uses being possibly at different rates; how should these rates be adjusted in order that the decrement of utility may be a minimum? [Ramsey, 1927, p.47].

$$\frac{\theta}{c_t} w_t \left(1 - \tau_t \left(e_t(1-p\chi) + 1 - \omega_0 e_t^{\gamma}\right) = \left(\frac{1-\theta}{1-n_t}\right)\right.$$

$$e_t = \left[\frac{\tau_t(1-p\chi)}{\gamma\omega_0}\right]^{1/\gamma-1}$$

Definition 5 *A Ramsey allocation is a set of individual allocations* $\Gamma \equiv \{c_t, n_t, h_{t+1,} k_{t+1}, e_t\}_{t=0}^{\infty}$ *which is according with a Ramsey Policy.*

Definition 6 *A Ramsey problem is a Ramsey Policy and Ramsey allocation which satisfies any competitive equilibrium.*

In order to analyse this problem, we followed the primal approach [see Atkinson and Stiglitz (1980), Jones, Manuelli and Rossi (1997), and Sargent (2001)]. The primal approach is very useful for classifying the Ramsey problem as Ramsey allocations and Ramsey policy. The following proposition describes the set of Ramsey allocations and the optimal Ramsey policy that characterised the optimal policy enforcement.

Lemma 7 *Any Ramsey allocation that solves the household problem should satisfy the following implementability constraint:*

$$\begin{aligned} u_c(0)c_0 - u_n(0)\, n_0 h_0 \quad &= \quad u_c(0)\left[(1-\delta_k) + r_0\left(1 - \tau_0\left(e_0(1-p\chi_0) + 1 - \omega_0 e_t^{\gamma}\right)\right]k_0\right. \\ &\quad +h_0\left[(1-\delta_h) + \left(1 - \tau_0\left(e_0(1-p\chi_0) + 1 - \omega_0 e_0^{\gamma}\right) w_0\right]\right. \end{aligned}$$

Proof. *See technical appendix.* ■

5.1 Complete and Incomplete Taxation

In this subsection, we discuss the question raised in the introduction: How does implemented optimal tax policy against income tax evasion? As showed in this section, the problem could be approximate follow second best Ramsey approach. In the specifically case, the main characteristic is that a share of total income is not taxed, hence, the tax code is incomplete. Note, the share of income not taxed is endogenously determined by household optimal behaviour, therefore the incompleteness is endogenous.

As presented in following proposition, the Ramsey policy is determined in two cases: First, is when the income tax is zero which lead to zero income tax evasion, in this case the fine not apply. The second case is constraint for income tax values distinct to zero. The main result is that the optimal value of income tax is determined for level of public good supply in the economy. When public good supply rises, the tax burden falls, which leads to income tax evasion decreases. Similarly, optimal fine level is determined by public good supply and probability of caught. The effect of the public good supply depend on probability level. If the public good supply rises the effect over fine is ambiguous, but if the public good supply guarantees $\left(\frac{1+\eta_{c_t}}{2+\eta_{c_t}} = \frac{p}{\left(1-e_t^{\gamma-1}\right)\gamma\omega_0}\right)$, the fines tend to zero.

Proposition 8 *The Ramsey Policy with income tax evasion is given by :*

- *Complete Taxation:*

$$\tau_t^* = 0;\ \chi_t^* : n.d \Rightarrow e_t^* = 0$$

- *Incomplete Taxation:*

$$\tau_t^* > 0, \tau_t^* = \left(\frac{e_t^{\frac{\gamma}{1-\gamma}}}{1-e_t^{\gamma-1}}\right)\left(\frac{1+\eta_{ct}}{2+\eta_{ct}}\right) \Rightarrow e_t^* > 0$$

$$\chi_t^* = \frac{1}{p} - \left(1-e_t^{\gamma-1}\right)\gamma\omega_0\left(\frac{2+\eta_{ct}}{1+\eta_{ct}}\right) \Rightarrow e_t^* > 0$$

Proof. See technical appendix. ■

Proposition 9 *If the Ramsey allocation satisfies the following properties:*
i) implementability constraint :

$$\sum_{t=0}^{\infty} \beta^t \left[u_c\left(t\right) c_t - u_l\left(t\right) n_t\right] = u_c\left(t\right) \left\{ \begin{array}{c} u_c\left(0\right)\left[\left(1-\delta_k\right) + r_0\left(1-\tau_0\left(e_0(1-p\chi_0) + 1 - \omega_0 e_t^{\gamma}\right)\right] k_0 \\ +h_0\left[\left(1-\delta_h\right) + \left(1-\tau_0\left(e_0(1-p\chi_0) + 1 - \omega_0 e_0^{\gamma}\right) w_0\right] \end{array} \right\}$$

iii) marginal consumption and labour substitution rates:

$$\frac{1}{c_t} = \beta\left[\frac{1}{c_{t+1}}\left(\left(1-\delta\right) + \left(1-\tau_t\left(e_t(1-p\chi) + 1 - \omega_0 e_t^{\gamma}\right) r_t\right)\right]$$

$$\frac{\theta}{c_t} w_t\left(1-\tau_t\left(e_t(1-p\chi) + 1 - \omega_0 e_t^{\gamma}\right) = \left(\frac{1-\theta}{1-n_t}\right)$$

iv) optimal income tax evasion:

$$e_t = \left[\frac{\tau_t(1-p_t\chi_t)}{\gamma\omega_0}\right]^{1/\gamma-1}$$

v) period resource constraint:

$$C_t + K_{t+1} - \left(1-\delta\right) K_t + G_t \leq Y_t^d$$

vi) The government budget constraint (26) is satisfied.

