A Model of State Infrastructure with Decentralized Public Agents:

Theory and Evidence

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April 2004

This paper presents an alternative macroeconomic framework for the aggregate production function. I consider the local monopoly power of public agents through the explicit separation of the private versus public sector production functions. Productivity is estimated of private versus public capital, of private versus public labor, and of public infrastructure.

Results differ significantly from those found using the standard APF approach. They suggest that the elasticity of public infrastructure is significantly lower than previous estimates. They confirm that private infrastructure productivity is country specific. The theoretical results suggest that public agent decentralization, intuitively greater in less developed countries, is associated with steeper growth paths leading to lower steady-state capital-labor ratios and consumption levels. Thus, shocks to developing countries should have more dramatic effects but shorter life spans than shocks to industrialized countries.

"Sure there are dishonest men in local government. But there are dishonest men in national government too." - Richard M. Nixon (1913-1994)

Keywords: decentralization, infrastructure, aggregate productivity, growth

JEL Classifications: O41, O47, H1, H4, E1

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The author is grateful to Patricia Sourdin for research assistance, the University of Adelaide, School of Economics for invaluable discussion, and The Australian Research Council for financial support.

The imperative of the state, irrespective of its organization, is to provide law and order, contract enforcement, and property rights. All of the goods and services provided by government, from alleviation of market failures to the enforcement of standards, fall into these three broads categories. The government provides the foundation for markets to function and for society to prosper. A common thread in any government is that, for a given society, the state holds a monopoly over its services. The fact that there is no competition in the provision of most state services breeds its own type of inefficiency that in most societies is typically accepted, if not actually expected, by the private sector. The following paper considers the broad implications of the state's monopoly over its services on growth and productivity.

Although the government is generally the sole provider of its services, it does not behave like a traditional monopolist. The goods and services provided by the public sector are various and not always clearly defined. Furthermore, the ability of the government to centrally coordinate its services breaks down as one moves from mandate to actual provision.

The central theme in this paper is that the government actually provides its services through a network of decentralized self-seeking agents. Government employees, i.e. public agents, each have a degree of power over some aspect of what the state provides. As a result, the provision of public services, at the most basic level, is carried out by individuals who find themselves in positions of power and, bowing to incentives, consequently maximize their respective welfare functions.

An important distinction is between publicly produced goods and pure public goods.[1] I assume that pure public goods are a subset of publicly produced goods. Non-rival and non-exclusive goods production falls into the state's categorical role of property rights delimitation. In fact, many publicly produced services such as law and order as well as contract enforcement, are to a large degree quite rival and exclusive. Even infrastructure, such as transportation and communication, is not really a pure public goods due to being subject to user fees. Therefore, the Samuelson (1954) condition over optimal provision of pure public goods is limited to a subset and does not adequately describe the state's broader control over law and order, property rights, and contract enforcement.

There is a rich literature on decentralization of governance, most often referred to as "fiscal federalism." In general, the concern is the manner in which public goods are disseminated across society and the efficiency aspects of authority in the hands of local government versus central government. Most authors analyze fiscal federalism as an alternative mechanism to provide efficient provision of public goods and the preservation of market incentives.[2] A review of the issues with particular regard to developing countries appears in Bardham (2002).

The contribution of this paper, distinct from previous work, is my regard for the representative government agent, who exists irrespective of structural organization of the state, in the context of a generalized macroeconomic model. I liken the representative public agent to a local monopolist who is able to exploit her market power due to a lack of proper accountability. Her incentives exist irrespective of the degree of fiscal federalism within her

[1] Throughout the paper, I use a rather loose definition of public goods. By public goods, I really mean publicly provided goods, services, and infrastructure.

society. Although several authors have explored aspects of accountability and fiscal federalism,[3] none have considered the more general macroeconomic implications of decentralization of public authority amongst the agents of the government.

The implications of the principal agent problem that stems from decentralized control over government services by agents are great. Policy makers, especially those in developing countries, are keenly aware their importance. The impact of decentralization on development is most readily apparent in large developing countries such as Brazil, India, Indonesia, and China. In such places, neither the central nor the local government is capable of exercising the optimal degree of control over its representatives. Public agents may act unilaterally or could be subject to capture by elites, as suggested by Bardhan and Mooherjee (2001). Although specific reasons for this lack of control vary from country to country, some generalizations may be made. Government agents expect to personally profit from their positions in the public sector. It is one of the main incentives to seek public employment. Furthermore, the general population may expect a certain degree of dishonesty or corruption of public servants. In many places, there exist long and rich histories of public sector corruption. Its existence is treated as a fact of life that all must be resigned to endure.[4]

Every single element that makes up the macroeconomy, irrespective of its role, may be classified as either a public or a private asset. This important accounting fact allows us to

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[2] For example, see Qian and Weingast (1997). Fiscal federalism in the United States is considered in the four papers that comprise the "Symposium on Fiscal Federalism" which appears in the JEL, Autumn 1997.

[3] Most notably Seabright (1996) discusses political accountability in a theoretical model with central and locally elected officials competing over control rights. Other papers addressing different dimensions of this principal-agent problem include Besley and Case (1995), Beasley and Coate (1999) and Tommasi and Weinschelbaum (1999).

[4] For example, in the Middle East, payment of *Baksheesh* is part of the expected protocol when dealing with a public official. Similarly in Paraguay, where cronyism and corruption were institutionalised during a 35 dictatorship, private citizens expect to pay speed money, known there as *coima*, for anything from

classify each and every good or service available to society by its state of nature. The importance of the accounting constraint that governs both labor and capital justifies the explicit consideration of an aggregate private sector production function versus an aggregate public sector production function.

The difference in market organization between the public and private sectors combined with the binding accounting constraint over all resources are of the utmost importance to understanding the equilibrium price and quantity of labor, capital, public goods and private goods. The overwhelming majority of research on the role of public infrastructure within an aggregate production function framework assumes that public capital is an unpaid input to production. Most authors assume that public capital is trivially produced from tax revenue. This wholly inadequate convention conveniently sidesteps the issue of a public sector production function as well as any issues relating to the resource accounting constraint while maintaining the real cost, in taxes, of public goods and services. Furthermore, the trivializing assumption that $G = tY$ does not account for the significant labor and capital inputs in public sector production.

This paper presents an alternative methodology to modeling the state's role as the producer of public infrastructure within the context of an aggregate production function that explicitly considers the government's natural monopoly as well as the general equilibrium constraint on resources. The basic framework is well documented in both the theoretical and applied literatures on public capital. One of the simplest treatments of public goods from growth theory is Barro (1990).[5] Papers that estimate the return to public infrastructure within

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phone service to passports. Examples in the literature that document this phenomenon include Geddes and Neto (1992), Barreto (1996), and Crook and Manor (1998).

[5] Other papers that explicitly consider infrastructure in the context of an endogenous growth model include Barreto (2000), Dasgupta (1999), Turnofsky (1996), and Futagami, Morita, & Shibata (1995).

an aggregate production function generally follow Aschauer (1989,1990).[6] A common assumption adopted to capture non-exclusivity, which is shared by all of the theoretical models that underlie this research, is that public capital is an unpaid input within the production process. In other words, the cost function faced by firms resembles the following, $Y(K, L, G) = P_G G + wL + rK = wL + rK$ where $P_G = 0$ and all public services, *G*, are paid for by tax revenues. This framework fails to account for either the inherent difference in market structure between the public sector and the private sector or the public-private accounting constraint over resources.

I consider an aggregate production function where public infrastructure is a productive input along with private capital and labor. Although subsidized by tax revenues, public goods must still be explicitly paid for via user fees,[7] such that the demand for public goods is downward sloping.[8] But unlike capital and labor, which by assumption, are provided competitively, government services are sold at an endogenously determined premium. The premium is a function of the tax rate, the marginal product of public capital and the marginal product of private capital.

Previous econometric studies generally regress output on total capital, total labor, and total government to yield a higher than expected estimates of public infrastructure

[6] Other notable papers that estimate the positive impact of public capital include Morrison and Schwartz (1996), Nadiri and Mamuneas (1994), Berndt and Hansson (1992), Lynde and Richmond (1992), and Munnell (1990). However, papers that find a negligible impact of public capital include Holtz-Eakin (1994), Garcia-Mila and McGuire (1992), and Hulten and Schwab (1984, 1991).
[7] User fees for public services are described analytically by Samuelson (1954). More recent examples include Barreto (2000) in the context of an endogenous growth model with corruption and Bardhan and Mookherjee (2001) in the context of fiscal federalism within developing countries.
[8] Dasgupta (1999) considers non-rival infrastructure inputs that are also purchased by the private sector. Similar to the approach taken here, the price of infrastructure is determined by equating the supply by the government to the demand by the private sector.

productivity. By not explicitly considering the accounting constraint, the relative size of government is smaller and its estimated productivity is consequently higher.[9]

The paper is organized as follows. Section 2 describes the theoretical framework that describes the allocation of labor and capital across the public and private sectors. The resulting allocations determine the price and quantity of provision of public goods. Section 3 estimates the productivity of capital, labor, and public infrastructure within the context of the allocative framework. The results, exclusive to OECD countries, suggest the relatively high productivity of public labor and of private capital. Section 4 considers the model in the purely theoretical context of endogenous growth. The decentralized agent model predicts that the steady state effective capital-labor ratio is much greater than traditional growth models that account for a public sector might predict. Section 5 presents some concluding remarks.

2. The Model

I consider an aggregate production function model where the state is an intermediate factor of production which not only requires real resources but is also administered by self-seeking, albeit decentralized, public agents. The representative firm pays user fees in addition to income taxes to the government in return for service. The user fee, i.e. the price of government services, has a limit of zero as the tax rate approaches its optimum. In other words, a society with perfect information could set taxes exactly right such that the user fee

[9] Estimates vary widely from very high to low to insignificant. Aschauer (1989, 1990) estimates values for b from .38 to .56. Subsequent literature pushed this estimate down to something closer to .30. The lowest estimate belong to Eberts (1990), who found b to be .03.

for public goods is truly zero[10] and consequently the model collapses to the more traditional treatment of public goods within the private production process. In practice therefore, the second best outcome implies the user fees for public goods must be positive.

As long as user fees are positive, the representative private agent faces a downward sloping demand for public goods. The demand for a service is unaffected by the good's nature in terms of its non-rivalry. On the other hand, the quantity demanded certainly changes in reaction to the supply of the good, which of course is a function of the good's nature and the tax rate. Previous work in the corruption literature that assumes a downward sloping demand for public goods includes Schliefer and Vishny (1990), Barreto (2000), and Bardhan and Mookherjee (2001).

In a perfect world, the selfless government employs labor and capital at their competitive rates as well as collects taxes in order to provide society with its services as efficiently as possible. Optimal provision of any normal good, such as G, from the theoretical point of view, is equivalent to the perfectly competitive equilibrium. We may therefore consider a government that employs resources to produce an intermediate good where deviation from the perfect competition benchmark represents allocative inefficiency.

Irrespective of whether one believes in the validity of the aggregate production function, it has several unequivocally useful applications.[11] Modern macroeconomic growth theory, on the other hand, is largely based the use of aggregate production functions that are conceptually derived from microeconomic foundations. Theoretical research that follows

[10] This is the market solution to the principal agent problem between agents. As P_G approaches zero, so do the monopoly rents available to the public agent.

[11] Criticisms of the APF generally follow Fisher (1965, 1969, 1971, 1983) who developed a set of theoretical conditions that are so rigid that successful aggregation, as implied by the APF framework, is all but impossible. Fisher's criticism, if taken to heart, completely discounts the use of the aggregate production function in order to estimate the productivity of infrastructure. See Felipe (2001).

