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Should countries control international profit shifting?

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Abstract

This paper presents a fiscal competition model in which policy decisions are not only corporate taxes but also whether or not to control the multinational firms'(MNF) profit shifting activities. MNFs manipulate transfer prices as a means to shift profits from high to low tax countries. National governments may hinder such a behavior by monitoring the MNF's accounts. We show that a country may optimally decide not to monitor the MNF for two different reasons. On the one hand, that makes it an attractive location for the MFN even if the corporate tax is high. On the other hand, not monitoring increases the mobility of the MFN's profits. This shifts the focus of tax competition in that corporate taxation then influences not only the MNF's location as the place where it declares its profits.
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1 Introduction

Competition to attract mobile firms is known to create a downward pressure on their profit taxation. A likely consequence of such a fiscal competition is a fiscal burden transfer out of mobile capital into immobile labor and domestic small and medium firms, less likely to move in response to changes in taxation. Both the European Union and the OECD have reacted to this tendency by recommending both tax coordination efforts among countries and the non-use of discriminatory taxation (European Communities (1992, 1998), OECD (1998)). One of the reasons why countries accept to commit to this non discrimination principle is that it increases the fiscal cost of attracting firms via tax cuts, thereby alleviating the downward pressure on profit taxation. However, each country would still like to offer targeted fiscal breaks. Therefore one may expect fiscal authorities use other instruments than the tax rate in trying to discriminate. The present paper builds on this intuition by considering the case where mobile firms are multinational ones and the alternative fiscal instrument is the control of international profit shifting.

Multinational firms, as they own fiscal entities at different locations, may take advantage of tax differentials by manipulating local profits in various ways, the so-called *profit shifting* behavior. Among the tools at the firms' disposal, the manipulation of transfer prices (i.e. the prices that are used for intrafirm international trade in goods and services) is a widely used one. The specific rule firms should comply with is the "arm's length principle", recommended by the OECD (1995), according to which intrafirm trade should be priced as if it were conducted between independent firms (i.e. use the market price). The consequences of transfer price manipulation seem to be a concern for some governments as the following quotation from The Economist illustrates: *In theory the transfer price is supposed to be the same as the market price between two independent firms (...). So multinationals spend a fortune on economists and accountants to justify the transfer prices that suit their tax needs. Increasingly, firms try to restructure their operations to get their tax bill down as far as possible. There are plenty of opportunities: according to the OECD, around 60% of international trade involves*

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transactions between two related parts of multinationals. But tax authorities are increasingly looking out for such wheezes. In America, in particular, the taxman has been putting the squeeze on companies, which have responded by allowing more of their taxable profits to arise there to keep him happy. This is prompting other countries to get tougher, too. (The Economist, 2000)

There is extensive evidence of transfer price manipulation. See Hines (1997,1999) for comprehensive surveys of the empirical literature. The more direct evidence to date is presented by Clausing (2003). She uses explicit observations of both intrafirm and non intrafirm (market) prices of US international trade. She concludes that export (import) intrafirm prices do increase (decrease) with the tax rate of the destination (origin) country as compared to the market ones.

A region may be appealing to multinational firms simply because it offers them a greater latitude to manipulate transfer prices. In particular, it may attract the firm even with a high profit tax if it is totally loose in this respect, allowing the firm to shift all its profits to lower tax regions. There is evidence that countries do differ in how much they enforce transfer pricing rules, as is clear from Table 1, borrowed from Bartelsman and Beetsma (2003). The table shows the date at which different transfer pricing related policies have been introduced.[1] Moreover, the enforcement seems to play a role in the transfer pricing behavior of firms. The authors perform a multi-country analysis of profit shifting among 16 OECD countries and they conclude that (i) there is robust evidence of transfer pricing manipulation in response to tax differences and (ii) profit shifting decreases with the degree of enforcement of the country (where the enforcement variable is taken from Table 1 and ranges from 0 to 3 according to the number of enforcement policies in place).

