Quantifying the Costs of Investment Limits for Chilean Pension Funds[1]

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Abstract

Since its creation in 1981, Pension Funds Administrators in Chile were not free to choose optimal investment portfolios because of a stringent regulation on investment limits. The diagnosis implicit with the imposition of limits was that the Chilean capital market was not deep and that there was an important demand for funds to finance the expansion of the productive sector. As this regulation entails an inefficient combination of risk and return, this paper quantifies its costs.

    Key Words: Pension Funds, Portfolio Choice, VaR.
    JEL Classification: G11, G23, C63.

1 Introduction

Many countries have conducted reforms of their social security systems in recent years, switching from Pay As You Go (PAYG) to Fully Funded (FF) systems with individual accounts. One of the main reasons for such a reform is that the demographic transition observed around the world implies declining birth rates and declining workers to retired ratios.[1]

Usually run by the government, PAYG systems are not generally efficient. Furthermore, due to political pressures, in many cases the funds are used for different purposes. On the other hand, the administration of FF systems have usually been delegated to private firms known as Pension Funds Administrators (PFAs). Competition among them is expected to lead to the efficient investment of the resources and provision of pensions. However, in response to the principal-agent problem that might arise in this market, regulations impose important restrictions on investments. This market is also characterized by compulsory contributions and government guarantees, aspects that might induce a lack of interest on pension products by customers, weakening market competition.

Regarded as a pioneering example of this transition, Chile started its pension fund reform in 1981.[2] Since then, several regulations have been adopted and changed. This paper focuses on the effects of one of them: Namely, the regulation that has prevented PFAs from freely choosing their portfolio allocations. We analyze the potential costs of these limits on the pensions received by those retiring, on the profile of risks and returns that they have faced, and on the welfare consequences of the regulation.[3]

However, any general equilibrium implications that the implementation of the FF system or its regulation might have had are absent from our analysis. Therefore, in quantifying the costs, we build counterfactual scenarios by imposing restrictions that make our analysis as realistic as possible but we do not address some possible benefits of the regulations that might arise in general equilibrium. Corbo and Schmidt-Hebbel (2003) quantify the effects of the Chilean pension fund system on the development of capital market, on resource allocations, and growth.[4]

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[1] For a detailed analysis of pension fund reforms see Valdés (1997).

[2] See Cheyre (1991) and Superintendence of Pension Funds Administrators (2003).

[3] Cardinale (2003) attempts to find the optimal portfolio in the absence of limits for investment abroad, but does not consider the specific investment regulation in Chile.

[4] See Vittas (1996).

The paper is organized as follows: Section 2 briefly describes the regulation on investment limits that PFAs have faced. Section 3 discusses the methodology used to evaluate the consequences of the regulation. Section 4 presents the results. Finally, Section 5 concludes.

2 The Chilean Pension Fund Industry

Being a market in which workers are compelled to contribute (i.e., to buy the product) and may not be well informed about the specific characteristics of what they are buying, the Pension Fund Industry is subject to heavy investment regulation.[5] The services that the PFAs provide to their contributors are also regulated (the pensions that they can offer and the information they have to provide). Prices are somehow regulated as well, in the sense that they have to be a fixed amount per contribution and/or a percentage of the taxable income per contribution.

The underlying assumption for these regulations is that agency problems require the implementation of mechanisms intended to ensure the safe and adequate management of the funds. When the system was designed, competition was expected to lead to adequate risk-return combinations offered, low prices, and efficient provision of services. Nevertheless, intending to protect uninformed customers, the regulation limited differentiation and competition, considered necessary in the case of a compulsory product with government guarantees involved.

2.1 Investment Regulation

Intended to guarantee the safekeeping of Pension Funds, the regulation states that resources must be invested only in instruments that are authorized by Law (DL 3.500 of 1980). These instruments are financial assets on public offer, if not issued by the government or the Central Bank of Chile or other country, their issuers are supervised by some government agency like the

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[5]Even after 20 years of existence of the Chilean pension system, the affiliates appear not to be well informed about prices, returns and other important variables. According to surveys conducted in 2001 (Barómetro-CERC) and 2002 (Encuesta HLSS), more than 90% of the affiliates did not know how much they were charged by PFAs as administration fees. Lack of information on pensions or financial education in general appear not to be a Chilean but a worldwide characteristic (Bernheim, 1998; D'ambrosio, 2003).

Superintendence of Securities and Insurance, Superintendence of Banks and Financial Institutions or their similar in other country.[6]

Among the financial instruments that are currently authorized by the Pension Fund Law we find the following: State-issued titles of the Central Bank of Chile and the General Treasury of the Republic, Previsional Past-Service Bonuses, instruments issued by financial institutions (deposits, promissory notes, mortgage notes, bonds and stocks), corporate bonds, stocks, shares of investment funds and foreign instruments. Within this last category there are instruments issued by States, Governments, and corporate bonds and stocks. Pension Funds are also allowed to carry out hedging operations by using derivatives in domestic and international markets.

Pension Funds are allowed to invest on an extensive list of instruments. However, each of them must be classified and approved as valid investment instruments by a Risk Rating Commission (CCR). In the case of debt instruments, they are required to have a rating between AAA and BBB or equivalent, except in the case of National State Issued instruments which are authomatically approved. In the case of stocks, they have to be explicitly approved by the CCR, or meet some specific requirements with respect to results and assets.

Moreover, the Law specifies a range for the maximum percentage of the fund that can be invested in each instrument and the Central Bank sets the actual limit within this range. There are limits per instrument, per issuer, per risk, per group of instruments, and some specific limits for issuers that have property relations with the pension fund manager.

The limits per instruments have been slackening significantly over time (see Table 1). In 1981 investment was allowed only on national fixed income instruments. The maximum limits for state issued instruments, instruments of financial institutions (such as mortgage notes, deposits and bonds) were of 100%. In the case of corporate bonds, the limit was of 60% and no variable income investment or investment abroad was accepted. As the local capital market developed, investment on some stocks was allowed, with a maximum limit by Law of 30% in 1985. Investments on real estate stocks and stocks from corporations with concentrated property were authorized in 1989 with an overall limit for stocks that was still of 30%. In 1990 a new instrument was introduced: shares of investment funds, with a limit that went up to 20%.