Ramsey (1928), Cass (1965), Koopmans (1965) and later Lucas (1988), implicitly adopts Samuelson's (1962) view that the aggregate production function is a useful parable to illustrate important facets about the production process and the consequent growth of output per capita. I adopt this latter view. Although I estimate the model given its aggregate production function framework, I make no claim about the models validity for estimation purposes.[12]

I consider the two sectors, public and private, separately. Given the non-trivial accounting constraint, the two sectors compete over both labor and capital. The inherent difference in market structures between sectors justifies the explicit consideration of the equilibrium wage and rental rate as the result of competition between the two separate aggregate production processes. Therefore, the model may be expressed generally as follows.

$$Y = F\left(K_Y, L_Y, G\right) \qquad \text{and} \qquad G = H\left(K_G, L_G, t\right)$$

subject to

$$\bar{K} = K_Y + K_G \qquad \text{and} \qquad \bar{L} = L_Y + L_G$$

Suppose that the public production function, which requires both capital and labor, produces public infrastructure. The private production function, which requires capital, labor, and public infrastructure, produces consumable goods on which all agents within society, public and private, ultimately depend. The privately employed representative laborer is paid his marginal product according to the demand from the competitive representative firm and the opportunity cost of private versus public employment. Perfect mobility of

[12] The econometric exercise found herein compares the results from this specification to those of the traditional aggregate production function estimations that generally follow Aschauer (1989, 1990).

resources guarantee the representative public agent is also paid her marginal product, which must equal the private sector wage.

The government is a monopoly whose services are centrally mandated while decentrally administered by its agents. The relationship between the government and its agent is governed by the assumption that the public agent can neither directly affect the marginal product of the public good or service, i.e. its price, nor the marginal product of her work effort, i.e. her wage. Instead the user fee for public goods is determined by the endogenous allocation of labor and capital across the public and private sectors. The second best resource allocation is a proxy for the less than optimal effort exerted by public agents.

Suppose the private sector produces all of society's consumable output through a production function that is homogeneous of degree one in capital (K_Y), labor (L_Y), and infrastructure (G).

$$Y = F\left(K_Y, L_Y, G\right) = AK_Y^a G^b L_Y^{1-a-b} \tag{2.1}$$

The central government collects taxes proportional to final production and hands over the revenue,[13] T, to its agents who produce and then sell the public good to the private sector. The public sector production function is defined as follows.[14]

$$\begin{aligned} G &= H\left(t, K_G, L_G\right) = \tilde{G}\left(K_G, L_G\right) + \frac{T}{P_G} \\ &= AK_G^g L_G^{1-g} + \frac{\left[1 - t - \dfrac{tb}{2\left(1-t\right)}\right]Y}{P_G} = \frac{2b\left(1-t\right)K_{Gt}^g\left(A_t L_{Gt}\right)^{1-g}}{2\left(1-t\right)\left(b-t\right) - bt} \end{aligned} \tag{2.2}$$

[13] Total tax revenue, $T = \left(Y_t + rK_{Gt} + w_t L_{Gt}\right)t = Y_t\left[t + tb/2\left(1-t\right)\right]$.

[14] Taxes lower the marginal cost of government such that $\lim_{t \to b} P_G = 0$. Furthermore, at the optimal tax rate of $t^* = b$ the government is perfectly efficient as $\lim_{t \to b} G = tY$ and the model collapses to the more typical framework where public capital is an unpaid input.

Both sectors are subject to the accounting constraints, $\overline{K} = K_Y + K_G$ and $\overline{L} = L_Y + L_G$ respectively. The central government would prefer its agents work such that G is provided as efficiently as possible. Thus the competitive and therefore optimal price of G is determined by $\Psi = P_G G - wL_G - rK_G = 0$. Competitively produced G is the optimum and thereby serves as a benchmark. The optimal tax rate in the above framework is simply $t^* = b$ where $\lim_{t \to b} P_G = 0$. In the perfect world benchmark, the elasticity of infrastructure is exactly known, taxes are set optimally, and government services, provided at the competitive equilibrium, are consumed at zero cost.

In an imperfect world, the elasticity of public infrastructure is not exactly known such that $t < b$.[15] Furthermore, if the decentralized public agents accept the value of public capital as well as the wage rate as given, competitive resource markets insure that their opportunity costs equal the private sector rental rate of capital and wage respectively. Therefore, the decentralized public agent maximizes her welfare function given competitive resource markets. The superscript bar over the variable denotes the public agent's inability to directly affect its value.

$$\begin{aligned}\Psi &= P_G G - wL_G - rK_G \geq 0 \\ &= \left[MPL_G - MPL_Y\right] L_G + \left[MPK_G - MPK_Y\right] K_G \geq 0\end{aligned} \tag{2.3}$$

$$\begin{aligned}\frac{\partial \Psi}{\partial L_G} &= \overline{P}_G \frac{\partial G}{\partial L_G} - \overline{w} = \frac{\partial Y}{\partial L_Y} \\ &= MPL_G - \overline{MPL_Y} = \overline{MPL_Y}\end{aligned} \tag{2.4}$$

[15] It is important to note that the optimal tax rate of $t^* = b$ insures that government services are consumed at zero cost. This is because $\lim_{t \to b} P_G = 0$ is independent of how close government production can approximate the competitive equilibrium. This corner solution strictly limits consideration of the tax rate to $0 \leq t < b$.

$$\frac{\partial \Psi}{\partial K_G} = \overline{P}_G \frac{\partial G}{\partial K_G} - \overline{r} = \frac{\partial Y}{\partial K_Y}$$
$$= MPK_G - \overline{MPK_Y} = \overline{MPK_Y} \tag{2.5}$$

Notice that the competitive private wage rate, w, is the marginal labor cost to the public agent. The marginal benefit of working in the public sector is the competitive wage plus the premium available to the public agent. Therefore, resources allocation is determined by marginal benefit equalizing marginal cost. The allocation rules governing labor and capital may be defined as follows.[16]

$$k_G{*} = \frac{K_G{*}}{L_G{*}} = \frac{g}{(1-g)} \cdot \frac{2(1-a-b)(1-t)+b(1-g)}{2a(1-t)+bg} \cdot \frac{K}{L} \tag{2.6}$$

$$k_Y{*} = \frac{K_Y{*}}{L_Y{*}} = \frac{a}{(1-a-b)} \cdot \frac{2(1-a-b)(1-t)+b(1-g)}{2a(1-t)+bg} \cdot \frac{K}{L} \tag{2.7}$$

It is important to note that equations (2.6) and (2.7) are indeed socially sub-optimal allocation algorithms. One way for the social planner to overcome the principle agent problem is to provide public services as if they were competitive. Therefore, the benchmark rules for optimal resource allocation are defined as follows.

$$k_G{}^{pc} = \frac{K_G{}^{PC}}{L_G{}^{PC}} = \frac{g}{(1-g)} \cdot \frac{(1-a-b)(1-t)+b(1-g)}{a(1-t)+bg} \cdot \frac{K}{L} \tag{2.8}$$

$$k_Y{}^{pc} = \frac{K_Y{}^{PC}}{L_Y{}^{PC}} = \frac{a}{(1-a-b)} \cdot \frac{(1-a-b)(1-t)+b(1-g)}{a(1-t)+bg} \cdot \frac{K}{L} \tag{2.9}$$

The only difference between the allocation algorithms is the constant, *2*, that appears in (2.6) and (2.7) but is absent in (2.8) and (2.9). Therefore, it is possible to analyze resource allocation more generally as follows: $k_{Gt} = \overline{q}_G(a, b, g, t, f) \cdot k_t$ and

[16] See Appendix 1 for the derivation.

$k_{Yt} = \overline{q}_Y \left(a, b, g, t, f \right) \cdot k_t$ where q_Y and q_G are the fixed proportions of capital per unit of labor and $f \geq 1$ determines the degree of allocative inefficiency due to decentralization.[17]

I contend that (2.6) and (2.7) represent second best equilibrium allocations resulting from local welfare maximization by decentralized public agents while (2.8) and (2.9) represent the first best socially optimum allocations. Figure 1 illustrates the two benchmark equilibria. Although $\{k_G,k_Y\}^*$ is the second best equilibrium allocation of capital given decentralized agents and $\{k_G,k_Y\}^{pc}$ is the equilibrium allocation of capital given the competitive optimum, that is not to say that other resource allocations may not be considered. In fact, one could analyze the implications of any resource allocation of k_Y versus k_G. For example, perfect information as well as complete impunity of public agents might leads to an allocation of L^m. The problem with L^m is that the public agent effectively gets a larger slice of a smaller pie. Relative to her private sector counterpart, the public agent may be wealthier, but she would still have been better off in absolute terms at L^*. Irrespective of whether the reader believes that L^* is indeed an equilibrium, it is nonetheless clear that L^{pc} is the best that society can hope to achieve and consequently any movement away from L^{pc} is sub-optimal.

There are three important observations regarding how the model can and should be practically applied. First, the model predicts rates of resource usage as a function of the production elasticities and the tax rate. Along these lines, the model also predicts that optimal resource usage is defined by **Error! Reference source not found.** through (2.9) versus second best equilibrium resource usage which is defined by

[17] Any resource allocation may be represented by changing this constant in which a $f = 1$ yields the competitive equilibrium and a $f = 2$ yields the decentralized public agent equilibrium. In Figure 1, any

Error! Reference source not found. through (2.7). Therefore, for any reasonable set of values for a, b, g and t, it is always the case that $z_L* < z_L^{pc}$ and $z_K* < z_K^{pc}$. Thus the further the economy is from optimal allocation, the lower is the relative resource usage by government.[18]

The implications of this observation are relatively straightforward. One might hypothesize that industrialized countries are closer to L^{pc} than developing ones and movement away from L^{pc} represents the prevalence of public inefficiency through corruption. This view that corruption stems from the monopolistic exploitation of public services is discussed in Schliffer and Vishny (1990). But one must take care when making a comparison of this type because the results, $z_L^{LDC} < z_L^{IND}$ and $z_K^{LDC} < z_K^{IND}$ as reflecting greater corruption in the LDC, only holds strictly when a, b, g and t are equivalent across the two countries in question.

The second observation pertains to the coefficient values. They may, and in fact are quite likely to be, different across countries. For the theoretical model to be consistent with the stylized fact that the average government of a developing country employs relatively more of society's labor and relatively less of society's capital than the average government of an industrialized economy, barring differences in tax rate across countries, at least one of the three elasticities, a, b or g, must differ between representative countries.

Consider the simplest scenario first. Assume that the production elasticities of public versus private capital, a and b, and the degree of allocative inefficiency, f, are the same across countries, but the elasticity on capital in the public production function, g, is different.

value $f > 1$ is equivalent to movement to the left of L^{pc} and any value $0 < f < 1$ is equivalent to movement right of L^{pc}.

[18] I only consider allocations to the left of L^{PC}, but the model is not limited thus.

If the productivity of public capital is greater in industrialized countries than in LDC's, the implication is that infrastructure development in industrialized countries is more capital intensive than in LDC's. In terms of the model, $g^{IND} > g^{LDC}$, given $a^{IND} = a^{LDC}$, $b^{IND} = b^{LDC}$ and $f^{IND} = f^{LDC}$, insures that both inequalities, $z_K{}^{IND} > z_K{}^{LDC}$ and $z_L{}^{IND} < z_L{}^{LDC}$, hold.