This paper takes this possibility of differing in the control of transfer prices into account. We assume that countries abide by the code of conduct of the European Union and thus cannot directly discriminate profit taxation of mobile and immobile firms. Being unable to do so, the cost to attract the firm through lowering taxes is high, since it consists of (almost) completely

[1]Ernst and Young (2000) (as cited in Bartelsman and Beetsma (2003)) defines policies as follows: *(i)*"explicit TP rules" means "transfer pricing regulatory provisions exist"; *(ii)* "formal TP documentation rules" refers to governing tax authorities requires or recommends that taxpayers prepare and maintain written documentation to confirm that the amounts charged in related party's transaction are consistent with the arm's length principle" and *(iii)* "TP specific penalties" indicates "tax authority will impose a transfer pricing specific penalty if the taxpayer is found not to be in compliance with the transfer pricing rules imposed by the country".

Country	Explicit TP rules	Formal TP documentation rules	TP specific penalties
Australia	07/83	09/95	07/83
Austria	-	-	-
Belgium	07/99	07/99	-
Canada	-	01/99	01/99
Denmark	01/99	01/99	01/99
Finland	01/31	-	-
France	09/85	04/96	04/96
Germany	02/83	-	-
Italy	12/86	-	-
Japan	04/86	-	-
Netherlands	-	-	-
Portugal	-	-	-
Spain	01/96	-	-
Sweden	-	-	-
UK	07/99	07/99	07/99
US	01/28	01/94	01/94

Source: Ernst and Young (2000) cited in Bartelsman and Beetsma (2003).

giving up on profit taxation from other (immobile) sources. In a model with two almost symmetric regions, we show that one government may decide not to enforce the arms' length principle in order to host a multinational firm while setting high profit taxes on domestic firms. At such an equilibrium, the other country does not enjoy the benefits from the localization of the multinational but taxes its profit. We consider a multistage game where before deciding on profit tax rates, governments decide how much to control the firms' transfer pricing behavior. Announcing a loose monitoring policy in the first stage of the game is a way for governments to change the focus of tax competition in the second one, since if the firm is allowed to run away with its tax payments, it no longer bases its location decision on tax differences. Governments do however compete for the multinational's profit taxation, given its location. The choice of the control policy determines whether countries will compete for the firm or for its profit. Out of the possible gains linked to localization (increase in the wage bill, decreasing unemployment, agglomeration externalities,...), our model relies on savings in transport costs. Households of the country hosting the firm enjoy a higher consumer surplus as no transport cost have to be incurred.

Our paper features three aspects which have not, to the best of our knowledge, been simultaneously studied in the literature. One is more general: we provide a detailed analysis of the nature of tax competition when fiscal authorities cannot discriminate among tax bases with different elasticities. In a way, tax competition becomes less intense, though more unstable. Janeba and Peters (1999), already pointed out this implications of non-discrimination. However, they provide a partial analysis of the tax equilibrium. Their objective, as that of Keen (2001) is different from ours in that they compare the relative merits of discriminatory vs non-discriminatory taxation.[2] The two others contributions are specific to the transfer pricing literature. On the one hand, while we fix a very simple transfer pricing rule (equal to marginal cost of production), **we let the countries decide on whether or not to enforce it**. On the other hand, **location is not given** and it is not trivial due to the existence of a **transport cost**.

We show that in a subgame perfect equilibrium, at most one country is loose on control. Typically, the one that benefits from a "location" advantage. Countries do not engage in a run to the bottom on this policy instrument, unlike the widely known result on tax competition (see e.g. the survey by Wilson, 1999) nor do they always engage in a run to the top, unlike usual results on transfer pricing regulations for immobile multinationals (Raimondos-Møller and Scharf (2002), Mansori and Weichenrieder (2001)). E.g., the former paper takes location of the multinational as given and takes for granted that it will respect the transfer pricing rule fixed by the government. In this context, governments set transfer pricing rules which lead to excess effective taxation and depressed international trade - a race to the top. The fact that when competition in tax rates is banned, countries may have an incentive to be less stringent in the application of tax laws has been obtained by Cremer and Gahvari (2000). They model countries competing both in indirect taxation of a consumption good (collected by firms) and on audit probabilities and show that auditing probabilities are cut in response to a ban in tax competition.