$$g_t = \eta_{ct} c_t \leq \tau_t Y_t \left[\nu_0 \tau_t^{\varepsilon-1}\left(1-p\chi_t\right) + p\chi_t\right]$$

then there be tax policy π *and relative prices* $\{r_r, w_t\}_{t=0}^{\infty}$ *such that the Ramsey allocation is a competitive equilibrium.*

6 Calibration

The model is calibrated using DANE[4] National Account data for 2000. Specifically, we calculated the parameters using the first order conditions which are evaluated in the steady state and other quantitative parameters. The parameter values were chosen so that the model would replicate the data observed in Colombia. For calibration purposes, the model was transformed into effective unit terms terms. This approach follows studies about the tax incidence on economic growth for Colombia [see Suescún 2001, Fergunsson 2002].

The parameters are divided into the macroeconomic and the evasion blocks. Table 1 describes the main macroeconomic variables which are taken as benchmark values. These values are taken as a percentage of 2000 GDP.

6.1 Benchmark Parameters

Table 1

c/y	i/y	k/y	g/y
0.651	0.137	2.4	0.212

Table 2

β	α	δ	θ	λ
0.932	0.379	0.05	0.33	0.24

The consumption, investment and production values are consistent with DANE National Account Data for 2000. Income tax revenues are taken from DIAN[5] data for 2000. Using the first order conditions of representative agent, the human capital growth rate is $\Delta_0 = 0.042$. In the case of exogenous growth, the long term growth rate is 0.035. (k_0, I_0) are calibrated using the following equations:

$$k_0 = \frac{VK}{r + \delta}$$

[4] DANE: Colombian National Statistics Administration Department

[5] DIAN: Colombian National Tax and Customs Administration.

$$I_0 = (\Delta_0 + \delta)\, k_0$$

VK reflects the value of income capital for 2000, according to DANE National Account data.We take a capital rental price of 10%. The table 3 describes the main parameters for the evasion block:

Table 3

	Value	Source
Probability of detection (p)	0.11	DIAN
Income Tax Rate (τ)	0.20	DIAN
Punishment of Caught (χ)	1.50	Fullerton and Karayannis (1994)
Income tax evasion in the Benchmark (e_0)	33.7	DIAN
Cost evasion for Households $(\omega_0 e)^\gamma$	0.004	Chen (2002) and Author's Calculation
Cost of detection for Tax Authority (h_0/y)	0.002	Author's Calculation
Elasticity of Revenue (ε)	0.004	National Planning Department
Efficiency of tax authority (υ_0)	0.001	National Planning Department

We calculated the cost evasion for households and the tax authority following the first order condition for income tax evasion. Other parameters were taken as exogenous, according to DIAN data and other studies related with this.

7 Experiments and Results

Within this theoretical framework, we evaluated different enforcement policies in exogenous and endogenous growth environments. In both cases, we analysed successive increments of probability of detection, income taxes and fines. The first group of simulations showed the short term effects and transitional dynamic of macroeconomic variables when different experiments were carried out. The second group analysed the effects of the different tax evasion and policy scenarios on long term economic growth.

7.1 Exogenous Growth

7.1.1 *Changes in the Probability of Detection*

The results show a positive effect of increases probability of detection on growth. Graph 4 shows the positive response of output, capital accumulation and consumption. The quantitative effects oscillate around 0.2 percentage points and 1 percentage point during 15 years. This measure is equivalent to 0.05 percentage points per year when the probability of detection changes over an interval [0.11 to 1]. When the probability of detection increases, the current capital cost decreases, and the saving and capital accumulation go up. As theoretical ideas suggest, consumption and capital have substantially increased, by 0.1 and percentage points 0.21 year respectively.

7.1.2 *Changes in Fines*

On the other hand, the similar results are supported by positive changes in fines. When fines increase[6], the saving increases because the present of consumption value decreases for given income. Tax burden over the time is reduced when fines are increased (see Euler equation for consumption). The graph 5 shows these effects, namely the quantitative response to growth of output is 0.03 percentage points on average. Consumption and capital show a similar response as they rise by 0.06 and 0.18 percentage points per year as shown in the graph 5.