Alternatively, the output elasticities, a and or b, may also differ across countries. If we assume the elasticity on public capital, b, is constant and that $a^{IND} < a^{LDC}$, then homogeneity requires that $\left(1 - a^{IND} - b\right) > \left(1 - a^{LDC} - b\right)$. This implies that private capital in industrialized countries is less productive than private capital in developing countries.[19] If b actually decreases with development as the endogenous and traditional growth theories suggest, and $a^{IND} < a^{LDC}$, then the inequalities, $z_K{}^{IND} > z_K{}^{LDC}$ and $z_L{}^{IND} < z_L{}^{LDC}$, hold so long as the elasticity of capital in the public sector production function is restricted by $g^{IND} > g^{LDC}$. In other words, while the relative return on private capital is relatively lower in industrialized countries than in developing ones, the relative return on public capital is greater in industrialized economies than in developing ones. Intuitively, this suggests that factories are relatively more productive in developing countries while labor is relatively more productive in industrialized ones. Simultaneously, a courthouse is relatively more effective in industrialized countries while a police officer is relatively more effective in developing ones.

The final observation pertains to the tax rate, t. If a, b, g, and f are equal across countries, then differing tax rates across countries yields $z_L{}^{low\ t} < z_L{}^{high\ t}$ and $z_K{}^{low\ t} < z_K{}^{high\ t}$. Moreover, the further the tax rate is below the optimal rate, the higher is the effective price of public capital. Since the effective tax rate in LDC's is generally less than the effective tax rate in industrialized countries,[20] the model predicts that LDC's will suffer less growth effects due to taxation but at the cost of lower welfare and greater income inequality.

Table 1 presents the relative resources usage rates and the average tax rates for a selection of countries. Given the three observations from above, several testable hypotheses may be drawn from the data. First, compare the USA to the UK where we observe that $z_L^{USA} < z_L^{UK}$ and $z_K^{USA} > z_K^{UK}$. Recall that if a, b, g, and f are equal across these two countries and the only difference is average tax rates, then both the public labor and the public capital usage rates should be lower in the UK. Since they are not, then one or all of the coefficients must be different between the two countries. Second, comparing the USA to France, we observe that $z_L^{USA} < z_L^{France}$ and $z_K^{USA} < z_K^{France}$. If the difference in the average tax rate between these two countries accounts for observed resource usage, then it should be that all of the other coefficients are the equal. Third, comparing the USA to Australia (or Canada), we observe that $z_L^{USA} < z_L^{Australia}$ and $z_K^{USA} < z_K^{Australia}$. Although the observed inequalities are consistent with the USA's lower tax rate, the difference is so marginal that it would seem unlikely to be the root of the difference in labor usage. Fourth, compare the USA to Japan where we observe $z_L^{USA} > z_L^{Japan}$ and $z_K^{USA} < z_K^{Japan}$. Since Japan has a lower average tax rate than the USA, unless the coefficients are different, both usage rates should be lower in the USA. Fifth, compare the USA to Portugal (or Italy).

[19] Nourzad (2000) found no difference in the marginal productivity of public capital between industrialized and developing countries but found that the marginal productivity of private capital to be relatively higher in developing economies.

[20] Average tax rate in European countries is strictly greater than 30% versus the average tax rate in Mexico and Turkey are 17% and 15% respectively. [so. World Bank Indicators, 1994]

Although the usage rates are almost equal, what if the government of Portugal is less efficient than that of the USA due to corruption? If so, then one or all of the coefficients must differ for the usage ratios to be equal. These hypotheses are the subject of the next section.

3. Empirical Estimates of the Coefficients within the Model

The following econometric exercise estimates the values of the coefficients, a, b and g, for a panel of 16 OECD countries. I estimate the three coefficients using the dynamic equivalents to equations (2.1) and (2.2). This is convenient because it is not only theoretically justifiable, but also removes any unit roots in the data. I estimate the following four equations.

$$\frac{\dot{Y}}{Y} = a\frac{\dot{K}_Y}{K_Y} + b\frac{\dot{G}}{G} + (1 - a - b)\frac{\dot{L}_Y}{L_Y} \tag{3.1}$$

$$\frac{\dot{G}}{G} = g\frac{\dot{K}_G}{K_G} + (1 - g)\frac{\dot{L}_G}{L_G} \tag{3.2}$$

$$\frac{\dot{Y}}{Y} = a\frac{\dot{K}_Y}{K_Y} + bg\frac{\dot{K}_G}{K_G} + b(1 - g)\frac{\dot{L}_G}{L_G} + (1 - a - b)\frac{\dot{L}_Y}{L_Y} \tag{3.3}$$

$$\frac{\Delta GDP}{GDP} = a\frac{\dot{K}}{K} + b\frac{\dot{G}}{G} + (1 - a - b)\frac{\dot{L}}{L} \tag{3.4}$$

Equation (3.3) is the simply equation (3.1) with the growth rate of G, equation (3.2), imbedded into it. Equation (3.4) is estimated for comparison purposes. It represents the more typical experiment, following Aschauer (1989), to estimate the elasticity of output with respect to public infrastructure versus private capital. The relationship between the data used to estimate this last equation and those used in the previous three estimations may be summarized by the following accounting constraints, $GDP=Y+G$, $L=L_G+L_Y$, and $K=K_G+K_Y$.

It is preferable to estimate the levels growth of K_G, K_Y, G, and Y than their per capita equivalents for three reasons. First, the theoretical values of the coefficients from the equations defined in levels are identical to those values from the equations in intensive form. Second, modern econometric software allows one to easily impose restrictions, such as constant returns to scale, on the coefficient estimates. Third, it is easier to estimate equation (3.3) in levels. In theory, the growth rates of public and private labor are equal, but in practice they are not. Therefore, the per capita version of equation (3.3) would need to also contain a term for the difference in the rates of growth of L_Y and L_G. Again, given the ability to easily restrict coefficients, estimation of per capita data is unnecessary and, in this case, cumbersome.

The data is an annual panel of 16 countries from 1970 until 1996.[21] The data is from the OECD, who conveniently separates output, fixed capital formation, and employment into the two categories, total industries and producers of government services.[22] Unfortunately, data on the stocks of public and private capital do not exist in a comparable series across countries. This problem has been addressed in many previous studies. The closest paper to this one in terms of data is Ford and Poret (1991), who use essentially the same OECD data that I do.

The econometric methodology employed is similar to the panel study by Nourzad (2000). Table 2 presents the results of the five benchmark estimations, (3.1)

[21] The panel is not exactly balanced. Canadian data is from 1970-93. Danish data is from 1970-95. German (west) data is from 1970-92. Luxemburg's data is from 1970-91. Dutch data is from 1970-93. Portuguese data is from 1977-93. Swedish data is from 1970-94. British data is from 1971-95. Using a smaller balanced panel of 15 countries from 1971-95 does not significantly change any of the results.

[22] The data source is various years of the OECD, *National Accounts, Detailed Tables, Volume II*. The relevant tables are 4, 12 and 15.

through (3.4) plus one additional, i.e. equation (3.1) with a fitted value from equation (3.2).[23] They are specified as classical linear regression models [CLRM] with no control for country or period specific effects. Table 3 presents the results of the five equations specified as one way fixed effects models with controls for country specific effects [FE-country] and period specific effects [FE-period].[24] The capital and labor usage ratios reported in the previous section suggest likely differences in coefficients values across countries. In light of this, I interact the coefficient estimates of a, b and g for the entire panel with the country dummies. Table 4 reports the results of the five equations when the coefficient estimates are allowed to vary across countries.

The results from all of the experiments suggest that only the productivity of private capital differs across countries. The best estimates of those differences result from estimation of equation (3.3):[FE-period + FE-alpha] on table 4. Furthermore, the productivity of government represented by b, which is statistically the same across OECD countries, is significant and equal to anywhere from .11 to as high as .37. In general though, the data suggest that business cycles are indeed important and the coefficient on G is closer to the lower estimate. This is in sharp contrast to the most of the literature that follows Ashauer (1989), which finds much higher elasticities of output with respect to public infrastructure. It is interesting to note that the highest estimate for b results from estimation of equation (3.4) on Table 2. Recall that this equation represents the more typical experiment

[23] This effectively makes public infrastructure endogenous in the private production function and thereby allows some feedback from equation (3.2) to equation (3.1). See Flores de Frutos and Pereira (1993) for additional discussion.

[24] The corresponding random effect model specification of the five equations in Tables 2 and 3 were also tested. Based on Hausman's specification test, in all cases, the random effect models may be rejected at high levels of significance in favour of the corresponding fixed effect formulations. A two way fixed effects model with controls for country as well as periods was also estimated. The results suggest that in the presence of period effects, country effects are not jointly significant. In the presence of country

to estimate the productivity of public infrastructure. The estimates of g suggest that the relative productivity of public capital also does not vary across OECD countries and is equal to something between .06 and .22. Unfortunately, this is by no means a strong result.

Using the estimates from table 4, we may now address the hypotheses of the previous section. As the empirical results suggest, I assume that b and g are constant across countries while a varies. We observe that $z_L^{USA} < z_L^{UK}$ and $z_K^{USA} > z_K^{UK}$. The tax effect can explain the labor inequality but not the observed capital inequality. Using the estimates from equation (3.3):[FE-period + FE-alpha], the elasticity of private capital in the USA is .14 versus that of the UK is .22. This is exactly what we should expect to find.

The second observation is that $z_L^{USA} < z_L^{France}$ and $z_K^{USA} < z_K^{France}$. Since the tax rate of the US is lower than that of France, we would have expected that the value of a to be very close to that of the USA. In fact, it seems that α_{FRA}=.19, which is above the US estimate. Since the difference in private capital productivity between France and the USA is smaller that that between the UK and the USA, it is still quite possible that the theory is consistent with the empirical evidence.

The third observation is that $z_L^{USA} < z_L^{Australia}$ and $z_K^{USA} < z_K^{Australia}$. The estimates of a suggest that α_{AUT}=.18, which is again higher than in the USA. Since the tax rate difference is so small, the higher productivity of private capital in Australia is to be expected.

The fourth observation is that $z_L^{USA} > z_L^{Japan}$ and $z_K^{USA} < z_K^{Japan}$. Given the tax rate differential alone, we would expect both usage rates to be lower in Japan. The

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effects, period effects are still jointly significant, but less so that the one way fixed effects model with controls for period.

estimate of private sector productivity in Japan is α_{JAP}=.27, which is much higher than the US estimate. In this case, the theory fails somewhat since the higher private capital productivity in Japan should raise public labor usage and lower public capital usage.

The final observation considers the usage rates between the USA and Portugal which are almost equal. The higher tax rate in Portugal, ceteris paribus, drives the expected usage rates of both public capital and public labor up. The observed productivity of private capital in Portugal, which is α_{POR}=.18, reinforces higher public labor usage but implies lower public capital usage. The only possible offsetting factor that might explain the observed similarity in the public labor usage rate is if Portuguese government is less efficient due to corruption than that of the US, i.e. $f^{POR} > f^{USA}$. It is interesting to note that of the OECD countries sampled, the average bureaucratic efficiency is 8.90.[25] Only in Italy (BI_{ITA}=6.33) and in Portugal (BI_{POR}=5.58) are the indices lower than in France (BI_{FRA}=8.25). In other words, if greater of autonomy of public agents leads to more corruption, and if that is indeed what is captured by these bureaucratic efficiency indices, then again, the theory is quite consistent with the evidence.

In summary, the data, for the most part, support the explicit specification of the private aggregate production function versus the public aggregate production function. Given further study with other data, the next step is to explicitly consider developing countries versus industrialized ones. I leave that to the future.

[25] Business International (BI) corruption indicator average 1980-1993, collected by Mauro (1995): 10 (lowest corruption), 0 (highest corruption)

4. Theoretical Implications of the Model

The theoretical implications of the model are far reaching. Almost any endogenous growth model that is based on an aggregate production function framework, albeit with microeconomic foundations, can be easily altered to reflect a productive government with real resources costs. By doing so, one can compare the original work which reflects a trivially produced public sector where $G = tY$ to a more robust version that includes a decentralized public sector as a departure from the utopian optimum. Although there is a wealth of literature to choose from, some well known papers on endogenous growth that readily lend themselves to this analysis include Lucas (1988), Barro (1990), Romer (1986, 1990), and Aghion and Howitt (1992).