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[6]Walker and Valk (1995) analyze investment regulations and their performance.

Table 1: Changes in the Investment Limits

Instrument	1981	1985	1989	1990	1994	1996	1999
State Issued	100%	50%	50%	45%	35% / 50%	35% / 50%	35% / 50%
Mortgage Notes	40% / 100%	40% / 100%	40% / 100%	40% / 100%	35% / 50%	35% / 50%	35% / 50%
Fixed income issued by Financial Inst.	40%[1] / 100%	40%[1] / 100%	40%[1] / 100%	40%[1] / 100%	30% / 50%	30% / 50%	30% / 50%
Bonds public & private corporations	60% / 100%	30% / 100%	30% / 100%	30% / 100%	30% / 50%	30% / 50%	30% / 50%
Stocks Open Corp.	Not Allowed	10% / 30%	-	10% / 30%	30%[2] / 40%[2]	30% / 40%	30% / 40%
Real State Corp. Stock	Not Allowed	Not Allowed	10% / - / 30%	10% / 20% / 30% / 40%	10% / 20%	10% / 20%	10% / 20%
Stocks Open Corp. (Concentrated)	Not Allowed	Not Allowed	10%	10% / 30%	-	-	-
National Investment Funds	Not Allowed	Not Allowed	Not Allowed	10% / 20%	5% / 10%	5% / 10%	5% / 10%
International Investment Fund	Not Allowed	Not Allowed	Not Allowed	Not Allowed	Not Allowed	3%[3] / 6%[3]	3%[3] / 6%[3]
Fixed Income Foreign Instruments	Not Allowed	Not Allowed	Not Allowed	1%[4]	6% / 6% / 12% / 12%	6% / 6% / 12% / 12%	10% / 10% / 20% / 20%
Variable Income Foreign Instruments	Not Allowed	Not Allowed	Not Allowed	Not Allowed	3% / 6%	3% / 6%	5% / 10%
Hedging Instruments	Not Allowed	Not Allowed	Not Allowed	Not Allowed	5% / 15%	5% / 15%	10% / 25%
Others approved by the Central Bank	Not Allowed	Not Allowed	Not Allowed	Not Allowed	1% / 5%	1% / 5%	1% / 5%

Source: Superintendence of Pension Funds Administrators (SAFP).

[1] The limit is reduced to 30% if the duration is shorter than 1 year.

[2] The distinction between corporations with concentrated property and unconcentrated property was eliminated.

[3] This limit is joint with the variable income investment abroad.

[4] A 1% limit for 1 year was established and then it was increasing by 1% each year up to 5 years. After the fifth year it was supposed to be increased to 10%.

It was not until 1990 that investment abroad was allowed. Since then, the limits have been increasing steadily. They were set by the Central Bank in 2.5% in January of 1992 and increased to 3% in October of the same year, and in 1995 they went up to 6%. However, only investment on fixed income securities was allowed until May of that year, when the limit for fixed income instrument was increased to 9% and the limit for variable income was set in 4.5%, with an overall limit for investment abroad of 9%. In 1997 the limit for foreign variable income investment was increased to 6% and the limit for investment abroad as a whole was increased to 12%. In January (April) 1999 the limits for variable income were further relaxed to 8% (10%) and to 16%(20%) for fixed income, with a limit of 16% (20%) for investment abroad as a whole. Finally, in 2002 the limits were set at 15% and 20%, for variable and fixed income respectively.[7]

The limits per issuer are expressed as a percentage of the fund and as a percentage of the assets of the issuer. The first intends to achieve a higher diversification of pension funds investments and the latter to avoid the possibility of having a pension fund manager as controller of a specific issuer. Nevertheless, these limits are significantly decreased when the issuer has a property relationship with the pension fund manager. For example, in the case of stocks, the limit determined as a percentage of the issuer's assets is downsized from 7% to 2%.

Additionally, Pension Funds are subject to a minimum return regulation. It establishes that the managers are responsible of ensuring an average real return over the past twelve months that must exceed the average return of all the Funds minus two per cent or fifty per cent of the average return of all funds, whichever is lower.[8] For this purpose, PFAs must keep 1% of the value of the fund they manage (called the cash reserve). These resources are used if the returns go below the lower bound. When the difference is not covered by the reserve or the funds of the administrator, the authority must do it. However, in this case or when the cash reserve is not restored after being used, the PFA is liquidated.

The regulations described above, have had effects on the way PFAs have chosen their portfolios and by doing so on the risk and return of the invest-

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[7]Investment limits abroad have continued to increase. By Law this limit can go up to 30% with no distinction between variable and fixed income instruments. The limit set by the Central Bank until november 2003 was of 25%.

[8]The average rate of return to compute minimum return was changed from the last twelve months to the last 36 months in August 2002.

ments that the affiliates make and the pensions received once they retire (Arrau and Chumacero, 1998; Valdés and Ramirez, 1999; Walker 1993a; Walker 1993b).

Recently, there has been an important amendment to the Law which allows Pension Funds to invest in five different portfolios, from which affiliates have to choose the one that better suits their risk-return preferences. However, the regulation to which these Funds are subject is still in terms of investment limits with a similar structure to the one that prevailed in the case of one Fund. Additionally, there is a minimum return that it is now computed for each Fund with a band width that is larger for riskier Funds. These changes in the regulation may reduce the costs that are computed here, but the analysis of the effects of this change in the regulation goes beyond the scope of the paper.

2.2   Evolution of the PFAs Portfolio

As mentioned above and observed in figure 1, investment limits have changed significantly over time and PFAs have taken advantage of this increased flexibility. In fact, in the case of variable income instruments in Chile, its share steadily increase since 1985, when it was allowed, until 1991. In that year, the limit was close to be binding and it was increased from 30% to 40% in the case of stocks and incorporated investment fund shares as instrument of investment, rising the limit to 50% in variable income as a whole.