7.1.3 *Changes in Income Tax Evasion*

Graph 6 depicts income tax evasion scenarios which are characterised by two effects: Substitution and income. The substitution effect typically involves increases in the capital rental price, therefore capital accumulation decreases when the capital cost rises (Substitution effect). In contrast, if revenue is used for public output supply the level of welfare increases (Income effects). In the both cases, the results show that the positive negative on growth is around -0.06 percentage points per year for tax rate values between 20% and 100 %.

[6] Changes in fines are simulated as discrete intervals of 5 per cent.

7.2 Endogenous Growth

In order to understand the long run effects of income tax evasion and policies enforcement, we simulate different scenarios in a endogenous growth context. Specifically, the simulations scenarios are classifying in low, medium and high which considers different values for evasion, income tax and probability of detection.

7.2.1 *Changes in Probability of detection*

Table 4

Probability	Steady-State $\left(v_0^h\right)$
Benchmark (p=0.11)	3.58%
Low (p=0.20)	3.76%
Medium (p=0.40)	4.17%
High (p=0.90)	5.18%

The results are described in Table 4. The probability of caught has a positive effect over long-term economic growth, the results suggests that the changes of probability of detection has a positive effect on steady state growth rate of economy. The quantitative response show that the gains when the probability rises could be oscillate between 0.18 percentage points and 1.6 percentage points.

7.2.2 *Changes in Fines*

Table 5

Probability	Steady-State $\left(v_0^h\right)$
Benchmark (χ=1.5)	3.58%
Low (χ=1.64)	3.58%
Medium (χ=2.0)	3.60%
High (χ=2.5)	3.63%

As illustrated in the Table 5, the response of fines is not significantly sensitive, for different scenarios of rises fines the gains on economic growth is only 0.05 percentage points. The results is very intuitively because when the fines rises, the current consumption and disposable income fall. In contrast, the tax burden for the representative agent fall. The net effect in our simulation is a few response of saving and capital accumulation.

7.2.3 *Changes in Income Tax Evasion*

Table 7

Probability	Steady-State $\left(v_0^h\right)$
Benchmark (e^*=36%)	3.58%
Low ($e^* = 36\%$)	3.10%
Medium ($e^* = 50\%$)	2.42%
High ($e^* = 66\%$)	1.67%

As suggest theoretical model, the effect on economic growth play fundamental role in the performance, of macroeconomic variables, the results presented in table 7 show that when the evasion rises, the tax burden increases and the returns of physical and human capital descreases which lead to a reduction of economic growth rate in 2 percentage points when the income taxes is increased substantially.

8 Concluding Remarks

We constructed a dynamic computable model to explain the relationship between economic growth and evasion decisions by economic agents. The model established direct and indirect links between different enforcement policies and changes in the implicit income tax rate. The theoretical framework shows that optimal tax evasion is affected in proportional to the income tax rate and is inversely related to fines and the probability of detection.

The concept of optimal policy against evasion was introduced. Based in the Ramsey policy approach, we reformulated the Ramsey problem including income tax evasion. We show that optimal taxes and fines are determined by the public provision supply level, which lead to changes to optimal level of income tax evasion.

The main results show positive effects on welfare the probability of being caught and fines increases. In contrast, when the income tax rate goes up, the labour supply is reduced, income tax evasion rises and the tax burden as well.

This model could be improved with another framework. For example, income tax evasion could be analysed of the heterogeneous agents approach, where the agents have differing income levels and the intertemporal wealth distribution changes over time. The results could address the following question: how does income tax evasion affect the intertemporal income and wealth distribution?.