The paper by Lucas (1988) is arguably a synthesis of the research pioneered by Ramsey (1928), Cass (1965), and Koopmans (1965) and the model attributed to Solow (1956) and Swan (1956). Lucas describes the basic endogenous growth model where output is a function of technology augmented labor and capital. The rate of return on investment and the consequent saving are independent of the growth rate which is described by the Euler equation, *growth_rate=IES*×(*MPK-discount_rate*)-*tech_growth_rate*, where *IES* is the intertemporal elasticity of substitution.

In an extension to this line of research, Barro (1990) introduces government as a productive input, which is homogenous of degree one, within the dynamic production function. Public services are trivially produced and completely funded by a proportional income tax. For all tax rates, $t > 0$, the private return on investment is lower than the social return and the decentralized economy grows slower than is socially optimal. The optimal tax rate and consequent size of government, using the notation from above, is simply $t^* = g/y = b$.

Appendix 2 describes the analytical solution of a productive government with decentralized agents within an endogenous growth model. The model has labor and capital as in Lucas (1988) while public services are subject to diminishing returns as in Barro (1990). Therefore, suppose there exist two separable agents with identical preferences whose competition results in the allocation of the economy's available labor and capital. The agents maximize welfare, which is a function of consumption per worker. Upper case characters represent levels and lower case represent per effective capita. n is the exogenous growth rate of population and c is the exogenous growth rate of technology.

$$\begin{aligned} Max\ W &= \int_{t=0}^{\infty} \left[U\left(\frac{C_{gt}}{L_{Gt}}\right) \cdot L_{Gt} + U\left(\frac{C_{yt}}{L_{Yt}}\right) \cdot L_{Yt} \right] e^{-rt} dt \\ &= \int_{t=0}^{\infty} \left(\frac{c_{gt}^{1-q} - 1}{1-q} \cdot A_0^{1-q} L_{G0} + \frac{c_{yt}^{1-q} - 1}{1-q} \cdot A_0^{1-q} L_{Y0} \right) e^{\left[(1-q)c+n-r\right]t} dt \end{aligned} \tag{4.1}$$

The representative private agent, denoted by subscript y, owns the representative firm and receives income from wages, rental capital and sales of output to the entire economy. Taxes are assessed on wage and capital income. I assume that all capital is ultimately owned by the private sector. After tax income from final good sales accounts for private sector resource income but does not account for capital rental by the public sector.

$$Y_t = K_{Yt}^a G_t^b \left(A_t L_{Yt}\right)^{1-a-b} = r_t K_{Yt} + w_t L_{Yt} + P_{Gt} G_t \tag{4.2}$$

$$\left(Y_t + r_t K_{Gt}\right)\left(1-t\right) = P_{Yt} \cdot C_{Yt} + S_{Yt} \tag{4.3}$$

The price of y must be explicitly specified since it is possible to have greater dollar income than physical output. Thus $P_Y = \dfrac{Total\ Economywide\ Income}{Y}$.[26]

The representative public agent, denoted by subscript g, receives wage income which is taxed plus whatever public goods premium that she can garner from the administration of the public sector. Public goods are produced by the government at resource cost.

$$\Psi_t = P_{Gt}G_t - r_t\left(1-t\right)K_{Gt} - w_t\left(1-t\right)L_{Gt} \tag{4.4}$$

$$G_t = \tilde{G}_t + \frac{tY_t}{P_{Gt}} = \tilde{G}_t + \frac{tG_t}{b} = \frac{b}{b-t}\tilde{G}_t = \frac{b}{b-t}K_{Gt}^{g}\left(A_tL_{Gt}\right)^{1-g} \tag{4.5}$$

$$\Psi_t + w_t\left(1-t\right)L_{Gt} = P_{Yt}\cdot C_{Gt} + S_{Gt} \tag{4.6}$$

The two representative agents independently choose consumption and consequent saving to maximize their independent optimal consumption paths. Capital evolves as a result of the sum total of savings. Since the two agents have identical preferences, they necessarily have equal saving rates as well.[27] Thus the growth rate of consumption per effective capita for either agent is defined by a modified golden rule.

$$x = \frac{\dot{c}_y}{c_y} = \frac{\dot{c}_g}{c_g} = \frac{1}{q}\left[r\left(1-t\right) - r - qc\right] \tag{4.7}$$

$$\dot{K}_t = S_{Yt} + S_{Gt} \tag{4.8}$$

The dynamic adjustment process is similar to Lucas (1988). The results may be summarized as follows.

$$\frac{\dot{c}_{it}}{c_{it}} = \frac{\dot{k}_{it}}{k_{it}} = \frac{\dot{k}_t}{k_t} = x\ ,\ \frac{\dot{y}_t}{y_t} = \frac{\dot{y}_t}{y_t} = x\left(a + bg\right) \text{ and } \frac{\dot{g}_t}{g_t} = xg \tag{4.9}$$

[26] If taxes are set optimally at $t^* = b$, then the $P_Y = 1$.

There are some points worth noting. The relationship between the growth of output and that of consumption is independent of the effects of decentralized public agents. The further the country is from the competitive benchmark, the smaller is the proportion of public to private capital, and the lower is the marginal product of private capital. To understand the significance of equation (4.9), consider the Lucas (1988) model altered to include a Barro (1990) type government that is trivially produced from taxes such that $g = t\,y$. The dynamics of a Barro-Lucas framework are summarized follows.[28]

$$\frac{\dot{c}_t}{c_t} = \frac{\dot{k}_t}{k_t} = x \quad \text{and} \quad \frac{\dot{y}_t}{y_t} = \frac{\dot{g}_t}{g_t} = \frac{xa}{(1-b)} \tag{4.10}$$

The differences between the model with a productive government and Lucas-Barro model lie in the capital labor ratios at the steady state and in the rate of adjustment that leads to the steady state. For any set of values for a, b, g and f, the steady state capital labor ratio, given the explicitly specified public sector, is far greater than the Lucas-Barro framework predicts. For example, consider a hypothetical starting point of one unit of capital and one unit of labor with the technology parameter, A, also equal to one, such that the initial effective capital labor ratio is $K_0 / A_0 L_0 = k_0 = 1$. It is important to remember that that the transition to the steady state is unique.[29] Figure 2 plots the simulation results from three separate experiments, a Lucas-Barro model, a Lucas model with a productive albeit decentralized public sector (i.e. f=2) and a Lucas model with a decentralized competitive public sector (i.e. f=1). The coefficients on K and G are $a = .35$ and $b = .25$ respectively

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[27] See Appendix 2 for proof of this assertion.

[28] Equation (4.10) results from a single infinitely lived agent that maximizes $\int_{t=0}^{\infty} e^{-rt} U\left(\frac{C_t}{L_t}\right) L_t \, dt$ subject to, $Y_t = K_t^a G_t^b \left(A_t L_t\right)^{1-a-b}$, $G_t = t\,Y_t$, and $Y_t\left(1-t\right) = C_t + \dot{K}_t$.

[29] See the textbook by Romer (2001) for details.

and the tax rate is $t=.24$.[30] In each case, the heavy line is the unique saddle path to the steady state and the lighter line is the reference describing all points in which the growth of effective per capita capital is zero.

The models each converge to their respective steady states defined by modified golden rules. The simulation values at the steady state equilibrium are summarized in Table 5. Notice that the steady state effective capital labor ratio is $k^*=6.73$, $k^*=39.54$ and $k^*=56.16$ respectively. The saddle path is much steeper in the Lucas model and the steady state consequently occurs much sooner. Another way to compare transitions to the steady states is to consider the growth rates at any point away from the steady state. Figures 3 plots the changing growth rate of consumption per effective capita as each model converges to its steady state. Again, it is quite evident that the trivializing assumption that $G_t = t\,Y_t$ implies a much quicker transition to the steady state with a lower capital labor ratio and lower per capita consumption. A productive albeit decentralized government model predicts almost six times more capital per unit of effective labor at the steady state than the Barro-Lucas model. The competitive benchmark predicts over nine times more capital per unit of effective labor at the steady state. In general, this suggests that government efficiency is reflected by a higher effective capital labor ratio.

The empirical results herein reinforce the results of Nourzad (2001) in that only a differs across countries, while b and g are constant. Figure 4 depicts the saddle path with a lower elasticity of private capital ceteris paribus, a lower tax rate ceteris paribus and a lower private capital elasticity combined with a lower tax rate. As the productivity of private

[30] The other relevant coefficient values pertaining to figures 2, 3 and 4 are the coefficient on public capital, g=.5, the growth rate of technology, c=.03, the growth rate of population, n=.02, the coefficient of relative risk aversion, q=.99, and the discount rate, r=.03.

capital relative to private labor falls, so does the steady state effective capital labor ratio and the per capita consumption. Therefore, if developing countries suffer from lower private capital productivity, then one would expect tendency toward a lower steady state effective capital labor ratio and a quicker transition to the steady state. Furthermore, a shock to an LDC should have a more pronounced growth effect but a shorter life span.

Lower private capital productivity combined with a tax that is relatively further from its optimal rate reinforce each other by both implying lower steady state effective capital labor ratios. If one assumes that industrialized countries also have more efficient resource allocations between sectors (i.e. f is closer to 1 than to 2), then developing countries tend toward a steady state at far lower effective capital labor ratio and per capita consumption than do industrialized ones.

Finally, consider the steady state equilibrium values across the various scenarios summarized in Table 5. There are several points worth noting. The effective capital labor ratio in row 6 decreases as the scenario differs from the benchmark competitive equilibrium with optimal taxes. Although lower private capital productivity (i.e. columns 5 & 6) lowers the consumption per effective capita (i.e. row 3) and consequently the welfare per capita (i.e. rows 33 & 34), the implications of sub-optimal taxation are greater (i.e. compare columns 3 & 4 versus columns 3 & 5). This is also evident in the effect of taxation on the level of public goods per effective capita (i.e. row 7).

Conclusion

This paper brings to light the problems resulting from the generally accepted, although wholly inadequate convention of assuming that publicly produced goods are provided to the private sector at zero user cost and funded solely by tax revenues. By ignoring the public-private accounting constraint over resources, the theoretical proportion of government to capital is lower and estimates of government productivity are consequently higher. Alternatively, explicit consideration of the public private accounting constraint over resources yields empirical estimates of the elasticity of public goods at closer to .13. This is far lower than that found by Ashauer (1989) and others. From the theoretical standpoint, the assumption greatly skews the equilibrium capital labor ratio as well as the per effective capita consumption. The inclusion of the accounting constraint dramatically changes the relative price of private versus public goods and consequently the relative price of consumption and saving. In a growth framework, the transition to the steady state becomes more gradual and the resulting equilibrium is characterized by relatively more capital per unit of effective labor at a higher level of consumption per effective capita.

There are several conclusions that may be drawn from the preceding analysis. First, any degree of decentralization exploited by government agents will necessarily lead to sub-optimal allocations of labor and capital between the public and private sectors. Therefore less publicly produced goods will be offered to the private sector at a higher price. The resulting rent accrued by public agents is spent along with private sector income on private sector output. This effectively means more nominal income chasing less real goods. Second, the principle agent problem may be effectively addressed by a concerned central government by setting taxes exactly equal to the productivity of public goods. Unfortunately, this is highly unlikely.

There are two caveats to this conclusion. One, tax substitution and the ability to levy consumption taxes would likely lower the optimal income tax rate.[31] Two, the government can set the tax rate above the estimated optimum. Although inefficient, the principal agent problem effectively disappears irrespective of the degree of decentralization as the price of public goods approaches zero.