On the other hand, investment abroad was introduced as an eligible instrument in 1990. At the beginning it was circumscribed to investment on fixed income instrument and very few investment abroad occurred. The limit was increased steadily and investment overseas became more and more important, specially in variable income instruments, where the limit was almost binding since the year 2000.[9]

3   Characteristics of the Exercise

This paper intends to provide a quantitative approximation of the costs of investment limits. This task is not easy, as a plausible counter-factual scenario must be provided. That is, we have to evaluate how would the PFAs have chosen their investment portfolios in the absence of limits.

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[9]See Zurita and Jara (1999) for an analysis of the performance of pension funds.
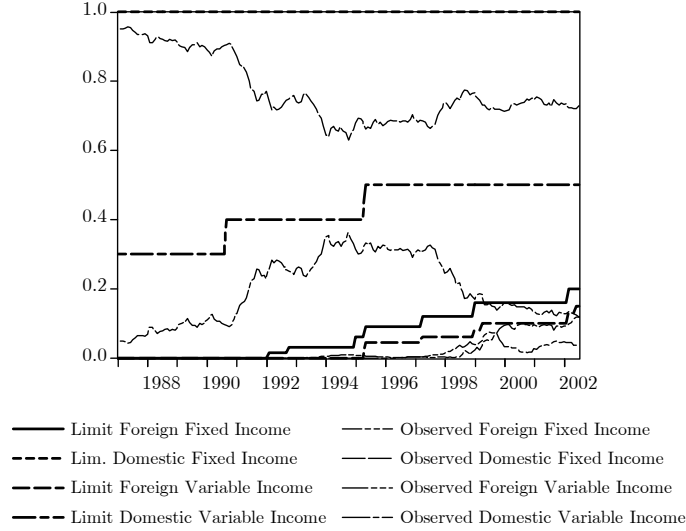
Figure 1: Investment Limits and Observed Portfolios

The basic premise that we try to follow is to prioritize the construction of realistic scenarios, and when in doubt, we choose to model decisions that lead to underestimation of the cost of these limits; thus, most likely providing lower bounds.

For the construction of the counterfactual scenario we need to make explicit:

a. The **instruments** in which PFAs could have had invested. PFAs have numerous instruments to choose from when making their portfolio decisions. We assume that they have only four, grouped in an equal number of categories: Chilean fixed income, Chilean variable income, foreign fixed income, and foreign variable income. Representative prices for these categories were proxied by: Promissory Notes of the Central Bank of Chile with maturity of 8 years (PRC8), the Chilean General Index of Stock Prices (IGPA) for the national variable income instruments, (PRC8), an index based on all US bonds, and the DOWJONES respectively.[10] As PFAs have a broader range of instruments to choose

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[10]Appendix A provides further details of the data.

from, we expect our simulated returns to be consistently below the ones actually realized.

b. The **returns** of each of these instruments. We assume that the returns are independent of the decisions taken by PFAs. In general equilibrium, the returns of some of these assets (particularly the Chilean ones) may have been affected by the decisions of PFAs and the investment restrictions they faced. If Chilean PFAs had market power, portfolio choices would have to internalize the effects of the actions on the prices of assets. We do not consider this possibility here and discuss some implications below.

c. The way in which the **portfolio** would have been chosen. This point deserves further scrutiny. As mention, what is intended here is to quantify the costs of a regulation. Thus, we need to compare what happened with what would have happened in its absence. To construct this comparison, we need to evaluate scenarios for the way in which PFAs would have chosen their portfolio. We approach this problem by considering several strategies that they may have had followed, the most popular being the construction of minimum-variance portfolios, however we also consider other cases such as variants of VaR (Value-at-Risk) efficient portfolios.[11]

d. The law of motion of the **assets** that would have been managed. The comparison of the performance of different investment strategies depend on at least two dimensions: the returns of a given portfolio and the total amount of assets invested. Denote by W_t^i to total assets available at t when portfolio strategy i is followed; its law of motion is given by:

$$W_t^i = W_{t-1}^i r_t^i + A_t^i, \tag{1}$$

where A_t^i is the amount of net inflows received in period t when portfolio strategy i is followed,[12] and r_t^i is the gross return of the portfolio chosen

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[11] Appendix B defines and briefly describes the different approaches considered for modelling the portfolio choice of PFAs.

[12] The net inflows are computed from (1) using the information of total assets and returns of the system reported by the Superintendence of Pension Funds every month. A methodological description of the valuation of assets can be found on Circular # 1216, Superintendencia de Administradoras de Fondos de Pensiones de Chile, July 2002.

in period t when strategy i is followed. This gross return is computed as:

$$r_t^i = x_t' w_{t-1}^i, \tag{2}$$

where x_t is the vector of gross returns of the k assets available for investment and w_{t-1}^i is the $k-$vector of portfolio shares chosen in period $t-1$. In each period, PFAs have to choose how they will invest the assets that they have available. Thus, given the instruments, expected returns and volatilities, constraints, and an objective function to optimize, PFAs are assumed to choose their portfolio.

The ground rules that we used to obtain these key inputs are: the initial total assets are fixed independently of i to the value observed in February 1987, which is the earliest period available for computing returns of the assets considered. The assets considered and their returns are discussed above. However, in constructing each portfolio strategy i, we assume that PFAs make their choices based on forecasts of returns and volatilities and not based on the returns observed ex-post. These expectations are computed using windows of different sizes. That is, in order to estimate the expected returns of the assets in period t, we compute the vector average returns and covariance matrix using information between periods $t-1$ and $t-H$ for $H > 1$. If H is large and the stochastic process followed by the returns is persistent, the estimated first moments will not be a good forecasters of the returns, but would arguably be better estimates of the second moments. We explore several settings for H.[13] Finally, we consider the sequence of A_t^i as deterministic and independent of i. Once again, this assumption would underestimate the cost of the investment limits regulation as net inflows by the affiliates would have, most likely, increased if the returns of the investment would have been greater than observed.

Prior to reporting the results of the exercises, we need to construct a useful benchmark. Given that in our simulations we assume that PFAs choose their portfolio from a subset of the assets considered in reality, we compute what we call our Simulated portfolio. It uses the observed portfolio weights, but constructs r in (2) by using only the observed returns of the four instruments considered in our exercise. Given a value for W_0, the sequence of A and x, we construct a sequence for r and W.

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[13] For the numerical exercises we set $H = 36$ (three years).