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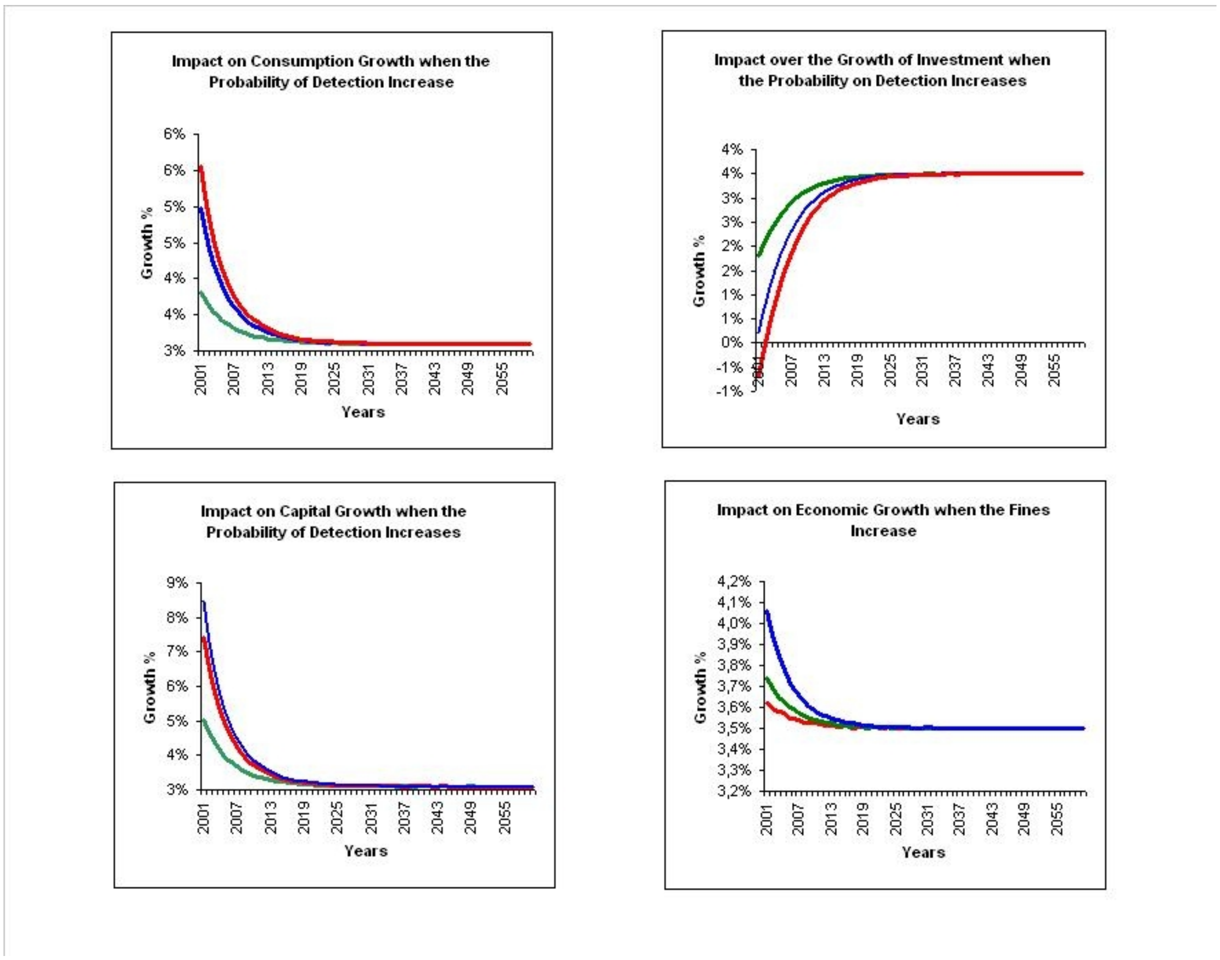
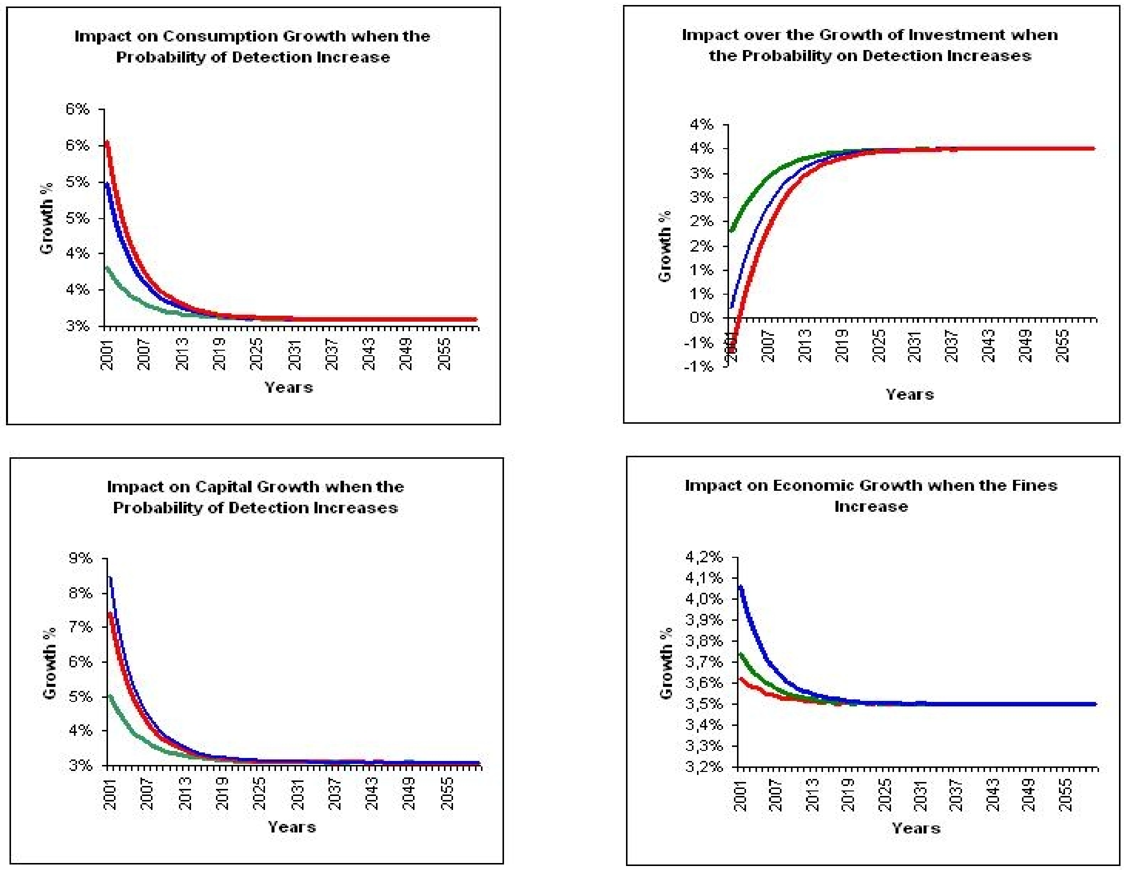
10 Graphics Appendix