The model predicts the more inefficient the allocation of resources and the further the tax rate is from optimal, the lower is the steady state effective capital labor ratio and the steeper is the saddle path to the equilibrium. Therefore, if governments in developing countries are less able or desirous to control their agents, they are effectively more decentralized and thus have greater freedom to exploit their positions of power. In such cases, we should expect a steeper saddle path to a steady state defined by less capital and consumption per capita. An intuitive interpretation is that business cycles in developing countries are shorter than those in industrialized countries but have higher amplitudes, or simply are more volatile.

The policy implications are quite straightforward. To an economy on the saddle path somewhere to the left of the steady state, an injection of either technology or labor would decrease the effective capital labor ratio, push the economy further left along the saddle path and manifest itself in the form of temporarily higher growth rates. The steeper the saddle path, the more pronounced is the growth effect of a shock but the shorter its life span. Therefore if a goal of an LDC is to lengthen the business cycle and decrease its amplitude, i.e. stabilization, then assuming the empirical results herein are correct, the only options are either to increase the productivity of private capital or set the tax rate at its optimum.

[31] See Barreto and Alm (2002).

The paper implicitly assumes that allocation of capital and labor across the two sectors is determined via some kind of long run institutional process. In the model, this is represented by the coefficient f approaching 1. Any improvement in resource allocation improves private agents' welfares at the expense of public agents' in the sense that public agents get a relative smaller piece of a bigger pie. Since only private sector output may ultimately be consumed, it is a positive sum gain for the whole economy. In other words, the model conforms to the stylized fact regarding institutional development in which governments of industrialized countries tend to better serve their respective populations while governments of developing countries tend to better serve themselves. Moreover, the model depicts how industrialized countries not only tend to a higher equilibrium capital labor ratio and per capita consumption, but they do so via business cycles that are longer and more stable.

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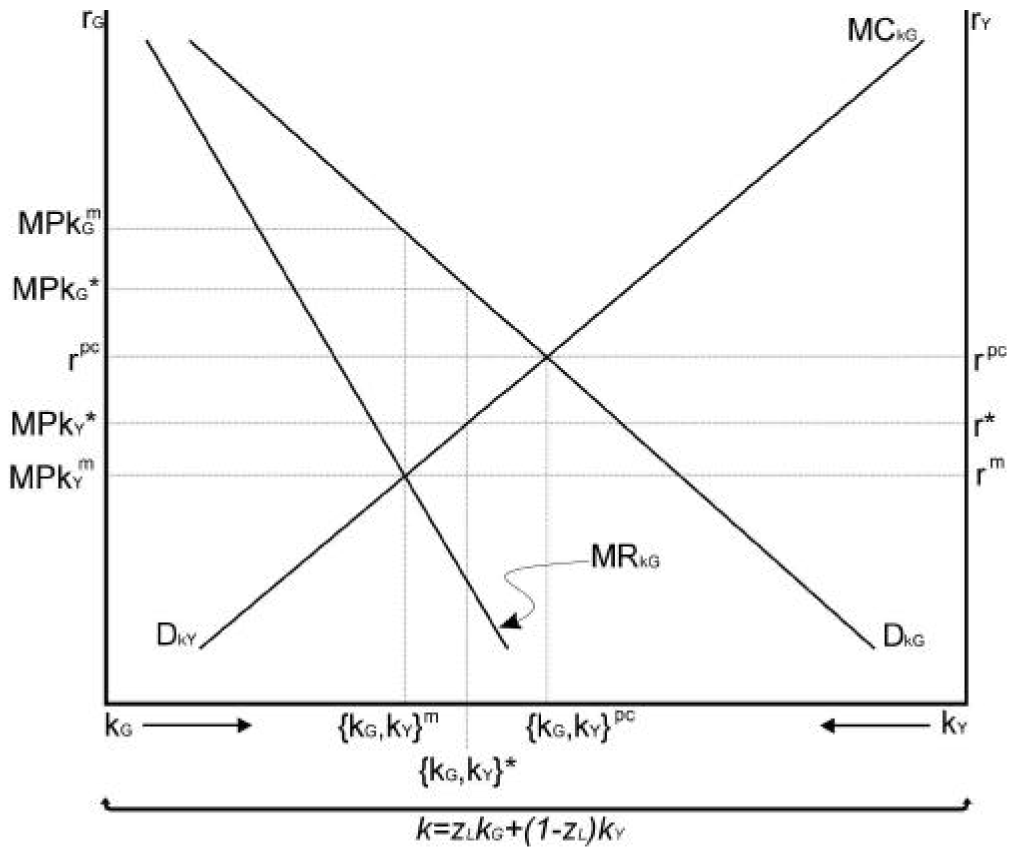
Figure 1



rG
rY
MCkG
MPkG m
MPkG*
r pc
r pc
MPkY*
r*
MPkY m
r m
MRkG
DkY
DkG
kG
{kG,kY}m
{kG,kY}pc
kY
{kG,kY}*
k=zLkG+(1-zL)kY

Figure 2

Simulation Results: Saddle Path Comparisons

$\{a = .35, b = .25, g = .50, t = .24\}$

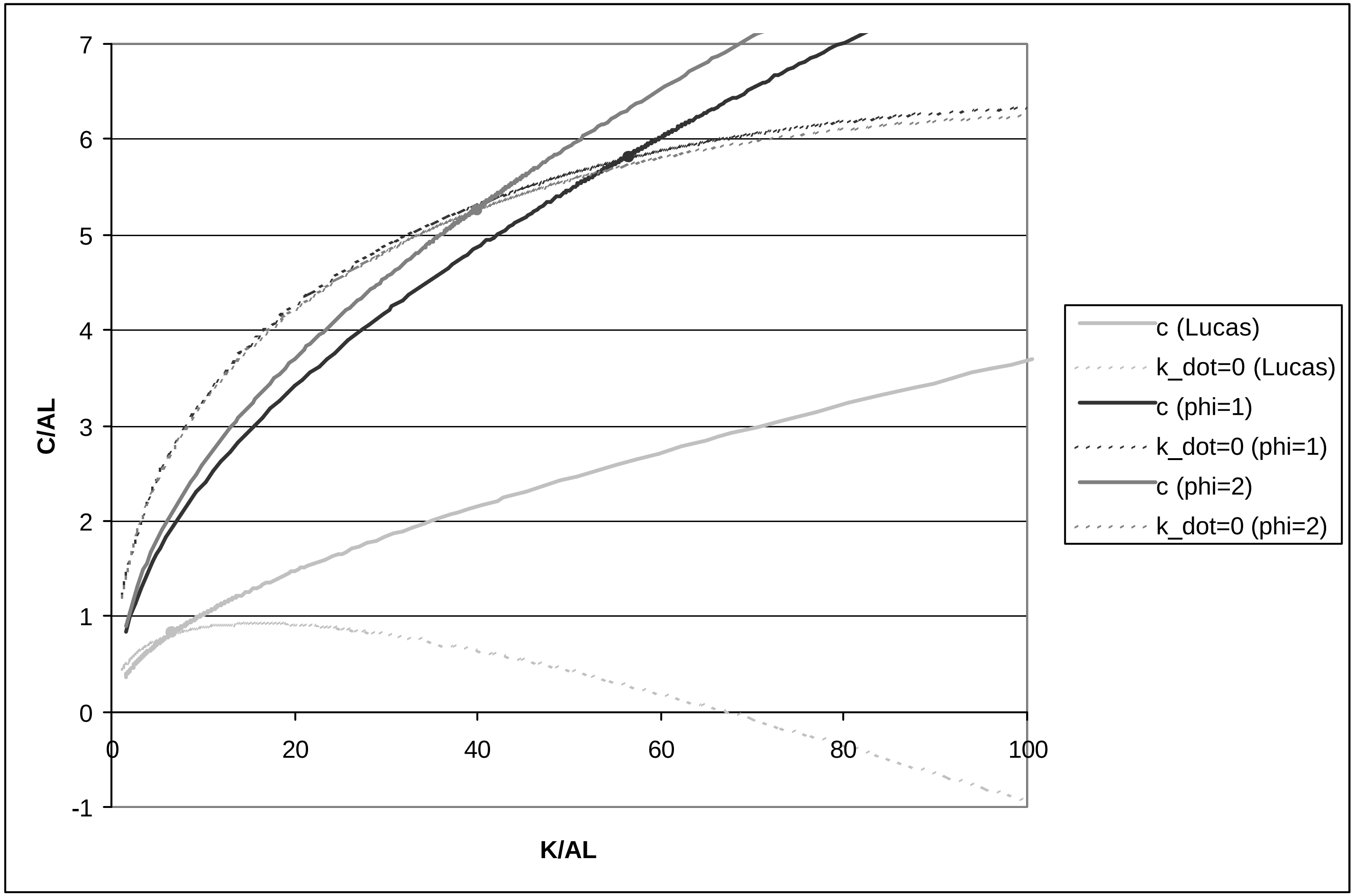


Figure 3
Simulation Results: Growth Path Comparisons
$\{a = .35, b = .25, g = .50, t = .24\}$

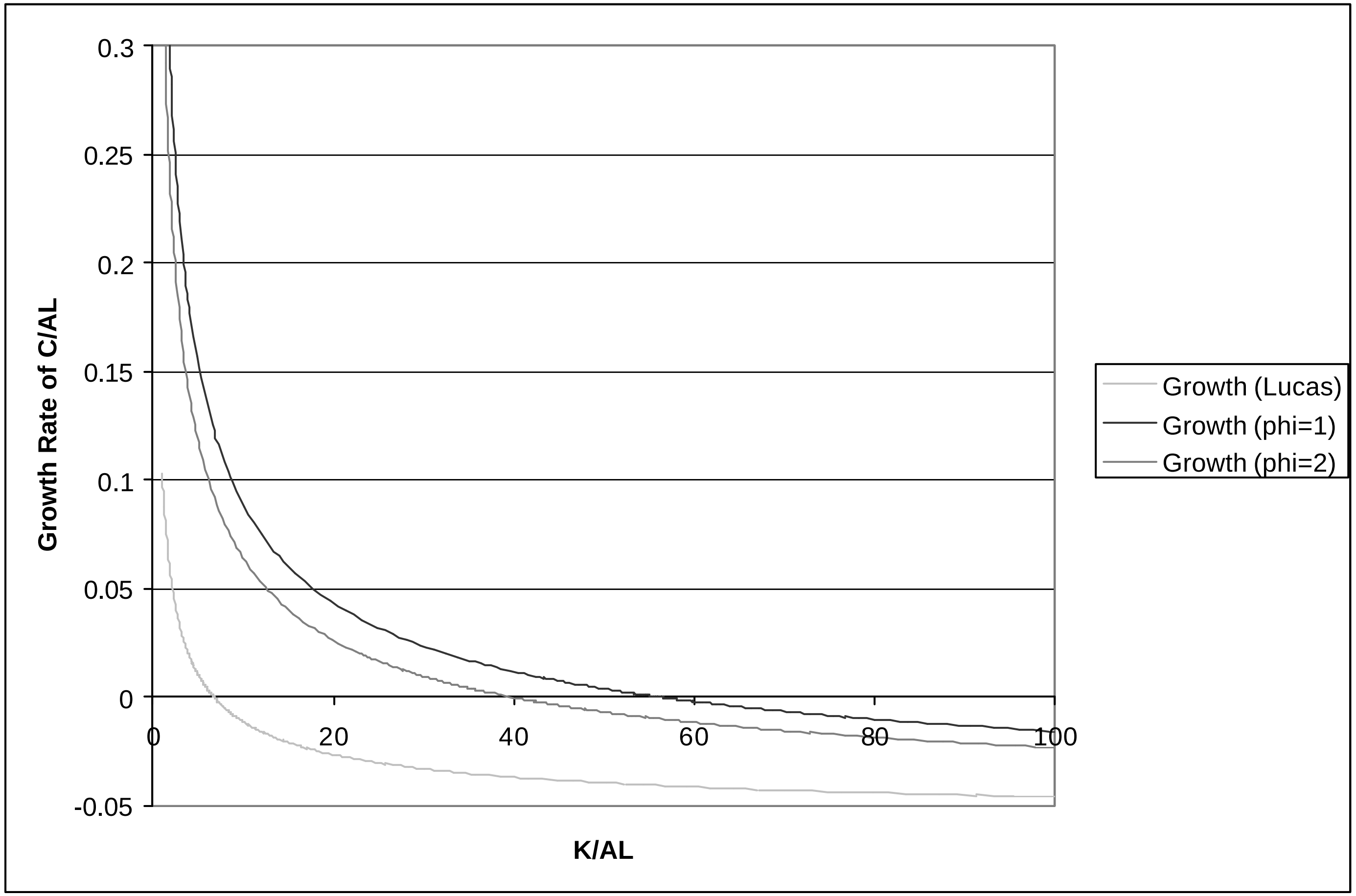


Figure 4
Simulation Results: Saddle Path Comparisons
$\{a = \hat{a}, b = .25, g = .50, t = \hat{t}, f = 2\}$