Figure 2 presents a comparison of the evolution of total assets and returns for the observed portfolio and the simulated portfolio. It shows that the pension funds effective returns are followed closely by the ones simulated using the same portfolio on our restrictive set of assets. As expected, the simulated total assets are always bellow the observed ones, but follow them. This accounts for the fact that PFAs have a larger set of instruments from which to choose their portfolio, so that our simulated returns are outperformed by the effective returns, which in fact was expected by construction.
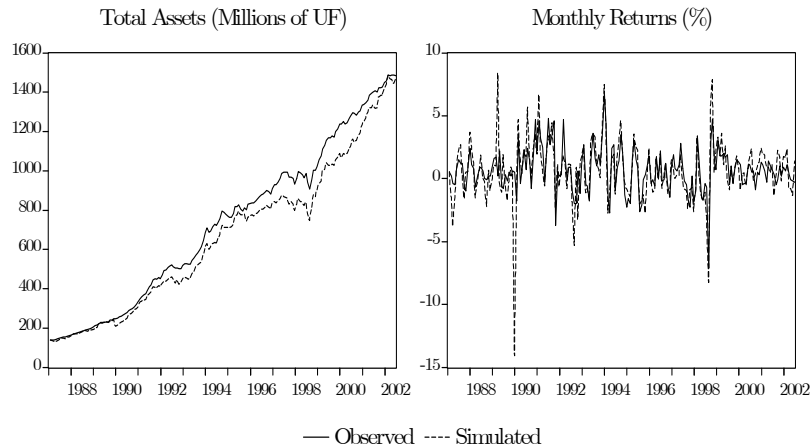

Figure 2: Observed and Simulated Assets and Returns

To level the field when assessing the effects of investment limits regulations, we will use the simulated assets and returns instead of the observed ones as a benchmark, because we are considering the evolution of these variables using a restricted set of assets from which to choose the portfolios.

In summary, when possible, we construct conservative estimates of the costs of investment limits. For example, we considered a very restrictive set of instruments from which PFAs can form their portfolios. The wider the variety of instruments considered, the heavier the costs associated with investment limits should be. Besides, we consider naive models for forecasting expected returns and volatilities. Furthermore, we do not allow for hedging operations, which, once again restricts the set of instruments. Finally, in our

counterfactual scenarios, we consider that the net inflows would not have been influenced by a better performance of the funds.

However, there are some aspects that may imply overestimation of the costs. For instance, we do not consider transaction costs. According to our model, in the absence of limits, PFAs would have changed their portfolios more frequently and more abruptly than with limits; thus, arguably incurring in higher transaction costs. Furthermore, some of the potential benefits of investment limits (particularly in early stages) may have been their beneficial effect on the development of local capital markets.

Finally, there are some factors associated with the size of pension funds in the local market that are not considered. If they had market power, some strategic behavior, not considered here, would have been possible. In this sense, prices are assumed to be unchanged, independently of the strategy followed by PFAs. The sign of this effect on our computations of the costs is ambiguous. On the one hand, pension funds could have taken advantage of their market power but on the other hand, this could have been an additional constraint for their portfolio selection.

4   Results

As discussed above, to quantify the cost of the regulation we intend to compute the portfolio that would have been chosen in the absence of investment limits. For that purpose, we need to take a stance regarding the way in which PFAs choose their portfolios. As no explicit mechanism is known, we use several standard models for portfolio selection with their respective optimization problems. The three strategies for selecting portfolios that were considered are (without short sales): the minimum-variance portfolio (portfolio p), the quadratic preferences portfolio (portfolio q), and the VaR efficient portfolio (portfolio v).[14]

For each of these problems we proceed as follows:

- First, select the portfolio that is consistent with the corresponding model subject to the constraints imposed by the investment limits. For each period, the expected return of the portfolio that is chosen replicates as closely as possible the average return of the simulated portfolio

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[14]See Appendix B for details and definitions.

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in a window of J periods.[15]

- In the case of the minimum-variance portfolio, there is no need for an additional constraint to the one that asks our hypothetical investor to match the expected return when there are investment limits.

- For the case of quadratic preferences portfolio, we calibrate the parameter B of (B.10) to replicate the average return observed on the window.

- For the VaR efficient portfolio, we set $\alpha = 0.05$ and calibrate the parameter $\overline{VaR}$ of (B.14) to replicate the average return observed on the window.

- Next, we consider the optimization problem without the investment limits constraints. As this problem has fewer constraints, it is expected to perform better. Thus, the portfolio chosen here matches the expected volatility of the portfolio that would have been chosen with limits. If risk is volatility, the portfolio without limits is chosen so that every period the portfolio is exposed to the same risk as the one with limits. This amounts to:

  - Finding the expected return that is necessary for the unconstrained portfolio to match the expected volatility of the constrained p portfolio.

  - Finding the value of B that is necessary to match the expected volatility of the constrained q portfolio.

  - Finding the value of $\overline{VaR}$ that is necessary to match the expected volatility of the constrained v portfolio.

As the strategy followed by Pension Fund Administrators is not necessarily the same as the ones used by us, we perform every exercise by asking our hypothetical PFA to face the same optimization problem with and without limits in order to isolate the cost of the restrictions from other variables that could affect performance.

Figure 3 presents a comparison of the evolution of the ratio between the total assets obtained without and with limits for each strategy. From it,

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[15]We set $J = 36$ (three years average).