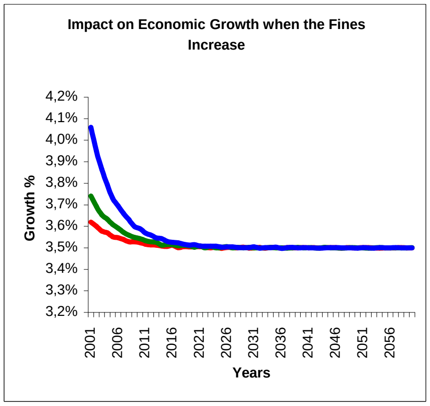
Graph 2

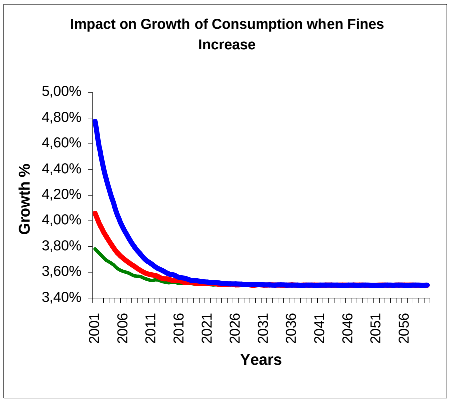


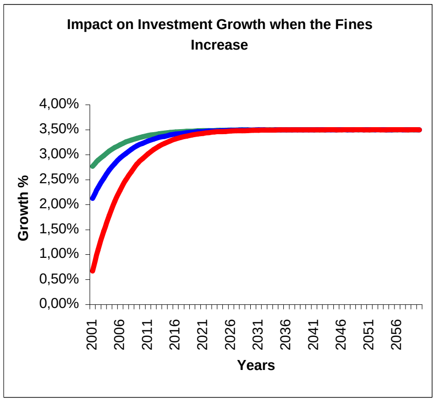


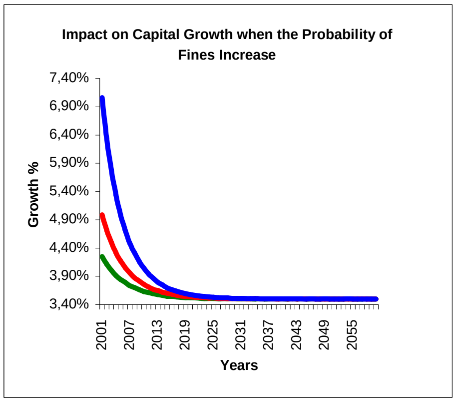
Graph 3

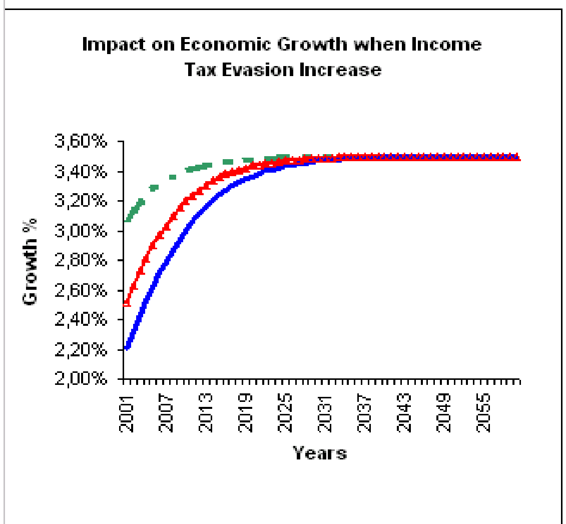
Graph 4

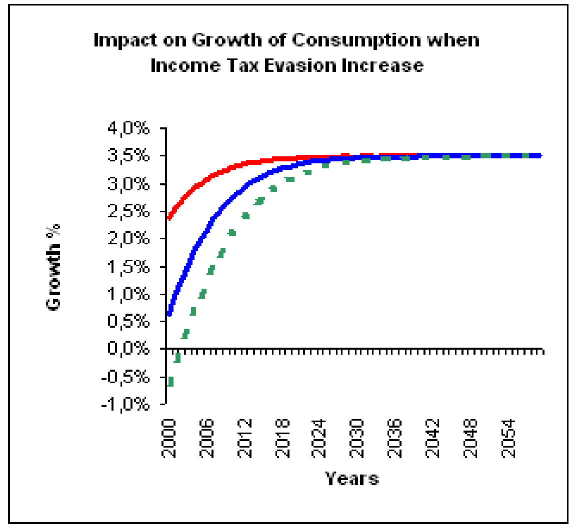

Impact on Economic Growth when the Fines Increase
Growth %
Years

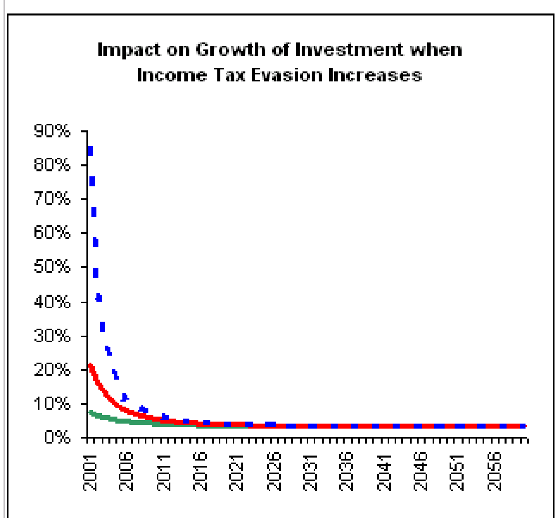

Impact on Growth of Consumption when Fines Increase
Growth %
Years

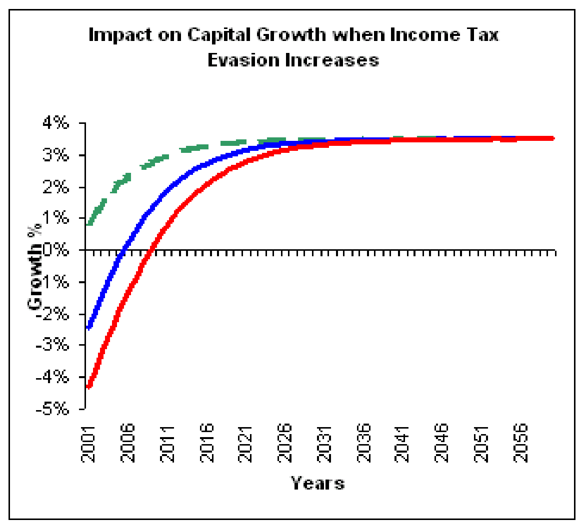

Impact on Investment Growth when the Fines Increase
Growth %
Years


Impact on Capital Growth when the Probability of Fines Increase
Growth %
Years


Impact on Economic Growth when Income Tax Evasion Increase
Growth %
3,60%
3,40%
3,20%
3,00%
2,80%
2,60%
2,40%
2,20%
2,00%
2001
2007
2013
2019
2025
2031
2037
2043
2049
2055
Years


Impact on Growth of Consumption when Income Tax Evasion Increase
Growth %
4,0%
3,5%
3,0%
2,5%
2,0%
1,5%
1,0%
0,5%
0,0%
-0,5%
-1,0%
2000
2006
2012
2018
2024
2030
2036
2042
2048
2054
Years


Impact on Growth of Investment when Income Tax Evasion Increases
90%
80%
70%
60%
50%
40%
30%
20%
10%
0%
2001
2006
2011
2016
2021
2026
2031
2036
2041
2046
2051
2056


Impact on Capital Growth when Income Tax Evasion Increases
