INDIVIDUALISM AND GROWTH VOLATILITY

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April 2004

Abstract: Quinn and Woolley (2001) provide empirical evidence that, democracies, compared to autocracies, generate less volatile growth. This paper identifies channels through which democracies can generate less volatile growth by incorporating Hofstede's (1980a,b) two cultural dimensions: 'individualism' and 'power distance'. In our theoretical analysis, we consider societies (1) with less power-distance oriented and individualistic division rules, and (2) with more power-distance oriented and collectivist ones. We find that (1) generate less volatile growth by encouraging at least some agents to invest even when the others don't invest due to unfavorable conditions, which would discourage investment totally in (2). In our empirical analysis it turns out that, individualism on which most democracies are based, is the crucial variable in explaining variations in the economic growth volatility. As predicted by our theoretical model, the relationship is negative, i.e. as individualism scores of countries increase economic growth becomes less volatile.

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I. Introduction:

Quinn and Woolley (2001), citing various micro survey data, pointed out that, in democracies, economic instability is one of the major fears of citizens. Their main thesis is that "the mechanism of democratic competition and the preferences of voters lead democracies to select away from very low and very high growth, and away from high volatility." The authors subsequently provide empirical evidence that, democracies, compared to autocracies, generate less volatile growth.¹

The aim of this paper is to identify theoretical and empirical channels through which democracies are able to generate less volatile growth. Our starting point is Hofstede's (1980a,b) analysis of different cultural values which we use to explain how certain institutional features can be crucial for less growth volatility since those institutional features may encourage, not only the large agents, but also small agents to undertake investments - and may encourage some at least some agents to invest even when the others do not find worth investing due to unfavorable conditions. Hofstede conducted questionnaires in 1968 and 1972 among 117,000 IBM employees; the initial 40-country sample was subsequently expanded to a total of 52 countries. It is still considered the most comprehensive comparative study in terms of both the range of countries and the number of respondents involved.

Hofstede based his analysis on four dimensions: '*individualism*', '*power distance*', '*masculinity*', and '*uncertainty avoidance*'. According to Hofstede, "individualism stands for a society in which ... everyone is expected to look after himself or herself and his or her immediate family only," and "collectivism stands for a society in which people from birth onwards are integrated into strong, cohesive in-groups, which throughout

¹ They trace the idea that 'economic policy in democracy seeks to moderate risks' back to Polanyi (1944). Polanyi maintained that the dynamics of market societies were governed by a double movement of two conflicting organizing principles: The first one is the principle of economic liberalism and the second one the principle of social protection. Quinn and Woolley contend that the latter principle is the fundamental source of the concern for economic stability.

people's lifetime continue to protect them in exchange for unquestioning loyalty."² In Hofstede's own words, "Power Distance indicates the extent to which a society accepts the fact that power in institutions and organisations is distributed unequally."³

In subsequent research, individualism/collectivism far exceeded the other dimesnions in popularity. Power distance comes a distant second. Consequently, we consider these two factors in our theoretical analysis. Although we consider all four dimensions in our empirical analysis, only individualism/collectivism emerges with the correct sign and statistically significant.

We first use a theoretical analysis. Agents (i.e., individuals and households) with different asset levels form links with each other. Agents can enhance their assets by investing in them at a cost. This cost and the rate of return to their investments can fluctuate and are influenced by government expenditures. We consider two types of societies. In one of them, less power-distant and individualistic agents adopt a division rule which yields a payoff ratio identical to the asset ratio of the agents such that the payoff of an agent will increase when his assets increase, but will not change when the other agent's assets increase. In the other one, more powerdistant and collectivist agents adopt a division rule which yields a constant payoff ratio independent of the asset ratio always favoring the agent with more assets such that the payoff of an agent will increase not only whenever his assets increase, but also will increase whenever the other agent's assets increase as well. We find that the former-type society generates less volatile growth by encouraging at least some agents to invest even

² As Kagitcibasi (1997) observes "[i]ndividualism/collectivism cleavage has long been of significance in social thought about ... the relationships among human beings. The basic question confronted by social philosophers and social scientists 'How is social order possible?' has been answered by stressing either the individualistic or the collectivistic elements in the human-society interface." In a sense, Hofstede's *Culture's Consequences* (1980a) revived the concepts of individualism and collectivism by providing a comprehensive empirical basis for them.