we gather that the costs in terms of the amounts of total assets managed by the PFAs appear to be substantial. In particular, the exercise suggests that with the minimum-variance portfolio, at least 30% more of assets would have been on the system by mid 2002 without increasing the volatility of the returns. Thus, investment limits may have not only been costly in terms of not allowing proper risk diversification, but also they could have had a cost in terms on foregone assets. The figures obtained with other investment strategies follow the same general pattern but imply lower costs. The reason is that the other types of investment strategies tend to be more conservative (particularly in the case of the VaR efficient portfolio). At any rate, according these exercises, for the end of the sample, the amount of foregone total assets may have been of at least 10%.
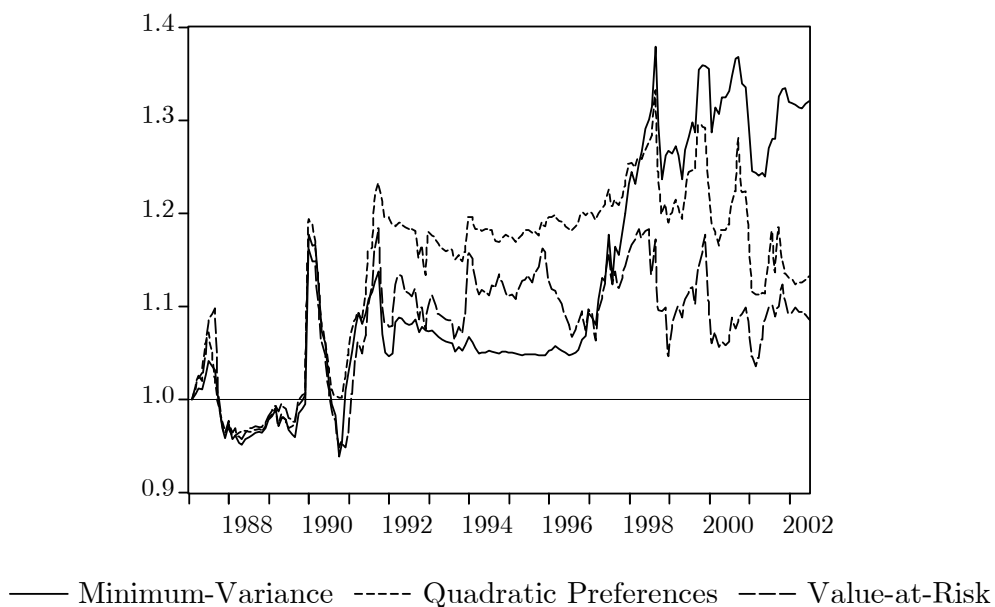

Figure 3: Ratio of Total Assets (No Limits/Limits)

In terms of the characteristics of the portfolios that would have been chosen in each case, the models predict a much heavier partition of fixed and variable foreign instruments than the one observed. More importantly, the

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choice of the portfolio allocations in the model with limits was set so that the expected returns (and thus total assets) be consistent with the observed trajectory of total assets and the portfolio selection strategy considered.

Table 2 presents other summary statistics that describe the nature of the results. They suggest that the costs of investment limits could have been substantial. In particular, the portfolios consistent with the limits have returns significantly below the ones obtained in the absence of limits without increasing risk (as the expected volatility is made to coincide). This is something that should not be surprising, as the models consider that the average share of investments abroad should have been of more than 30%, and independently of the strategy considered, the limits might have been binding 90% of the times.[16]

Table 2: Costs of Investment Limits

Portfolio	p	q	v
Average Monthly Return (without limits) (%)	0.84	0.75	0.70
Standard Deviation of Monthly Return (%)	2.46	2.38	2.57
Average Monthly Return (with limits) (%)	0.66	0.62	0.61
Share of Fixed National (%)	49.4	51.9	44.8
Share of Variable National (%)	17.9	15.5	18.9
Share of Fixed Foreign (%)	18.6	20.2	17.3
Share of Variable Foreign (%)	14.1	12.5	19.1
Probability of Binding Limit (Fixed National) (%)	0.0	0.0	0.0
Probability of Binding Limit (Variable National) (%)	15.1	14.0	9.1
Probability of Binding Limit (Fixed Foreign) (%)	62.4	66.7	61.3
Probability of Binding Limit (Variable Foreign)	90.3	91.9	86.6
Average Gap (UF per affiliate)	38	36	20
Standard Deviation Gap (UF per affiliate)	46	27	15
Maximum Gap (UF per affiliate)	141	106	58
Equivalent Tax (%)	6.5	9.9	5.8

In terms of costs, not only the total assets could have been at least 10% higher, but also depending on the investment strategy, the average affiliate may have had lost more than UF 20 (approximately US$500). Finally, in order to assess the welfare costs of the regulation we conducted a very simple

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[16]Cardinale (2003) finds that, on average, the optimal share of investment abroad would be around 20%.

first order approximation. As in both cases the average volatility is matched, we compute the present value of the utility of a representative affiliate when the PFA follows strategy i as:

$$U^i = \sum_{t=0}^{T} \beta^t u\left(W_t^i / N_t\right)$$

where β is a discount factor (that we set equal to 0.99), N_t is the number of persons affiliated to the system in period t, and $u\left(\cdot\right)$ is a utility function (that we assume logarithmic). The implicit subsidy (tax) that would make and affiliate indifferent between having the PFA follow strategy i or j is:

$$(1 + \tau) = \exp\left[\frac{1 - \beta}{1 - \beta^{T+1}}\left(U^i - U^j\right)\right] \tag{3}$$

If $\tau > (<)0$, the individual would need a subsidy (tax) of τ of its per capita wealth of strategy j is followed. In our case, we compute this value to be equivalent to taxing more than 5% of the per capita wealth of the affiliates. This number is probably a lower bound because of another important factor, (3) is constructed assuming that the sequence of per capita assets is deterministic. In reality, this variable is stochastic and non stationary. Thus, the welfare costs could be greater because this model matches the volatility of the returns and not of total assets.

Despite that we construct the optimal portfolio with limits so that the expected return matches the average return of the simulated portfolio, the composition of the portfolio selected may differ from the one observed in practice. Figure 4 presents a comparison of the evolution of the portfolio shares in each of the four instruments. For illustrative purposes, the limits and the actual portfolio shares are also included.[17]

Even though the portfolio shares obtained with the portfolio with limits follow the behavior of the shares that the PFAs actually invested, our shares are much more volatile. Candidates for explaining the excessive volatility of our portfolio shares are: In our exercise, transaction costs in adjusting portfolios are absent; this makes portfolio changes less costly. While transaction costs may explain some of the persistence of portfolio allocations, some other feature may be at play. One of them is the choice of the length of the window with which the vector of expect returns and the conditional covariance