Growth %
4%
3%
2%
1%
0%
-1%
-2%
-3%
-4%
-5%
2001
2006
2011
2016
2021
2026
2031
2036
2041
2046
2051
2056
Years

11 Technical Appendix

This appendix displays proof of the propositions made in the paper.

11.1 Proof of Proposition 3

Proof. We must prove that there is a level of capital k^*which satisfies 27 and that in $k^* = k$, consumption the level is not negative. On the other hand, we can show that a unique value exists for income tax evasion and tax income. Note that $\lim_{k\to 0} f'(k) > \left[\frac{\beta^{-1}-(1-\delta)}{1-\tau(e_t(1-p\chi_t)+1-\omega_0 e_t^\gamma}\right]$ and $\lim_{k\to\infty} f'(k) < \left[\frac{\beta^{-1}-(1-\delta)}{1-\tau(e_t(1-p\chi_t)+1-\omega_0 e_t^\gamma}\right]$.

Then for lower values of k, $\beta\left[(1-\delta)+f'(k^*)\left(1-\tau\left(e_t(1-p\chi_t)+1-\omega_0 e_t^\gamma\right)\right]>1$ and for higher values of k, $\beta\left[(1-\delta)+f'(k^*)\left(1-\tau\left(e_t(1-p\chi_t)+1-\omega_0 e_t^\gamma\right)\right]<1$. As $f'(\cdot)$ is a continous function , hence there exists k^*such that it is competitive equilibrium.

$$\beta\left[(1-\delta)+f'(k^*)\left(1-\tau\left(e_t(1-p\chi_t)+1-\omega_0 e_t^\gamma\right)\right]=1. \tag{27}$$