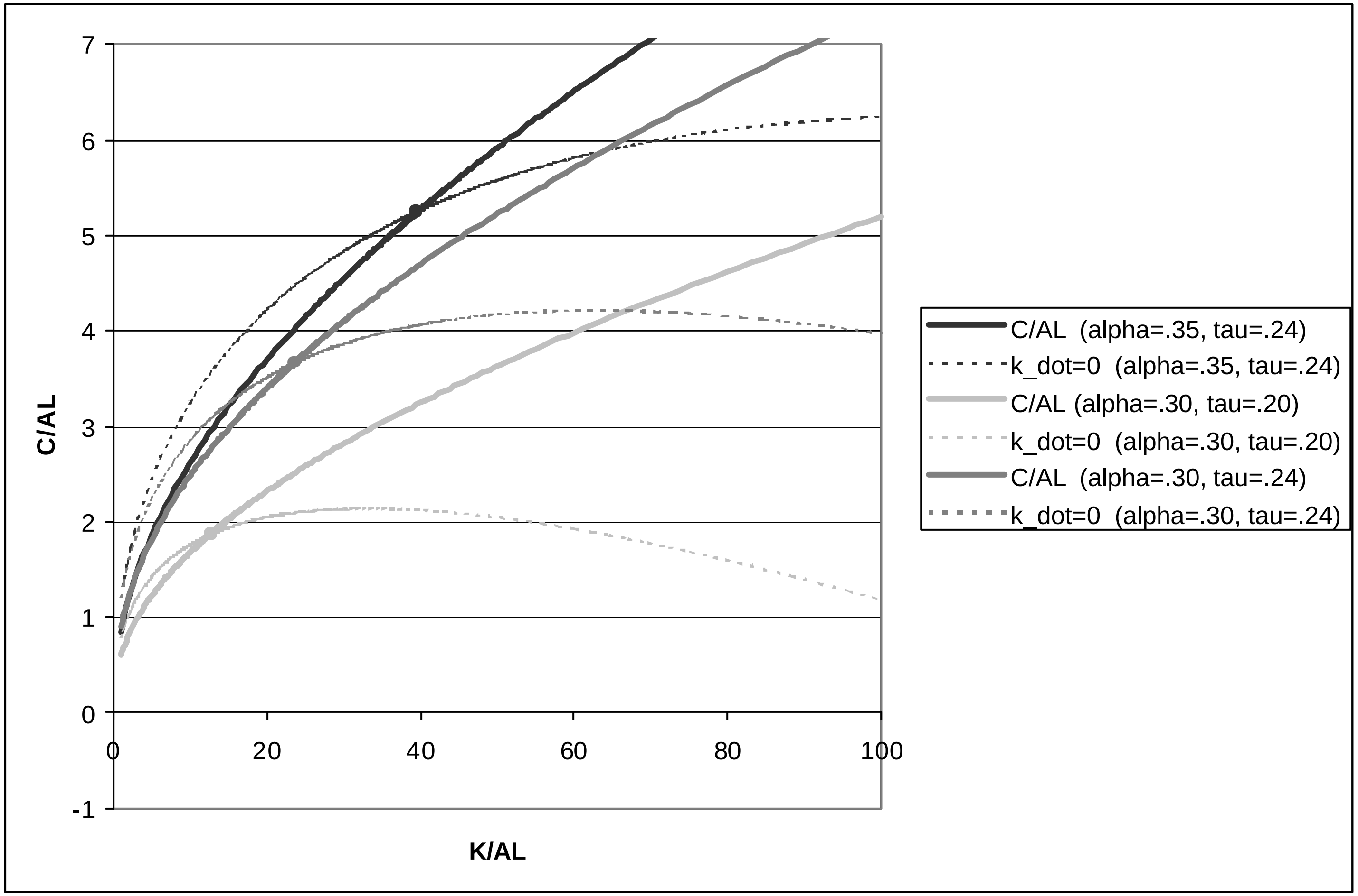


Table 1

	USA	CANADA	AUSTRALIA	MEXICO	JAPAN	KOREA
Ky/K	0.88	0.85	0.86	0.82	0.82	0.86
Kg/K	0.12	0.15	0.14	0.18	0.18	0.14
Ly/L	0.84	0.80	0.76	0.86	0.94	0.90
Lg/L	0.16	0.20	0.24	0.14	0.06	0.10
Avg. tax rate	0.22	0.26	0.24	0.17	0.13	0.19

	FRANCE	ITALY	PORTUGAL	UK	GREECE	TURKEY
Ky/K	0.86	0.90	0.88	0.92	0.76	0.73
Kg/K	0.14	0.10	0.12	0.08	0.24	0.27
Ly/L	0.78	0.85	0.88	0.81	0.82	0.92
Lg/L	0.22	0.15	0.12	0.19	0.18	0.08
Avg. tax rate	0.48	0.39	0.35	0.47	0.40	0.16

Ky/K = average across time of (gross fixed capital formation by total industries)/(gross fixed capital formation) & Kg/K = average across time of (gross fixed capital formation by producers of govt services)/(gross fixed capital formation). [so. *OECD National Accounts*, Vol II, Country Table 3, Items 46, 47 & 50, various years]. {* Data for Mexico and Turkey, [so. 2002, OECD, *Quarterly National Accounts*, country Table 5a]

Ly/L = average across time of (employment of persons by total industries)/(total employment of persons) & Lg/L = average across time of (government employment as percent of total employment)*, [so. 1996, OECD, *Historical Statistics, 1960-1994*, Table 2.15]. {* Data for Mexico = average across time of (employment of persons by producers of govt services)/(total employment of persons), [so. 1997, OECD, *Services, Statistics on value added and employment*, Country Table II]. Data for Korea and Greece = average across time (employment in public administration + defence + education + health + social work)/(total employment), [so. 2001, OECD, *Services, Statistics on value added and employment*, Country Table II]. Data for Turkey = share of public employment to total employment (1985-1999), [so. 2001, OECD Public Management Service].}

Avg. tax rate = average across time of (total tax revenue)/(output by total industries). [so. for tax data from World Bank Indicators, 1994, and for output data from OECD National Accounts, Vol II, Table 12, Items 46, various years]

Table 2

Equation	**(3.1)**	**(3.2)**	**(3.3)**	**(3.4)**	**(3.1) w/ fitted G from (3.2)**
Regression Type	[CLRM]	[CLRM]	[CLRM]	[CLRM]	[CLRM]
Dep. Var.	D**Y/Y**	D**G/G**	D**Y/Y**	D**GDP/GDP**	D**Y/Y**
constant	**0.0432**	**0.0554**	**0.0545**	**0.0365**	**0.0431**
	(0.0059)	(0.0030)	(0.0028)	(0.0051)	(0.0048)
alpha	**0.1666**		**0.1895**	0.0302	**0.1882**
	(0.0278)		(0.0279)	(0.0316)	(0.0280)
beta	**0.2182**		**0.2032**	**0.2839**	**0.2024**
	(0.0691)		(0.0551)	(0.0612)	(0.0525)
gamma		**0.2227**	0.1923		
		(0.0720)	(0.1071)		
Akaike Info. Crit.	-3.8983	-3.0282	-3.8460	-3.7626	-3.8443
Adj. R Sq.	0.2983	0.0305	0.2577	0.3156	0.2594
SEE	0.0343	0.0531	0.0353	0.0367	0.0353

Numbers in parentheses are White corrected standard errors. Numbers in bold typeface indicate significance with confidence of 95% or greater.

Table 3

Equation	(3.1)		(3.2)		(3.3)		(3.4)		(3.1) w/ fitted G from (3.2)*	
Regression Type	[FE-country]	[FE-period]	[FE-country]	[FE-period]	[FE-country]	[FE-period]	[FE-country]	[FE-period]	[FE-country]	[FE-period]
Dep. Var.	D**Y/Y**	D**Y/Y**	D**G/G**	D**G/G**	D**Y/Y**	D**Y/Y**	D**GDP/GDP**	D**GDP/GDP**	D**Y/Y**	D**Y/Y**
alpha	**0.1657**	**0.1224**			**0.1883**	**0.1277**	**0.1878**	**0.1480**	**0.1417**	**0.1187**
	(0.0285)	(0.0283)			(0.0286)	(0.0302)	(0.0329)	(0.0324)	(0.0299)	(0.0300)
beta	**0.2162**	0.1132			**0.2007**	**0.1184**	**0.2868**	**0.1979**	**0.3743**	0.2061
	(0.0705)	(0.0726)			(0.0597)	(0.0376)	(0.0623)	(0.0775)	(0.0544)	(0.3136)
gamma			**0.2159**	**0.0690**	0.1800	0.0679				
			(0.0738)	(0.0337)	(0.1130)	(0.1714)				
Akaike Info. Crit.	-3.8265	-4.0075	-2.9728	-3.3596	-3.7738	-4.0012	-3.6892	-3.7867	-3.8771	-3.9781
Adj. R Sq.	0.2740	0.4240	0.0133	0.3459	0.2329	0.4064	0.2908	0.3721	0.3098	0.3911
SEE	0.0349	0.0316	0.0536	0.0436	0.0359	0.0316	0.0374	0.0352	0.0340	0.0320
16 Countries [chi Sq]	4.2756		9.0659		4.7287		5.4106		13.8845	
26 Years [chi Sq]		**132.1326**		**324.0836**		**224.3183**		**118.7764**		**112.6317**

Numbers in parentheses are White corrected standard errors. Numbers in bold typeface indicate significance with confidence of 95% or greater.
* These two estimations use the fitted value for G from equation (3.2):[FE-period].