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[17]The results for the q and v portfolios are qualitatively similar and are not included but are available upon request.
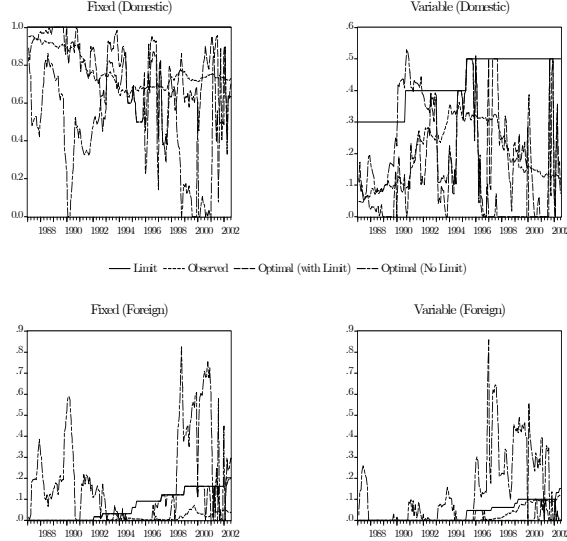
Figure 4: Portfolio Shares Implied by the $\boldsymbol{p}$ Model

matrix are computed (the parameters H and J discussed above); as we set them to 36 (three years), the information that is used to "forecast" future returns is not that precise. That is, if returns were relatively persistent, the average returns observed in the past three years may not be good candidates for forecasting next month's return.

As in practice, the portfolios actually selected have much smoother trajectories than the ones chosen by our model, next we consider how important is this feature. For that purpose, we consider a portfolio strategy in which PFAs choose their portfolio as a weighted average between the optimal portfolios (presented in figure 4) and the portfolio chosen the previous period. Thus, if we denote by w_t^i to the optimal portfolio chosen following strategy i in the period t, we now compute the smoothed portfolio as:

$$\widetilde{w}_t^i = \gamma \widetilde{w}_{t-1}^i + (1 - \gamma) \, w_t^i, \tag{4}$$

and set $\widetilde{w}_1^i = w_1^i$ (for $0 \leq \gamma \leq 1$). With $\gamma = 1$, the portfolio chosen is constant (and equal to the initial portfolio); with $\gamma = 0$, the portfolio chosen is equal to the optimal portfolio. Thus, the closer γ to 1, the smoother the portfolio allocation would be.

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Table 3: Costs of Investment Limits (with smoothed portfolios and minimum-variance objective function)

γ	**0.25**	**0.50**	**0.75**
Average Monthly Return (without limits) (%)	0.81	0.76	0.72
Standard Deviation of Monthly Return (%)	2.45	2.45	2.46
Average Monthly Return (with limits) (%)	0.63	0.59	0.54
Share of Fixed National (%)	49.4	49.5	49.8
Share of Variable National (%)	17.9	17.9	17.9
Share of Fixed Foreign (%)	18.6	18.5	18.3
Share of Variable Foreign (%)	14.1	14.1	14.1
Probability of Binding Limit (Fixed National) (%)	0.0	0.0	0.0
Probability of Binding Limit (Variable National) (%)	11.8	12.4	11.8
Probability of Binding Limit (Fixed Foreign) (%)	62.4	64.5	66.1
Probability of Binding Limit (Variable Foreign)	89.8	91.4	91.9
Average Gap (UF per affiliate)	37	37	37
Standard Deviation Gap (UF per affiliate)	43	40	40
Maximum Gap (UF per affiliate)	135	127	121

Table 3 presents the results the different values of γ. As expected, the smaller the value of this parameter, the closer the model to the optimal portfolio and the larger the costs of investment limits. Thus, the more sluggish the portfolio allocations, the lesser the costs of the regulations because both, expected returns and costs of investment limits, are decreasing functions of γ. At any rate, even with very sluggish portfolio allocations ($\gamma = 0.75$) the main results of the paper hold and the estimated costs of the limits appear to be substantial.[18]

5   Concluding Remarks

Since its creation in 1981, Pension Funds Administrators in Chile were not free to choose optimal investment portfolios because of a stringent regulation on investment limits. The diagnosis implicit with the imposition of limits was

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[18]In fact, as Table 3 suggests, the differences in portfolio allocations are not substantial and the trajectory of the ratio between assets with and without limits is in accordance with the one reported in Figure 3.

that the Chilean capital market was not deep, that there was an important demand for funds to finance the expansion of the productive sector, and that due to principal-agent problems, protection for uninformed affiliates was needed.

As this regulation entails an inefficient combination of risk and return, this paper intends to quantify its costs. For that purpose, we construct counterfactual scenarios for the evolution of the assets and returns that PFAs would have administered had this regulation been absent. In the construction of this counterfactual scenario we tried to be as conservative as possible. That is, we consider that the costs computed here are most likely lower bounds, because we have always tried to construct scenarios in which if a bias was present it would be towards underestimating the costs.

Our results suggest that the costs may have been substantial and that in the absence of limits, the total assets managed by PFAs could have been at least 10% larger, that pension fund affiliates might have been exposed to more volatility, that the investment limits may have been binding approximately 90% of the times, that on average each affiliate lost between US$500 and US$1,000, and that the regulation can be thought of as a tax of more than 5% on the wealth of the affiliates.

However, our analysis abstract from any possible endogeneity with respect to the role of pension funds on the development of local capital markets, which may have been crucial in early stages.

References

Arrau, P. and R. Chumacero (1998). "Tamaño de los Fondos de Pensiones en Chile y su Desempeño Financiero," *Cuadernos de Economía* 105, 205-35.

Bernheim, D. (1998). "Financial Illiteracy, Education, and Retirement Savings," in O. Mitchell and S. Schieber (eds), *Ling with Defined Contribution Plans*. University of Pennsylvania Press, 38-68.

Brandimarte, P. (2002). *Numerical Methods in Finance: A MATLAB-Based Introduction*. John Wiley & Sons.

Campbell, J., A. Lo, and C. MacKinlay (1997). *The Econometrics of Financial Markets*. Princeton University Press.

Cardinale, M. (2003). "Optimal Portfolio Allocations and Funded Pension Systems: The Case of Chile," Watson Wyatt Technical Paper 2003-GP02, Imperial College Management School.

Cheyre, H. (1991). *La Previsión en Chile Ayer y Hoy*. Second Edition. Centro de Estudios Públicos.