³ Masculinity points to the extent that the dominant values in society are assertiveness, the acquisition of money and things, and *not* caring for others, the quality of life, or people. Finally, "Uncertainty Avoidance, indicates the extent to which a society feels threatened by uncertain and ambiguous situations and tries to avoid these situations by greater career stability, establishing more formal rules, not tolerating deviant ideas and behaviors, believing in absolute truths and the attainment of expertise."

when the others do not find worth investing due to unfavorable conditions, which do not allow any agent to invest in the latter-type society.

In our empirical analysis, we first incorporate the above mentioned four cultural dimensions of Hofstede into a specification adopted by Quinn and Woolley. Our estimation results, which are free of heteroscedasticity, auto-correlation, non-normality and specification error, indicate that the relationship is negative, i.e. as individualism scores of countries increase economic growth becomes less volatile, as predicted by our theoretical model. To be specific, the variables that affect the volatility of economic growth with the correct sign and statistically significantly are (i) individualism dimension, and (ii) volatility of government expenditures. Index of democracy turns out to be less significant than the individualism dimension.

II. Theoretical Model:

Each individual's (or household's) income is generated through its interactions with others in the society (working for others, hiring others for their firms or for housekeeping help, trading goods and services etc). Such interactions (or links) require the consent of both parties. We will use the term agent instead of individual or household. There are two types of agents: the ones with low asset level, 1, and the ones with high asset level, a > 1; the assets could be human capital as well as physical capital. In short, we will refer to these agent types as low-asset types (or low types) and high-asset types (or high types). The numbers of agents of both types are equal. We will consider a bargaining setup where each high type interacts with one low type and likewise each low type interacts with one high type. For simplicity we will use a linear Pareto frontier. In particular, the bargaining set will be the convex hull of {(0,0),(1,0),(0,a)}. In other words, each party's ideal payoff will the level of their asset ownership.⁴

⁴ The direct link between the ideal payoffs and the agents' asset levels calls for an explanation. Suppose agents bargain over, say, \$10,000. Suppose one agent has a much higher asset (e.g., human or physical) level. Ideal payoff of an agent indicates the maximum that can be achieved with the entire surplus by that agent. Clearly the agent with more assets can achieve a higher maximum with the entire surplus than the agent with less

Growth in our setup will come from increases in agents' assets (and thus ideal payoffs). Such increases (be they human or physical capital - as well as technological - enhancements) will be achieved by agents' investments that will entail time and effort cost. In particular, given a level of ideal payoff b_i , each Agent i can enhance their b_i to kb_i at a cost c > 0 where 2 > k > 1.

Specifically, at period one, agents will start with their initial asset levels, and they will make their investment decisions individually considering their second period potential payoffs. Each agent will decide whether or not such a first-period investment is worth taking.

We will compare two types of societies. In one of these societies, less power-distant and individualistic agents will adopt a division rule which yields a payoff ratio identical to the asset ratio of the agents such that the payoff of an agent will increase when his assets increase, but will not change when the other agent's assets increase. Given a linear Pareto frontier, most of the prominent solution concepts in bargaining theory yield the same outcome (the Nash solution (Nash, 1950, 1953), the Kalai/Smorodinsky solution (Kalai and Smorodinsky, 1975), the Equal Area solution (Anbarci and Bigelow (1994), Anbarci (1993)).⁵ We will call any representative these solution concepts as the "Individualistic outcome" and we will denote the Individualistic outcome payoffs of the low and high agents by $I_i(b_pb_h)$, and $I_h(b_pb_h)$ where b_i denotes the ideal payoff of Agent i. The outcome with this solution is such that $I_h(b_pb_h)/I_1(b_pb_h) = b_h/b_p$. That is, when $b_h = a$ and $b_l = 1$, $I_h(b_pb_h)/I_1(b_pb_h) = a$. Let $I^*(b_pb_h) = (I_1(b_pb_h) + I_h(b_pb_h))$ be the total income of any pair with the Individualistic outcome given (b_pb_h) .