The objective is to show that k is the only value that satisfies 27. As it is a concave function, then $\beta\left[(1-\delta)+f'(k^*)\left(1-\tau\left(e_t(1-p\chi_t)+1-\omega_0 e_t^\gamma\right)\right]$ is a function decreasing in k, as shown above, there is just one value of k that satisfies 27. Now, we must prove consumption is not negative, that is $f'(k^*) > \delta k$, as $f'(k) = \left[\frac{\beta^{-1}-1+\delta}{\left(1-\tau(e_t(1-p\chi_t)+1-\omega_0 e_t^\gamma\right)}\right] > \delta$ then $f'(k) > \delta$ therefore consumption is positive. From equation 22 we have that $e_t^* = \left[\frac{\tau_t(1-p\chi)}{\gamma\omega_0}\right]^{1/\gamma-1}$ then given p^*, ω_0, h_0 there is a unique value of e_t^* because there is a single value that satisfies 27, therefore competitive equilibrium is unique. ■

11.2 Proff of Lemma 7

Proof. The household budget constraint for each period is:

$$c_t+k_{t+1}-\left(1-\delta^k\right)k_t+h_{t+1}-\left(1-\delta^h\right)h_t=\left(1-\tau_t\left(e_t(1-p\chi_t)+1-\omega_0 e_t^\gamma\right)\right)\left(w_t(1-l_t)h_t+r_t k_t\right) \tag{28}$$

in t=0 we can obtain:

$$c_0+k_1-\left(1-\delta^k\right)k_0+h_1-\left(1-\delta^h\right)h_0=\left(1-\tau_0\left(e_0(1-p\chi_0)+1-\omega_0e_0^{\gamma}\right)\right)\left(w_0(1-l_0)h_0+r_0k_0\right) \tag{29}$$

in t=1, the household constraint is expressed as follows:

$$c_1+k_2-\left(1-\delta^k\right)k_1+h_2-\left(1-\delta^h\right)h_1=\left(1-\tau_1\left(e_1(1-p\chi_1)+1-\omega_0e_1^{\gamma}\right)\right)\left(w_1(1-l_1)h_1+r_1k_1\right) \tag{30}$$

From 30 :

$$k_1=\frac{c_1+k_2+h_2-\left(1-\delta^h\right)h_1-\left(1-\tau_1\left(e_1(1-p\chi_1)+1-\omega_0e_1^{\gamma}\right)\right)\left(w_1(1-l_1)h_1\right)}{1+r_1-\delta^k} \tag{31}$$

Replace this equation in 29 equation:

$$\begin{aligned}&\frac{c_1+k_2+h_2-\left(1-\delta^h\right)h_1-\left(1-\tau_1\left(e_1(1-p\chi_1)+1-\omega_0e_1^{\gamma}\right)\right)\left(w_1(1-l_1)h_1\right)}{1+r_1-\delta^k}\\ =\;&\left(1-\tau_0\left(e_0(1-p\chi_0)+1-\omega_0e_0^{\gamma}\right)\right)\left(w_0(1-l_0)h_0+r_0k_0\right)-\left(1-\delta^k\right)k_0+h_1-\left(1-\delta^h\right)h_0-c_0\end{aligned}$$

Simplify and use that $1+r_1-\delta^k=J=\frac{p_t^*}{p_{t+1}^*}$ for all t, where are the J Arrow-Debreu Prices:

$$\begin{aligned}\sum_{t=0}^{\infty}J_tc_t &= \sum_{t=0}^{\infty}J_t\left[\left(1-\tau_t\left(e_t(1-p\chi_t)+1-\omega_0e_t^{\gamma}\right)\left(w_t(1-l_t)h_t\right)-h_{t+1}+\left(1-\delta^h\right)h_t\right]+\\ &\quad\sum_{t=0}^{\infty}J_t\left[\left(1-\tau_t\left(e_t(1-p\chi_t)+1-\omega_0e_t^{\gamma}\right)r_tk_t-k_{t+1}+\left(1-\delta^k\right)k_t\right]\end{aligned}$$

which is equivalent to:

$$\begin{aligned}\sum_{t=0}^{\infty}J_tc_t &= J_0\left[\left(1-\delta^h\right)+\left(1-\tau_0\left(e_0(1-p\chi_0)+1-\omega_0e_0^{\gamma}\right)\left(w_0(1-l_0)\right)\right]h_0+\\ &\quad J_0\left[\left(1-\delta^k\right)+\left(1-\tau_0\left(e_0(1-p\chi_0)+1-\omega_0e_0^{\gamma}\right)r_0\right]k_0\end{aligned}$$

If we choose J as numerarie J=1 we can obtain:

$$
\begin{aligned}
\sum_{t=0}^{\infty} J_t c_t &= \left[\left(1-\delta^h\right) + \left(1-\tau_0\left(e_0(1-p\chi_0)+1-\omega_0 e_0^{\gamma}\right)\left(w_0(1-l_0)\right)\right] h_0 + \\
&\quad \left[\left(1-\delta^k\right) + \left(1-\tau_0\left(e_0(1-p\chi_0)+1-\omega_0 e_0^{\gamma}\right) r_0\right] k_0
\end{aligned}
$$

for t=0:

$$
c_0 = \left[\left(1-\delta^h\right) + \left(1-\tau_0\left(e_0(1-p\chi_0)+1-\omega_0 e_0^{\gamma}\right)\left(w_0(1-l_0)\right)\right] h_0 +
$$