Table 4

Equation	(3.1)		(3.2)	(3.3)			(3.4)		(3.1) w/ fitted G from (3.2)*	
Regression Type	[FE-period + FE-alpha]	[FE-period + FE-beta]	[FE-period + FE-gamma]	[FE-period + FE-alpha]	[FE-period + FE-beta]	[FE-period + FE-gamma]	[FE-period + FE-alpha]	[FE-period + FE-beta]	[FE-period + FE-alpha]	[FE-period + FE-beta]
Dep. Var.	DY/Y	DY/Y	DG/G	DY/Y	DY/Y	DY/Y	DGDP/GDP	DGDP/GDP	DY/Y	DY/Y
alpha	0.1517	**0.1159**		**0.1437**	**0.1355**	**0.1289**	**0.2064**	**0.1401**	0.1310	**0.1128**
	(0.1122)	(0.0321)		(0.0522)	(0.0335)	(0.0332)	(0.1022)	(0.0351)	(0.1174)	(0.0316)
beta	0.0960	**0.1154**		**0.1260**	**0.1074**	**0.1090**	**0.2013**	**0.2114**	0.1684	**0.1128**
	(0.0912)	(0.0576)		(0.0418)	(0.0231)	(0.0495)	(0.0844)	(0.0668)	(0.3461)	(0.0316)
gamma			0.2301	0.0606	-0.2100	-0.2593				
			(0.1684)	(0.1743)	(0.1892)	(0.5863)				
*F.E. on coef. rel. to USA***	*alpha*	*beta*	*gamma*	*alpha*	*beta*	*gamma*	*alpha*	*beta*	*alpha*	*beta*
AUT	-0.0367	0.0644	0.4876	0.0383	-0.1600	0.1980	-0.0580	0.1118	-0.0303	-0.0138
	(0.1109)	(0.1231)	(0.3168)	(0.1163)	(0.2968)	(1.2677)	(0.1058)	(0.1116)	(0.1191)	(0.0862)
BEL	0.0262	0.0464	-0.1354	0.0407	-0.0375	0.1733	-0.0213	0.0493	0.0520	0.0418
	(0.1157)	(0.0692)	(0.1922)	(0.0578)	(0.1299)	(0.8482)	(0.1110)	(0.0667)	(0.1215)	(0.0966)
CAN	0.0750	-0.2072	-0.4642	0.0942	-0.2840	1.5149	0.0523	-0.1632	0.0759	-0.0248
	(0.1192)	(0.1174)	(0.3828)	(0.0702)	(0.3244)	(1.4273)	(0.1086)	(0.1041)	(0.1301)	(0.0659)
DNK	-0.0807	-0.0209	-0.1081	-0.0800	-0.0762	0.1796	-0.1397	-0.0270	-0.0656	0.0216
	(0.1087)	(0.0464)	(0.1771)	(0.0554)	(0.0612)	(0.5953)	(0.1000)	(0.0545)	(0.1181)	(0.0641)
FIN	0.0192	0.0202	-0.1127	0.0326	0.0769	-0.0526	-0.0445	0.0105	0.0366	0.0949
	(0.1170)	(0.0564)	(0.1733)	(0.0615)	(0.1820)	(0.8989)	(0.1081)	(0.0541)	(0.1222)	(0.0694)
FRA	0.0247	0.0120	-0.3089	0.0450	0.0639	0.2048	-0.0256	-0.0027	0.0547	0.0705
	(0.1191)	(0.0369)	(0.2238)	(0.0606)	(0.0504)	(0.6047)	(0.1079)	(0.0423)	(0.1240)	(0.0625)
GRW	0.0310	0.0530	-0.1047	0.0448	0.1081	0.4390	0.0040	0.0615	0.0468	0.0916
	(0.1252)	(0.0510)	(0.1665)	(0.0745)	(0.1486)	(0.8380)	(0.1195)	(0.0522)	(0.1322)	(0.0719)
ITA	-0.1092	-0.0285	-0.0985	-0.0965	-0.0082	0.0988	-0.1488	-0.0143	-0.0915	0.0450
	(0.1109)	(0.0522)	(0.1772)	(0.0490)	(0.1646)	(0.7323)	(0.1017)	(0.0590)	(0.1176)	(0.0902)
JAP	0.0949	0.0479	-0.0291	**0.1257**	-0.4790	1.1721	0.0554	0.0499	0.1162	0.1018
	(0.1129)	(0.0562)	(0.1710)	(0.0560)	(0.3365)	(0.9741)	(0.1043)	(0.0552)	(0.1189)	(0.0676)
LUX	-0.1019	-0.0696	-0.0567	-0.0797	0.5671	-0.8561	-0.0357	-0.0134	-0.0802	-0.0597
	(0.1366)	(0.1679)	(0.1763)	(0.0914)	(0.5018)	(1.1774)	(0.1740)	(0.2404)	(0.1409)	(0.2023)
NLD	-0.0866	0.0068	-0.2341	-0.0858	0.0168	0.2125	-0.0738	0.0493	-0.1094	-0.0141
	(0.1253)	(0.0882)	(0.1855)	(0.0799)	(0.1532)	(0.8343)	(0.1223)	(0.0869)	(0.1336)	(0.0745)
NOR	-0.0646	0.1142	0.1357	-0.0495	-0.1529	0.9218	-0.0908	0.1158	-0.0405	0.0981
	(0.1296)	(0.1005)	(0.2576)	(0.0832)	(0.1049)	(1.0244)	(0.1202)	(0.0975)	(0.1342)	(0.1320)
POR	0.0314	0.0556	-0.1255	0.0343	-0.0607	0.6167	-0.0399	0.0226	0.0670	**0.1619**
	(0.1250)	(0.0504)	(0.1809)	(0.0615)	(0.0898)	(0.8276)	(0.1132)	(0.0615)	(0.1212)	(0.0804)
SWE	-0.0992	-0.0527	-0.1510	-0.0922	-0.1582	0.1586	-0.1609	-0.0412	-0.0776	0.0026
	(0.1225)	(0.0743)	(0.1772)	(0.0722)	(0.1324)	(0.8779)	(0.1147)	(0.0731)	(0.1282)	(0.0812)
GBR	0.0652	-0.0189	-0.1915	0.0787	0.0481	0.2603	0.0049	-0.0110	0.0796	0.0353
	(0.1716)	(0.0838)	(0.1726)	(0.1343)	(0.1167)	(0.5972)	(0.1499)	(0.0732)	(0.1787)	(0.0933)
16 Countries [chi Sq]	**38.4624**	11.2940	18.7737	**46.3232**	9.0580	5.0314	**39.5032**	11.7772	**47.8395**	11.3069
Akaike Info. Crit.	-3.9654	-3.9702	-3.3437	-3.9758	-3.9721	-3.9643	-3.7375	-3.7391	-3.9497	-3.9225
Adj. R Sq.	0.4045	0.4073	0.3583	0.4120	0.4098	0.4052	0.3631	0.3641	0.3951	0.3784
SEE	0.0316	0.0315	0.0432	0.0314	0.0315	0.0316	0.0354	0.0354	0.0319	0.0323
26 Years [chi Sq]	**102.6835**	**137.6130**	**143.2692**	**177.4288**	**203.2585**	**189.2812**	**97.2618**	**125.1998**	**104.9775**	**115.5420**

Numbers in parentheses are White corrected standard errors. Numbers in bold typeface indicate significance with confidence of 95% or greater.

* These two estimations use the fitted value for G from equation (3.2):[FE-period].

** Coefficient estimates are relative to the coefficient estimate for the USA (i.e. from equation (3.1):[FE-period + FE-alpha], a_{AUT}=.1150=.1517-.0367).

Table 5

Simulation results at the steady state

		1	2	3	4	5	6
	Coeficient values	α=.35 β=.25 τ=.24	α=.35 β=.25 γ=.50 τ=.24 φ=1	α=.35 β=.25 γ=.50 τ=.24 φ=2	α=.35 β=.25 γ=.50 τ=.20 φ=2	α=.30 β=.25 γ=.50 τ=.24 φ=2	α=.30 β=.25 γ=.50 τ=.20 φ=2
1	ccg = CG / (A * LY)		2.46	5.70	2.74	4.38	2.24
2	ccy = CY / (A * LG)		7.17	5.14	2.46	3.51	1.79
3	cc = C / (A * L)	0.81	5.79	5.24	2.51	3.65	1.85
4	kkg = KG / (A * LG)		61.62	44.11	22.06	32.53	17.28
5	kky = KY / (A * LY)		53.92	38.60	19.30	21.68	11.52
6	kk = K / (A * L)	6.73	56.16	39.54	19.75	23.36	12.37
7	gg = G / (A * LG)	0.36	196.24	166.04	23.48	142.58	20.78
8	yy = Y / (A * LY)		12.10	8.68	4.12	5.69	2.87
9	Y / (A * L)	1.51	8.57	7.20	3.45	4.81	2.44
10	Y/LY		4856	3484	1655	2282	1151
11	Y/L	607	3441	2890	1385	1929	981
12	L	54.48	54.48	54.48	54.48	54.48	54.48
13	K	147119	1227792	864407	431879	510707	270429
14	A	401.32	401.32	401.32	401.32	401.32	401.32
15	Growth Rate	0.00	0.00	0.00	0.00	0.00	0.00
16	Ψ		23375	29495	14132	19691	10013
17	LG		15.87	9.29	8.90	8.42	8.06
18	LY		38.60	45.19	45.58	46.06	46.42
19	LG/L		0.291	0.171	0.163	0.155	0.148
20	KG		392517	164461	78810	109877	55874
21	KY		835275	699946	353069	400830	214555
22	KG/K		0.320	0.190	0.182	0.215	0.207
23	Total Income	25136	189341	159010	79207	106158	56117
24	Public Income		23433	29519	14144	19708	10021
25	Private Income		165908	129491	65063	86451	46096
26	C	17766	126684	114531	54818	79780	40545
27	S	7371	61390	43334	21648	25580	13544
28	Y	33074	187466	157436	75435	105107	53445
29	r	0.08	0.08	0.08	0.07	0.08	0.07
30	w	720712	4.84	3.47	1.65	2.56	1.29
31	PG	1.04	0.04	0.06	0.22	0.05	0.20
32	G	7938	1250103	619049	83895	481641	67213
33	Pub Util per capita		107.14	108.04	107.25	107.76	107.04
34	Priv Util per capita	5.96	108.29	107.93	107.14	107.52	106.80
35	Pub Welfare		1701	1004	955	907	863
36	Priv welfare		4180	4877	4883	4952	4957
37	Total welfare	325	5881	5881	5838	5859	5820

In each of the above simulations, an initial effective capital labor ratio of 1 is imposed and the model is allowed to "grow" for 200 periods such that the level of labor and technology are predetermined by their exogenous growth rates while saving and consequently capital growth is endogenous.

Appendix 1
Derivation of Allocation Rules (2.6), (2.7), (2.8) and (2.9)

$Y = F(L, K, G) = AK_Y^a G^b L_Y^{1-a-b}$ subject to

$$G = H(t, K_G, L_G) = \tilde{G} + \frac{tY}{P_G} = \tilde{G} + \frac{tY}{b\frac{Y}{G}} = \frac{b}{b-t}\tilde{G} = \frac{b}{b-t} AK_G^g L_G^{1-g} \quad \text{where } \tilde{G} = AK_G^g L_G^{1-g}$$

1. Decentralized Government agent maximization problem:

$$\begin{aligned} Max\Psi &= P_G G - wL_G - rK_G > 0 \\ &= (MPL_G - MPL_Y) L_G + (MPK_G - MPK_Y) K_G > 0 \end{aligned}$$

Given the equilibrium condition: $\frac{\partial \Psi}{\partial L_G} = \frac{\partial Y}{\partial L_Y}$ and $\frac{\partial \Psi}{\partial K_G} = \frac{\partial Y}{\partial K_Y}$

$$\frac{\partial \Psi}{\partial L_G} = (MPL_G - MPL_Y) = MPL_Y = w(1-t)$$

$$MPL_G = 2MPL_Y \quad \Rightarrow \quad \frac{b(1-g)}{L_G} = \frac{2(1-a-b)(1-t)}{L_Y}$$

$$\therefore L_G^* = \frac{b(1-g)}{2(1-a-b)(1-t) + b(1-g)} \cdot \bar{L} \qquad \textbf{(2.6)}$$

$$\Rightarrow \quad \frac{L_G^*}{\bar{L}} = z_L^*(a, b, g, t, f = 2)$$

$$\frac{\partial \Psi}{\partial K_G} = MPK_G - MPK_Y = MPK_Y = r(1-t)$$

$$MPK_G = 2MPK_Y \quad \Rightarrow \quad \frac{bg}{K_G} = \frac{2a(1-t)}{K_Y}$$

$$\therefore K_G^* = \frac{bg}{2a(1-t) + bg} \cdot \bar{K} \qquad \textbf{(2.8)}$$

$$\Rightarrow \quad \frac{K_G^*}{\bar{K}} = z_K^*(a, b, g, t, f = 2)$$

$$\therefore k_G^* = \frac{K_G^*}{L_G^*} = \frac{g}{(1-g)} \cdot \frac{2(1-a-b)(1-t) + b(1-g)}{2a(1-t) + bg} \cdot \frac{\bar{K}}{\bar{L}}$$