Likewise, in a different type of society, more power-distant and collectivist agents will adopt a division rule which yields a payoff ratio always favoring the agent with more assets. But more importantly, to reflect the collectivist (or solidarity) nature of this society, with this division rule the payoff of an agent will increase

agents.

⁵ These solution concepts have also been advocated as social division rules by (see for instance, Binmore (1994) and Gauthier (1986)).

not only whenever his assets increase, but also will increase whenever the other agent's assets increase as well. We will call this solution, the "Power-solidarity" solution and will denote its outcome payoffs of the low and high agents by $P_1(b_1, b_h)$, and $P_h(b_1, b_h)$. The outcome with this solution is such that $P_h(b_1, b_h)/P_1(b_1, b_h) = m$ such that m = ka. Thus, although m is higher than a (by the rate k), it will not be outrageously higher than a, since the leaders of a collectivist society are supposed to care about the welfare of the others too. Let $P^*(b_1, b_h) = (P_1(b_1, b_h) + P_h(b_1, b_h))$ be the total income of any pair with the Power-solidarity outcome given (b_1, b_h) . The proof simply follows the definitions of I and P).

Lemma 0: (1)
$$I_1(1,a) = \frac{1}{2}, I_h(1,a) = \frac{a}{2}; I^*(1,a) = \frac{(1+a)}{2}.$$

(2) $P_1(1,a) = \frac{a}{(m+a)}, P_h(1,a) = \frac{ma}{(m+a)}; P^*(1,a) = \frac{a(m+1)}{(m+a)}.$

There are four possible outcomes: (1) both types of agents invest, (2) only the high types invest, and (3) only the low types invest, (4) no type invests. Most of these outcomes will be equilibrium outcomes, based on the agents' cost-benefit comparisons, considering the levels of c and k. Clearly, when no agent invests, there is no growth. As will be observed, when both agents invests, the growth rate will be k, regardless of the division rule used in the society. If these two outcomes turn out to be the only equilibrium outcomes, then the growth volatility will be the same regardless of the division rule adopted by the society. But first, we will establish the payoff levels for each of the above possible outcomes (their proofs simply follow from the definitions of I and P):

Lemma 2: Suppose only high types invest.

(1)
$$I_1(1,ka) = \frac{1}{2}, I_h(1,ka) = \frac{ka}{2}; I^*(1,ka) = \frac{(1+ka)}{2}.$$

(2) $P_1(1,ka) = \frac{ka}{(m+ka)}, P_h(1,ka) = \frac{mka}{(m+ka)}; P^*(1,ka) = \frac{ka(m+1)}{(m+ka)}.$

Lemma 3: Suppose only the low types invest.

(1)
$$I_1(k,a) = k\frac{1}{2}, I_h(k,a) = a/2; I^*(k,a) = (k+a)/2.$$

(2) $P_1(k,a) = \frac{ka}{(mk+a)}, P_h(k,a) = \frac{mka}{(mk+a)}; P^*(k,a) = \frac{ka(m+1)}{(mk+a)}.$

Corollary 1: (1)
$$I_1(1,a) = I_1(1,ka), I_1(k,a) = I_1(k,ka), I_h(1,a) = I_h(k,a), I_h(1,ka) = I_k(k,ka).$$

(2) $P_1(1,a) < P_1(1,ka), P_1(k,a) < P_1(k,ka), P_h(1,a) < P_h(k,a), P_h(1,ka) < P_k(k,ka).$

In other words, given I, a particular type of agent's investment decision does not depend on that of the other type of agent. Given P, a particular type of agent's investment decision does depend on that of the other type of agent.