Corbo, V. and K. Schmidt-Hebbel (2003). "Efectos Macroeconómicos de la Reforma de Pensiones en Chile. Resultados y Desafíos de las Reformas a las Pensiones," Federación Internacional de Administradoras de Fondos de Pensiones. Edited by CIEDESS.

D'ambrosio, M. (2003). "Ignorance is not Bliss: The Importance of Financial Education," *Contractual Savings Conference*: Supervisory and Regulatory Issues in Private Pensions and Life Insurance, The World Bank, Washington DC.

D.L. N° 3.500 (1980). Regulations D.L. 3.500 and D.F.L. N° 101 (Updated Text - February 2002)

Dowd, K. (1998). *Beyond Value at Risk: The New Science of Risk Management*. Wiley & Sons.

Gourieroux, C., J. Laurent, and O. Scaillet (2000). "Sensitivity Analysis of Values at Risk," *Journal of Empirical Finance* 7, 225-45.

Huang, C. and R. Litzenberger (1988). *Foundations for Financial Economics.* Prentice Hall.

Johnson, C. (2001). "Value-at-Risk: Teoría y Aplicaciones," *Estudios de Economía* 28, 217-47.

Lai, T., K. Wang, S. Chan, and D. Lee (1992). "A Note on Optimal Portfolio Selection and Diversification Benefits with Short Sale Restrictions on Real Estate Assets," *Journal of Real Estate Research* 7, 493-501.

LeRoy, S. and J. Werner (2001). *Principles of Financial Economics.* Cambridge University Press.

Shiller, R. (2002). "Indexed Units of Account: Theory and Assessment of Historical Experience," in F. Lefort and K. Schmidt-Hebbel (editors), *Indexation, Inflation, and Monetary Policy*, Series on Central Banking, Analysis, and Economic Policies 2, Central Bank of Chile.

Superintendence of Pension Funds (2003), *The Chilean Pension System Based on Individual Capitalization.* Fourth Edition. Edited by Alejandro Ferreiro.

Valdés, S. (1997). "Financing a Pension Reform toward Private Funded Pensions," in S. Valdés-Prieto (editor), *The Economic of Pensions*, Cambridge University Press.

Valdés, S. and M.C. Ramirez (1999). "Herding in DC Pension Funds: Regulation or Market Forces?," *Manuscript*, Instituto de Economía, Pontificia Universidad Católica de Chile.

Vittas, D. (1996) "Pension Funds and Capital Markets," *Financial Sector Development* Note N°71, The World Bank, Washington, DC.

Walker, E. (1993a). "Desempeño Financiero de las Carteras Accionarias de los Fondos de Pensiones en Chile. ¿Ha tenido Desventajas ser Grandes?," *Cuadernos de Economía* 89, 34-75.

Walker, E. (1993b). "Desempeño Financiero de las Carteras de Renta Fija de los Fondos de Pensiones en Chile. ¿Ha tenido Desventajas ser Grandes?," *Cuadernos de Economía* 89, 34-75.

Walker, E. and E. Valk (1995). "La Inversión de los Fondos de Pensiones. Historia, Normativa y Resultados." in S. Baeza and F. Margozzini (eds.) *Quince Años Después. Una Mirada al Sistema Privado de Pensiones.* Centro de Estudios Públicos, 131-164.

Zurita, S. and C. Jara (1999). "Desempeño Financiero de Fondos de Pensiones," *Estudios Públicos* 74, 227-54.

A The Data

Information of the portfolio of PFAs (by investment instrument) is available since June 1981. These instruments were grouped in four categories: national fixed income, national variable income, foreign variable income, and foreign fixed income. The first category includes state issued instruments, mortgage notes, deposits, bonds and titles guaranteed by financial institutions, and bonds from public and private corporations. The second incorporates stocks from open corporations, real state corporate stocks, and national investment funds. The third category comprises international investment fund, stocks issued by foreign companies, and international mutual funds. Under the fixed income foreign investment are included credit titles, securities and negotiable titles issued by foreign states, foreign banks or central banks, and bonds issued by overseas companies.

Representative prices for the four categories were proxied by: The Chilean General Index for Stock Prices (IGPA) for the national variable income instruments deflated by a unit of account indexed to past inflation (known as the Unidad de Fomento, UF).[19] In the case of national fixed income instruments, the prices were calculated by using the interest rate for Promissory Notes of the Central Bank with maturity of 8 years (PRC8). For foreign variable income instruments the reference price was the DOWJONES and for foreign fixed income we constructed and index based on all bonds for the US. In all cases, returns are expressed in terms of UFs; which for foreign assets mean that we also used the observed exchange rate figures of the Central Bank of Chile.

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[19]See Shiller (2002) for a discussion about the use of the Chilean UF and indexed units of account around the world.

B   Portfolio Selection

This Appendix defines the various strategies for portfolio selection used on the paper. We begin by considering the conventional CAPM model and other strategies derived from quadratic objective functions, and then consider the increasingly popular Value-at-Risk model.

B.1   Quadratic Loss Functions

Following Campbell et al (1997), let there be n risky assets with mean vector m and covariance matrix V. Define w_a as the n-vector of portfolio weights for an arbitrary portfolio a with weights summing to unity. Portfolio a has mean return $\mu_a = w_a'm$ and variance $w_a'Vw_a$.

Definition 1 *Portfolio $\boldsymbol{p}$ is the minimum-variance portfolio of all portfolios with mean return μ if its portfolio weight vector is the solution to the following constrained optimization:*

$$\min_{w} \frac{1}{2}w'Vw \tag{B.1}$$

subject to

$$w'm = \mu \tag{B.2}$$

$$w'\imath = 1. \tag{B.3}$$

To solve this problem, we form the Lagrangian function L_1, differentiate with respect to w, set the resulting equations to zero, and then solve for w.

$$L_1 = \frac{1}{2}w'Vw + \lambda_1\left(\mu - w'm\right) + \lambda_2\left(1 - w'\imath\right),$$

where $\imath$ is an n-vector of ones, and λ_1 and λ_2 are the Lagrange multipliers of (B.2) and (B.3) respectively.