Now as:

$$
J_t = \frac{u_n(t)}{u_c(t)} \beta^t
$$

we have:

$$
\begin{aligned}
u_c(0)c_0 - u_n(0)\, n_0 h_0 &= u_c(0)\left[(1-\delta_k) + r_0\left(1-\tau_0\left(e_0(1-p\chi_0)+1-\omega_0 e_t^{\gamma}\right)\right] k_0 \\
&\quad + h_0\left[(1-\delta_h) + \left(1-\tau_0\left(e_0(1-p\chi_0)+1-\omega_0 e_0^{\gamma}\right) w_0\right]
\end{aligned}
$$

■

12 Proff of Proposition 8.

Proof. As presented in Sargent (2001) the Ramsey problem is maximize the households utility function subject to implementability and feasibility constraint. We assume the following auxiliary variable:

$$
W\left(c_t, n_{t,}, \eta_{ct}, \Phi\right) = u(c_t, n_{t,}, \eta_{ct}) + \Phi\left[u_c(t)\, c_t - u_n(t)\, n_t h_t\right]
$$

Expressed this problem as Lagrangian:

$$
L = \sum_{t=0}^{\infty} \beta^t \left\{ W\left(c_t, n_{t,}, \eta_{ct}, \Phi\right) + \theta_t \left[\begin{array}{c} F\left(k_t, n_t h_t\right) + \left(1-\delta^k\right) + \left(1-\delta^h\right) \\ -c_t\left(1+\eta_{ct}\right) - k_{t+1} - h_{t+1} \end{array} \right] - \Phi\Omega \right\}
$$

where

$$\Omega = u_c(t)\left\{\begin{array}{c}\left[\left(1-\delta^h\right)+\left(1-\tau_0\left(e_0(1-p\chi_0)+1-\omega_0 e_0^{\gamma}\right)\left(w_0(1-l_0)\right)\right]h_0+ \\ \left[\left(1-\delta^k\right)+\left(1-\tau_0\left(e_0(1-p\chi_0)+1-\omega_0 e_0^{\gamma}\right)r_0\right]k_0\end{array}\right\}$$

Using the first order conditions we obtain:

$$W_c(t) = \theta_t\left(1+\eta_{ct}\right)$$

$$W_n(t) = -\theta_t F_n(t)$$

$$W_{\eta_{ct}}(t) = \theta_t c_t$$

$$\left(\frac{W_c(t)}{W_c(t+1)}\right)\left(\frac{1+\eta_{ct+1}}{1+\eta_{ct}}\right) = \left[F_k(t+1)+\left(1-\delta^k\right)\right] \tag{32}$$

$$-\left(\frac{W_n(t)}{W_c(t)}\right)\left(1+\eta_{ct}\right) = F_n(t) \tag{33}$$

Now, we compare the first order conditions 33 and 34 with 16 and 17 first order conditions of representative agent for obtain the optimal tax policy. Therefore, the optimal income tax is:

$$\tau^* = \left(\frac{1}{e_t\left(1-p\chi_t\right)+1-\omega_0 e_t^{\gamma}}\right) \tag{34}$$

Of Euler equation for income tax evasion, we can derive the optimal fine:

$$\chi_t^* = \frac{1}{p} - \frac{e_t^{\frac{\gamma}{1-\gamma}}\gamma\omega_0}{\tau p}$$

Solve this equations system we obtain:

$$\tau_t^* = \left(\frac{e_t^{\frac{\gamma}{1-\gamma}}}{1-e_t^{\gamma-1}}\right)\left(\frac{1+\eta_{ct}}{2+\eta_{ct}}\right) \tag{35}$$

$$\chi_t^* = \frac{1}{p} - \left(1-e_t^{\gamma-1}\right)\gamma\omega_0\left(\frac{2+\eta_{ct}}{1+\eta_{ct}}\right)$$

Note that condition is satisfied for two cases. Firstly the condition is satisfied if $\tau_t^* = 0$ therefore $e_t^* = 0$ and the fines χ_t^* are indeterminated. The second case is characterized because the income tax is positive $\tau_t^* > 0$ which implies $e_t^* > 0$ and $\chi_t^* > 0$.

13 Proof of Proposition 9.

Proof. The first part of proposition are proved by the Lema 7. The second part of proposition we prove that given a Ramsey allocation that satisfies the implementability constraint, and first order conditions, then the prices can be reconstructed using the first order conditions 8 and 9. The optimal tax policy is obtain as proposition 8. The Walras law is satisfied therefore, the aggregated constraint also has to be satisfied. The optimal tax policy is reconstructing as presented in the proposition 8. Then the Ramsey allocation constitute a competitive equilibrium. ■