When no one invests, the outcome is same as the one in Lemma 0. Our next result indicates the cutoff levels of costs, i.e., the cost levels at which agents are indifferent between investing and not investing. Clearly, if two agents, L and H, have differing cutoff cost level, the one, L, with the lower cutoff level will be able to invest whenever the one, H, with higher cutoff level invests *and* at some other higher cost levels at which H is not willing to invest. It will turn out that, given I, agents will make their investment decisions regardless of whether the other agent invests or not; given P, on the other hand, agents will have higher cost cutoff levels if the other agent invests too (and will have a lower cost cutoff level if the other agent does not invest). Our notation will reflect this important difference. Given I, c_1^N will denote the low types' cutoff level, and c_h^P will denote the high types' cutoff level of cost. Given P, if the other agent is not investing, c_1^P

other agent too is investing, c_1^{p*} will denote the low types' cutoff level, and c_h^{p*} will denote the high types' cutoff level. The proof of the next result is in the Appendix.

Proposition 1: Either
$$c_l^P < c_l^{P*} < c_l^N \le c_h^P < c_h^N < c_h^{P*}$$

or $c_l^P < c_l^{P*} < c_h^P \le c_l^N < c_h^N < c_h^{P*}$.

The next result points out an important insight provided by Proposition 1 (which was also hinted before Proposition 1):

Corollary 2: Given P, if agent types of i invest, the other type of agents has less incentives to invest then he/she would if agent types i did not invest.

Suppose that, the cost c of investments in human capital or technological enhancements is negatively related to the level of government expenditures and other favorable circumstances; on the other hand, the rate of return to these investments, k, is positively related to the level of government expenditures and enhancements is negatively related to the level of government expenditures and other favorable circumstances. Then, fluctuations in government expenditures and in other circumstances will lead to fluctuations in these investments year to year in both types societies, i.e., in societies adopting I and the ones adopting P. Our next two results follow directly from Proposition 1.

Theorem 1: Consider I.

- (1) When $c \le c_1^N$, both types will invest and the growth rate will be k.
- (2) When $c > c_h^N$, no type will invest and the growth rate will be zero.
- (3) When $c_l^{\,\rm N} < c \leq c_h^{\,\rm N},$ only the high type will invest and the growth rate will be

$$(1+ka)/(1+a)$$

Theorem 2: Consider P.

When c ≤ c₁^{P*}, both types will invest and the growth rate will be k.
 When c > c_h^P, no type will invest and the growth rate will be zero.
 When c₁^{P*} < c ≤ c_h^P, only the high type will invest and the growth rate will be (km+ka)/(m+ka).

The next one is our main result (its proof is in the Appendix):

Theorem 3: Suppose c is uniformly distributed over any spectrum that contains all c values from c_1^P to c_b^{P*} . The growth volatility is higher with P than with I.

III. Empirical Analysis:

The above-mentioned four dimensions are among the factors that shape individuals' political behaviors (including voting ones) and, as such, they determine the degree of democracy in a given society. In what follows, we econometrically test the following model:⁶

$$varq_{i} = \beta_{0} + \beta_{1}varq(-1)_{i} + \beta_{2}meanq_{i} + \beta_{3}inv_{i} + \beta_{4}y_{i} + \beta_{5}sec_{i} + \beta_{6}pop_{i} + \beta_{7}tra_{i} + \beta_{8}vargov_{i} + \beta_{9}nf_{i} + \beta_{10}dnf_{i} + \beta_{11}idv_{i} + \beta_{12}pdi_{i} + \beta_{13}uai_{i} + \beta_{14}mas_{i} + u_{i}$$