The first order conditions for this problem are:

$$Vw_p - \lambda_1 m - \lambda_2\imath = 0. \tag{B.4}$$

Combining (B.2), (B.3), and (B.4) we find the solution

$$w_p = G + H\mu, \tag{B.5}$$

where G and H are n-vectors,

$$
\begin{aligned}
G &= \frac{1}{D}\left[BV^{-1}\imath - AV^{-1}m\right] \\
H &= \frac{1}{D}\left[CV^{-1}m - AV^{-1}\imath\right],
\end{aligned}
$$

and $A = \imath'V^{-1}m$, $B = m'V^{-1}m$, $C = \imath'V^{-1}\imath$, and $D = BC - A^2$.

The optimal portfolio (B.5) admits short-sales (some of the weights may be negative). When short-sales are not allowed the Lagrangian function is:

$$
L_2 = \frac{1}{2}w'Vw + \lambda_1\left(\mu - w'm\right) + \lambda_2\left(1 - w'\imath\right) + w'\delta,
$$

where δ is an n-vector of Lagrange multipliers that imposes the constraints

$$
w \geq 0. \tag{B.6}
$$

The first order conditions of this problem:

$$
Vw_p - \lambda_1 m - \lambda_2 \imath + \delta = 0, \tag{B.7}
$$

along with (B.2), (B.3), and the slackness conditions

$$
\delta_i w_i = 0 \text{ for } i = 1, \ldots, n
$$

are used to solve for the minimum-variance portfolio when short-sales are not allowed (Lai et al, 1992).

If a risk-free asset is introduced, the portfolio weights of the risky assets are not constrained to sum to 1, since $(1 - w'\imath)$ can be invested in the risk-free asset.

Definition 2 *Given a risk-free asset return f, portfolio $\mathbf{c}$ is the minimum-variance portfolio of all portfolios with mean return μ if its portfolio weight vector is the solution to the minimization of (B.1) subject to*

$$
w'm + \left(1 - w'\imath\right)f = \mu \tag{B.8}
$$

To solve this problem, we form the Lagrangian function L_3, differentiate with respect to w, set the resulting equations to zero, and then solve for w.

$$
L_3 = \frac{1}{2}w'Vw + \lambda_1\left(\mu - w'm - \left(1 - w'\imath\right)f\right),
$$

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where λ_1 is the Lagrange multiplier of (B.8).

The first order conditions for this problem are:

$$Vw_c - \lambda_1 (m - f\imath) = 0,$$

which combined with (B.8) yield:

$$w_c = JV^{-1} (m - f\imath), \tag{B.9}$$

where,

$$J = \frac{\mu - f}{(m - f\imath)' V^{-1} (m - f\imath)}.$$

When short-sales the constrained optimization problem must be modified to guarantee that (B.6) and $w'\imath \leq 1$ hold.

If risk is equated with volatility, the minimum-variance portfolio problem is closely related with the optimization problem in which an agent maximizes expected utility with quadratic preferences (see Brandimarte, 2002; Huang and Litzenberger, 1988; or LeRoy and Werner, 2001).

Definition 3 *Portfolio q is the optimal portfolio with quadratic preferences if its portfolio weight vector is the solution to the following constrained optimization:*

$$\max_w w'm - \frac{1}{2}Bw'Vw \tag{B.10}$$

subject to (B.2) and (B.3).

In this case, B is a parameter linked to risk aversion, with higher values indicating more risk aversion. The solution to this problem is:

$$w_q = \frac{1}{B}V^{-1} (m - D\imath), \tag{B.11}$$

where

$$D = \frac{A - B}{C}.$$

If short-sales are not allowed, we proceed as discussed above.

Definition 4 *Given a risk-free asset return f, portfolio s is the optimal portfolio with quadratic preferences if its portfolio weight vector is the solution to the following constrained optimization:*

$$\max_w w'm + (1 - w'\imath) f - \frac{1}{2}Bw'Vw \tag{B.12}$$

subject to (B.8).

The solution to this problem is:

$$w_s = \frac{1}{B} V^{-1} (m - f\imath).$$

As in the previous case, when short-sales are not allowed, we proceed by minimizing (B.12) subject to (B.6).

B.2   Value-at-Risk

Value-at-Risk (VaR) has become a key tool for risk management of financial institutions. Usually defined as the maximum expected loss over a given horizon period at a given level of significance, it is intended to provide quantitative and synthetic measures of risk.[20]

Following Gorieroux et al (2000), if P_t is the conditional distribution of future asset prices given the information at time and a loss probability of α level is considered, the Value at Risk $[VaR(w, \alpha)]$ is defined as:

$$P_t [W_{t+1}(w) - W_t(w) + VaR_t(w, \alpha) < 0] = \alpha.$$

In particular if the VaR is computed under the assumption of normality of the returns, with conditional mean m_t and covariance matrix V_t, then:

$$VaR_t(w, \alpha) = -w'm_t + (w'V_tw)^{1/2} z_{1-\alpha},$$

with $z_{1-\alpha}$ being the quantile of level $1 - \alpha$ of the normal distribution.

Definition 5 *Portfolio* v *is the VaR efficient portfolio if its portfolio weight vector is the solution to the following constrained optimization:*

$$\max_w w'm \tag{B.13}$$

subject to (B.8) and

$$VaR(w, \alpha) = \overline{VaR} \tag{B.14}$$

This portfolio is a function of the loss probability α and the bound considered, $\overline{VaR}$, and it satisfies the following first order conditions:

$$
\begin{aligned}
m &= -\lambda_t \frac{\partial VaR_t}{\partial w}(w, \alpha), \\
VaR_t(w, \alpha) &= \overline{VaR}
\end{aligned}
\tag{B.15}
$$

---

[20] See Dowd (1998) and Johnson (2001) for details.

where λ_t is the Lagrange multiplier of the constraint (B.14) and if the assumption of gaussianity holds:

$$\frac{\partial VaR_t}{\partial w} = -m_t + \frac{V_t w}{(w'V_t w)^{1/2}} z_{1-\alpha}.$$

The conditions stated in (B.15) form a nonlinear system of equations that can be solved to obtain the VaR efficient portfolio. If short sales are not allowed, we solve a constrained optimization problem in which the constraints (B.3) and (B.6) are included.