2. Competitive equilibrium in G solution ➔ $\Psi = 0$

$$\Psi = P_G G - wL_G - rK_G = 0$$
$$= \left(MPL_G - MPL_Y\right) L_G + \left(MPK_G - MPK_Y\right) K_G = 0$$

Given the equilibrium condition: $\dfrac{\partial \Psi}{\partial L_G} = \dfrac{\partial Y}{\partial L_Y}$ and $\dfrac{\partial \Psi}{\partial K_G} = \dfrac{\partial Y}{\partial K_Y}$

$$\frac{\partial \Psi}{\partial L_G} = MPL_G - MPL_Y = 0$$

$$MPL_G = MPL_Y \quad ➔ \quad \frac{b\left(1-g\right)}{L_G} = \frac{\left(1-a-b\right)\left(1-t\right)}{L_Y}$$

$$\therefore L_G{}^{pc} = \frac{b\left(1-g\right)}{\left(1-a-b\right)\left(1-t\right) + b\left(1-g\right)} \cdot \bar{L} \qquad \textbf{(2.10)}$$

$$➔ \quad \frac{L_G{}^{pc}}{\bar{L}} = z_L{}^{pc}\left(a, b, g, t, f = 1\right)$$

$$\frac{\partial \Psi}{\partial K_G} = MPK_G - MPK_Y = 0$$

$$MPK_G = MPK_Y \quad ➔ \quad \frac{bg}{K_G} = \frac{a\left(1-t\right)}{K_Y}$$

$$\therefore K_G{}^{pc} = \frac{bg}{a\left(1-t\right) + bg} \cdot \bar{K} \qquad \textbf{(2.12)}$$

$$➔ \quad \frac{K_G{}^{pc}}{\bar{K}} = z_K{}^{pc}\left(a, b, g, t, f = 1\right)$$

$$\therefore k_G* = \frac{K_G*}{L_G*} = \frac{g}{\left(1-g\right)} \cdot \frac{2\left(1-a-b\right)\left(1-t\right) + b\left(1-g\right)}{2a\left(1-t\right) + bg} \cdot \frac{\bar{K}}{\bar{L}}$$

Appendix 2

Lucas (1988) Endogenous Growth Model with Decentralized Government

$$Max\ W = \int_{t=0}^{\infty} \left[U\left(\frac{C_{Gt}}{L_{Gt}}\right) \cdot L_{Gt} + U\left(\frac{C_{Yt}}{L_{Yt}}\right) \cdot L_{Yt} \right] e^{-rt} dt \text{ where } c_{gt} = \frac{C_{Gt}}{A_t L_{Gt}} \text{ and } c_{yt} = \frac{C_{Yt}}{A_t L_{Yt}}$$

If $L_t = L_{Yt} + L_{Gt}$, then one may define $z_L = \frac{L_{Gt}}{L_t}$, $(1-z_L) = \frac{L_{Yt}}{L_t}$ and $B_L = \frac{(1-z_L)}{z_L}$

$$Max\ W = A_0^{1-q} L_0 \int_{t=0}^{\infty} \left[\frac{c_{gt}^{1-q}}{1-q} \cdot z_L + \frac{c_{yt}^{1-q}}{1-q} \cdot (1-z_L) \right] e^{-[r-n-(1-q)c]t} dt \qquad \text{(A.2.1)}$$

subject to

$$Y_t = F(L_{Yt}, K_{Yt}, G_t) = K_{Yt}^a G_t^b (A_t L_{Yt})^{1-a-b} \qquad \text{(A.2.2)}$$

$$y_t = \frac{Y_t}{A_t L_{Yt}} = k_{yt}^a \left(\frac{g_t}{B_L}\right)^b$$

$$(Y_t + rK_{Gt})(1-t) = P_{Yt} \cdot C_{Yt} + S_{Yt} \qquad \text{(A.2.3)}$$

$$\left(y_t + \frac{r_t k_{gt}}{B_L}\right)(1-t) = P_{Yt} \cdot c_{yt} + s_{yt}$$

$$\Psi_t = P_{Gt} G_t - r_t K_{Gt} - w_t L_{Gt} \geq 0$$

(A.2.4)

$$y_t = \frac{\Psi_t}{A_t L_{Gt}} = B_L P_{gt} g_t - r_t k_{gt} - \frac{w_t}{A_t} \geq 0$$

$$\Psi_t + w_t (1-t) L_{Gt} = P_{Yt} \cdot C_{Gt} + S_{Gt} \qquad \text{(A.2.5)}$$

$$y_t + \frac{w_t (1-t)}{A_t} = P_{Yt} \cdot c_{gt} + s_{gt}$$

$$G_t = \tilde{G}_t + \frac{tY_t}{P_{Gt}} = K_{Gt}^g (A_t L_{Gt})^{1-g} + \frac{tG_t}{b} = \frac{b}{b-t} K_{Gt}^g (A_t L_{Gt})^{1-g} \qquad \text{(A.2.6)}$$

$$g_t = \frac{G_t}{A_t L_{Gt}} = \frac{b}{b-t} k_{gt}^g$$

$$K_t = K_{Yt} + K_{Gt}$$

(A.2.7)

$$k_t = \frac{K_t}{A_t L_t} = \frac{K_{Yt}}{A_t L_{Yt}}(1-z_L) + \frac{K_{Gt}}{A_t L_{Gt}} z_L = k_{yt}(1-z_L) + k_{gt} z_L$$

$$\dot{K}_t = S_{Yt} + S_{Gt} \tag{A.2.8}$$

$$\dot{k}_t = \left[y_t \left(1 - t\right) - P_{Yt} c_{yt} \right]\left(1 - z_L\right) + \left(B_L P_{gt} g_t - P_{Yt} c_{gt} \right) z_L - \left(c + n \right) k_t$$

$$L_t = L_{Yt} + L_{Gt} \tag{A.2.9}$$

$$L_t = L_0 e^{nt} \tag{A.2.10}$$

$$A_t = A_0 e^{ct} \tag{A.2.11}$$

Optimization:

$$L = U\left(c_g\right) + U\left(c_y\right) + m_t\left(\dot{k}\right) + l_t\left[k - k_y\left(1 - z_L\right) - k_g z_L\right]$$

$$\frac{\partial L}{\partial c_y} = \frac{\partial U\left(c_y\right)}{\partial c_y} - m_t P_Y\left(1 - z_L\right) = 0 \quad \Rightarrow \quad \frac{\dot{c}_y}{c_y} = -\frac{1}{q} \cdot \frac{\dot{m}_t}{m_t}$$

$$\frac{\partial L}{\partial c_g} = \frac{\partial U\left(c_g\right)}{\partial c_g} - m_t P_Y z_L = 0 \quad \Rightarrow \quad \frac{\dot{c}_g}{c_g} = -\frac{1}{q} \cdot \frac{\dot{m}_t}{m_t}$$

$$\frac{\P L}{\P k_t} = -m_t\left(n + c\right) + l_t = -m_t\left[n + \left(1 - q\right)c - r\right] - \dot{m}_t \quad \Rightarrow \quad \frac{l_t}{m_t} = r + qc - \frac{\dot{m}_t}{m_t}$$

$$\frac{\P L}{\P k_g} = m_t \frac{\partial y}{\partial k_g} z_L - l_t z_L = 0 \quad \Rightarrow \quad \frac{l_t}{m_t} = \frac{\partial y}{\partial k_g} = \frac{B_L bg\, y}{k_g} - r\left(1 - t\right)$$

$$\frac{\P L}{\P k_y} = m_t \frac{\partial y}{\partial k_y}\left(1 - t\right)\left(1 - z_L\right) - l_t\left(1 - z_L\right) = 0 \quad \Rightarrow \quad \frac{l_t}{m_t} = \frac{\partial y}{\partial k_y}\left(1 - t\right) = r\left(1 - t\right)$$

Growth:

$$\frac{l_t}{m_t} = \frac{\partial y}{\partial k_g} = \frac{\partial y}{\partial k_y}\left(1 - t\right) = r + qc - \frac{\dot{m}_t}{m_t}$$

$$\therefore x = \frac{\dot{c}_y}{c_y} = \frac{\dot{c}_g}{c_g} = \frac{1}{q}\left[r\left(1 - t\right) - r - qc\right] = \frac{1}{q}\left[\frac{B_L bg\, y}{k_g} - r\left(1 - t\right) - r - qc\right] \tag{A.2.12}$$

Dynamic Analysis:

$$\frac{\dot{G}}{G} = g\,\frac{\dot{K}_G}{K_G} + (1-g)(c+n) \qquad \text{(A.2.13)}$$

$$\frac{\dot{g}}{g} = g\,\frac{\dot{k}_g}{k_g} \qquad \text{(A.2.14)}$$

$$\frac{\dot{Y}}{Y} = (a+bg)\frac{\dot{K}}{K} + (1-a-bg)(c+n) \qquad \text{(A.2.15)}$$

$$\frac{\dot{y}}{y} = (a+bg)\frac{\dot{k}_i}{k_i} \qquad \text{(A.2.16)}$$

$$\frac{\dot{\Psi}}{\Psi} = \frac{\dot{Y}}{Y} = \frac{\dot{y}}{y} + c + n \qquad \text{(A.2.23)}$$

$$\frac{\dot{y}}{y} = \frac{\dot{y}}{y}$$

$$\frac{\dot{K}}{K} = x + c + n \qquad \text{(A.2.18)}$$

$$\frac{\dot{k}}{k} = \frac{\dot{k}_i}{k_i}\left[\frac{k_y}{k}(1-z_L) + \frac{k_g}{k}z_L\right] \quad \Rightarrow \quad \frac{\dot{k}}{k} = \frac{\dot{k}_i}{k_i} \quad \Leftrightarrow$$

$$\frac{k_y}{k}(1-z_L) + \frac{k_g}{k}z_L = 1$$

$$\frac{\dot{C}_i}{C_i} = x + c + n$$

(A.2.19)

Effective Per Capita Growth:

$$\therefore \frac{\dot{y}}{y} = \frac{\dot{y}}{y} = x(a+bg)$$

(A.2.20)

$$\therefore \frac{\dot{c}_i}{c_i} = \frac{\dot{k}_i}{k_i} = \frac{\dot{k}}{k} = x \qquad \text{(A.2.21)}$$

$$\therefore \frac{\dot{g}}{g} = xg \qquad \text{(A.2.22)}$$

Levels Growth:

$$\therefore \frac{\dot{\Psi}}{\Psi} = \frac{\dot{Y}}{Y} = x(a+bg) + c + n \qquad \text{(A.2.23)}$$

$$\therefore \frac{\dot{C}_i}{C_i} = \frac{\dot{K}}{K} = \frac{\dot{K}_i}{K_i} = x + c + n \qquad \text{(A.2.24)}$$

$$\therefore \frac{\dot{G}}{G} = xg + c + n \qquad \text{(A.2.25)}$$

Saving Rate:

$$\therefore \bar{s} = \bar{s}_Y = \bar{s}_G = \frac{\left[f a (1 - t) + bg \right](x + c + n)}{f(xq + r + cq) P_Y} \qquad \text{(A.2.26)}$$