⁶ The sample used in estimations includes the following countries, for which Hofstede's cultural dimensions data are available,: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Hong Kong, Ireland, Israel, Italy, Japan, Netherlands, New Zealand, Norway, Portugal, Singapore, Spain, Sweden, Switzerland, United Kingdom, United States, Argentina, Brazil, Chile, Czech Republic, Hungary, Malaysia, Mexico, Panama, Poland, South Korea, Taiwan, Turkey, Uruguay, Venezuela, Colombia, Costa Rica, Ecuador, El Salvador, Guatemala, Iran, Jamaica, Peru, Philippines, South Africa, Thailand, China, India, Indonesia, and Pakistan.

where varq: variance of per capita GDP growth that measures volatility (1974-1998),

varq(-1): variance of the previous period (1963-1973),

meanq: mean of per capita GDP growth (1974-1998),

inv: initial investment 1971,

y: initial GDP per capita 1971,

sec: secondary school enrollment (1973),

pop: population growth (1974-1998),

tra: trade openness (imports + exports as percentage of GDP, 1974-1998),

vargov: volatility of government expenditures (as percentage of GDP, 1974-1998),

nf: index of democracy, 1973, gathered by Freedom House (higher the value of this index lesser democratic the country is),

dnf: change in index of democracy (1974-1998),

idv: individualism-collectivism dimension (1972), higher the value of this variable more individualistic the country is,

pdi: power distance dimension (1972), higher the value of this variable more power distant the country is, uai: uncertainty avoidance dimension (1972), higher the value of this variable more uncertainty avoidant the country is,

mas: masculinity-feminity dimension (1972), higher the value of this variable more masculine the country is,

u: stochastic error term.

The specification of this equation, apart from idv, pdi, uai, and mas, is very similar to that of Quinn and Woolley (2001). The differences, in terms of explanatory variables, are due to the lack of data for the sample we use for estimations. But, the equation we use and theirs are basically the same. The main finding of Quinn and Woolley is that the coefficients of both nf and dnf are positive. Here, we claim that democracy is shaped by cultural values that are measured by Hofstede's dimensions. As our model implies, we expect a negative sign for idv and a positive sign for pdi. Table below reports the OLS estimation results of the above equation.

Insert Table about here

Estimation results which are free of heteroscedasticity, auto-correlation, non-normality and specification error, indicate that growth volatility is negatively related to idv (at 5.4% significance level). In accordance with Quinn and Woolley's finding, nf has a positive sign, although its significance level is slightly higher than the conventional 5% level. Both dnf and pdi's coefficients have incorrect signs and they are statistically insignificant. As to the mas and uai's coefficients, they may have correct signs, but they have no statistical significance. As our theoretical model implies, vargov affects positively growth volatility, i.e., as volatility of government expenditures increases so does growth volatility. The coefficient of vargov is statistically significant. All the remaining explanatory variables are statistically insignificant.

Taking into consideration the fact that incorrect signs of dnf and pdi might result from multicollinearity between dnf and nf on the one hand and between pdi and idv on the other, we run regressions omitting nf and idv and we still got same results (which are not reported here), i.e., both dnf and pdi have insignificant coefficients with incorrect signs.⁷

On the other hand, the zero-order correlation between nf and idv is –61.8% and that between nf and pdi is 51.5%. The interesting point is that, despite this correlation between nf and idv, both of these variables are statistically significant and the signs of their coefficient are correct. Yet, the significance level of nf is

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Indeed, zero-order correlation between nf and dnf is 68% and that between pdi and idv is 68.1%.

slightly greater than the conventional level. Furthermore, the partial correlation coefficient between varq and nf is 36.8% whereas that between varq and idv is –38.2%. Diagrams 1 and 2 display partial regression plots for nf and idv, respectively. These diagrams seem to indicate that idv is relatively more important than nf in explaining variations in varq.

Insert Diagrams 1 and 2 about here

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Appendix:

Proof of Proposition 1: $c_l^P < c_l^{P*}$: $c_1^{P^*} = P_1(k,ka) - P_1(1,a) = ka/(m+a) - a/(m+a) = (k-1)a/(m+a).$ $c_1^P = P_1(k,a) - P_1(1,a) = ka/(mk+a) - a/(m+a) = a^2(k-1)/(mk+a)(m+a).$ Thus, $c_1^{P^*} - c_1^{P} = (k-1)(a/(m+a))[1-a/(mk+a)]$. k > 1. 1 > a/(mk+a) since m = ka. Thus, $c_1^{P^*} - c_1^{P} > 0$. $c_{1}^{P^{*}} < c_{1}^{N}$: $c_1^N = I_1(k,a) - I_1(1,a) = k^{1/2} - \frac{1}{2} = (k-1)/2.$ $c_1^{P^*} = P_1(k,ka) - P_1(1,a) = ka/(m+a) - a/(m+a) = (k-1)a/(m+a).$ Thus, $c_1^N - c_1^{P^*} = (k-1)/2 - (k-1)a/(m+a) = (k-1)[\frac{1}{2} - a/(m+a)]$, which is positive since, by definition, $m = (k-1)[\frac{1}{2} - a/(m+a)]$ ka and k > 1, and thus m > a. Thus, $c_1^N - c_1^{P^*} > 0$. $c_{l}^{P*} < c_{h}^{P}$: $c_h^{P} = P_h(1,ka) - P_h(1,a) = mka/(m+ka) - ma/(m+a) = m^2 a(k-1)/(m+ka)(m+a).$ $c_1^{P^*} = P_1(k,ka) - P_1(1,a) = ka/(m+a) - a/(m+a) = (k-1)a/(m+a).$ Thus, $c_h^{P} - c_l^{P*} = (k-1)(a/(m+a))[m^2/(m+ka) - 1]$. Since m = ka, k > 1, a > 1, it follows that (k-1)a/(m+a) $[m^2/(m+ka) - 1]$ is positive. $c_l^N vs. c_h^P$: $c_h^{P} = P_h(1,ka) - P_h(1,a) = mka/(m+ka) - ma/(m+a) = m^2 a(k-1)/(m+ka)(m+a).$ $c_1^N = I_1(k,a) - I_1(1,a) = k^{1/2} - \frac{1}{2} = (k-1)/2.$ Thus, $c_h^P - c_1^N = (k-1)[m^2 a/(m+ka)(m+a) - \frac{1}{2}]$. Since k > 1, $c_h^P - c_l^N$ is positive if $[m^2 a/(m+ka)(m+a) - \frac{1}{2}] > 0$ and negative if $[m^2 a/(m+ka)(m+a) - \frac{1}{2}]$ < 0. It reduces to a-(k+1)/k. Thus, when a > (k+1)/k, then $[m^2 a/(m+ka)(m+a) - \frac{1}{2}]$ is positive, and when a < (k+1)/k, it is negative; when a = (k+1)/k, it is zero. Thus, we have all three possibilities: $c_1^N > c_1^N > c$ $c_{h}^{P}, c_{l}^{N} < c_{h}^{P}$ and $c_{l}^{N} = c_{h}^{P}$.

 $c_l^N < c_h^N$ and $c_h^P < c_h^N$:

First we will establish $c_l^N < c_h^N$.

$$c_h^{N} = I_h(1,ka) - I_l(1,a) = ka/2 - a/2 = (k-1)a/2.$$

$$c_l^{N} = I_l(k,a) - I_l(1,a) = k^{1/2} - \frac{1}{2} = (k-1)/2.$$

Thus, $c_h^N - c_l^N = (k-1)[a-1]/2$ which is positive since by definition k > 1, and a > 1.

Now, we will establish $c_h^{P} < c_h^{N}$.

$$c_h^N = I_h(1,ka) - I_l(1,a) = ka/2 - a/2 = (k-1)a/2.$$

 $c_h^{P} = P_h(1,ka) - P_h(1,a) = mka/(m+ka) - ma/(m+a) = m^2 a(k-1)/(mk+a)(m+a).$

Thus, $c_h^N \cdot c_h^P = (k-1)a[\frac{1}{2} \cdot \frac{m^2}{(mk+a)(m+a)}]$. Observe that $\frac{1}{2}$ vs. $\frac{m^2}{(mk+a)(m+a)}$ reduces to $k^3a^2 + ka^2 + a^2$ vs. k^2a^2 , which is positive since m = ka and 2 > k > 1.

$$\begin{split} c_h^{N} &< c_h^{P^*}: \\ c_h^{P^*} &= P_h(k,ka) - P_h(1,a) = mka/(m+a) - ma/(m+a) = (k-1)am(m+a). \\ c_h^{N} &= I_h(1,ka) - I_l(1,a) = ka/2 - a/2 = (k-1)a/2. \\ \end{split}$$
Thus, $c_h^{P^*} - c_h^{N} = (k-1)a[m/(m+a) - \frac{1}{2}]. \ k > 1. \ m = ka > a; thus, m/(m+a) > \frac{1}{2}.$ Hence,

Thus, $c_h^{P^*} \cdot c_h^N = (k-1)a[m/(m+a) \cdot \frac{1}{2}]$. k > 1. m = ka > a; thus, $m/(m+a) > \frac{1}{2}$. Hence, $c_h^{P^*} \cdot c_h^N > 0$. This completes the proof of Proposition 1.

Proof of Theorem 3: With both I and P, when both agents invest the growth rate is k, and when no agents invest the growth rate is 0. When one agent invests, the growth rate is a median one with both I and P. Thus, the division rule that has a larger range of one type investing will have less growth volatility. For I, that range is

 $\Delta^{\rm I} = c_{\rm h}^{\rm N} \cdot c_{\rm l}^{\rm N} = (k-1)(a-1)/2.$

For P, that range is

$$\Delta^{P} = c_{h}^{P} - c_{l}^{P*} = (k-1)(a/(m+a))[m^{2}/(m+ka) - 1]$$

Thus, $\Delta^{I} \cdot \Delta^{P} = (k-1)[(a-1)/2 \cdot (a/(m+a))[m^{2}/(m+ka) \cdot 1]]$. Since k > 1, a > 1 and m = ka, we have $\Delta^{I} \cdot \Delta^{P} > 0$. This completes the proof of Theorem 3.

Table:

Dependent Variable: varq

	coefficients	Sig.
(Constant)	1.844E-03	.140
meanq	3.832E-03	.672
рор	1.864E-02	.435
varq-1	139	.371
у	4.245E-10	.985
i	-2.767E-05	.317
tra	-3.212E-06	.352
vargov	1.733E-04	.007
SS	5.736E-04	.467
nf	2.405E-04	.065
dnf	-2.478E-04	.094
pdi	-1.348E-05	.076
idv	-1.777E-05	.054
mas	1.180E-06	.835
uai	1.532E-06	.789

Adjusted R²=0.577

Diagram 1:

Partial Regression Plot

Dependent Variable: VARQ

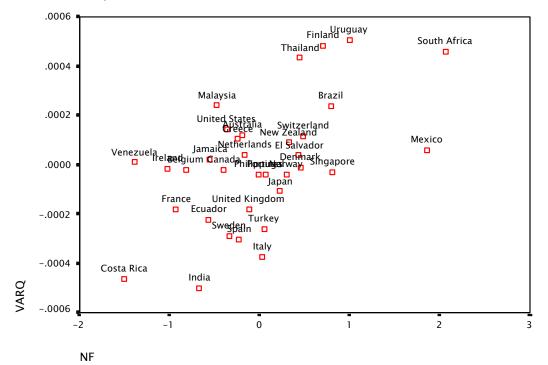


Diagram 2:

Partial Regression Plot