Other papers somehow linked to ours include Elitzur and Mintz (1996), who address tax competition when transfer prices are used both to shift profit and as a strategic device to give proper incentives to the affiliate. Haufler and Schjelderup (2000) show that full deduction of investment expenditures may not be optimal for an open economy hosting multinational corporations who shift profits by manipulating transfer prices. Two papers by Kind et al. (2001, 2002) introduce transport costs and/or foreign property of the firm and show under which circumstances economic integration

[2] The authors thank Ben Lockwood for attracting our attention to these references.

(a decrease in the former and/or an increase in the latter) leads to a decrease in corporate taxation.

The paper is organized as follows. Section 2 presents the model and basic derivations. We compute the subgame perfect equilibrium and present results in section 3. Section 4 extends the model in a few directions thereby testing the robustness of our results. Finally, we conclude in Section 5.

2 The Model

2.1 The Firm

We consider a partial equilibrium model in which two regions denoted 1 and 2 compete for a single multinational firm. The multi-national firm consists of one production plant and one sales office, each located in one region. For simplicity, we consider that the multi-national operates as a monopolist in each market, facing the following linear demand:

$$q_i(p_i) = \frac{\alpha - p_i}{\beta}, \quad i = 1, 2$$

where p_i and q_i are the price and quantity prevailing in region i.

The firm sets its production plant in one of the regions and a sales-office in the other. We will often refer to the production plant as the firm's headquarters or simply *the firm* and to its location as *home*. Each branch pays taxes locally. The plant produces a good at a constant marginal cost w and sells it both locally and to the sales-office located in the other region. Shipping the good from one region to the other costs τ per unit. The marginal cost of delivering the good to consumers is therefore equal to w at home and $w + \tau$ abroad. We adopt the following notation: q_{ij} (q_{ii}) and p_{ij} (p_{ii}) stand for, respectively, quantity sold and price set by the firm located in i for market j (i).

The firm sets quantity as to maximize its gross-of-tax profit. Accordingly,

$$\begin{aligned} q_{ii} &= \frac{\alpha - w - \tau}{2\beta} = \frac{A}{2\beta}, \quad i = 1, 2 \\ q_{ij} &= \frac{\alpha - w - \tau}{2\beta} = \frac{A\tau}{2\beta}, \quad i, j = 1, 2, \quad i \neq j \end{aligned}$$

and

$$
\begin{aligned}
p_{ij} &= \frac{\alpha + w}{2}, \quad i = 1, 2 \\
p_{ij} &= \frac{\alpha + w + \tau}{2}, \quad i, j = 1, 2, \;\; i \neq j
\end{aligned}
$$

We will often use A as shorthand for $\alpha - w$ and $A\tau$ for $\alpha - w - \tau$. Incidentally, these are respectively the social value of (the first unit of) the good at home and abroad.

The firm has to impute its cost of production between the headquarters and the affiliate. It does so by setting a constant transfer-price g_i at which the headquarters sell the good to the sales office. The "arm's length principle" recommends the transfer price to be equal to its marginal cost w. Nevertheless, the firm can try to manipulate it to shift profit from the high to the low tax region. If g_i is higher (resp. lower) than w, the firm is shifting profits out of (resp. into) the sales office into (resp. from) the production plant. This manipulation entails a cost equal to MC, which may either be an expected fine or payments to financial experts in charge of the firms' accounts. MC is influenced by the governments, as each of them may either control or not the transfer price decision of the firm. We use the terms control and monitor interchangeably to refer to the government's policy. When it does not control, we say it is loose. When it chooses to control, we say it is strict. This policy is assumed to refer only to outgoing profit and not to incoming one. That is, regional governments are always eager to have an enlarged tax base (through repatriated profits from foreign regions) and could only worry about seeing their tax base diminished. This implies that only the high tax region policy affects the firm's decision. We shall refer to this policy as the *effective policy*.

To simplify the analysis, we assume that if the effective policy is strict, the cost is dissuasive and therefore the firm does not manipulate the transfer price at all. If it is loose, there is no manipulation cost and the firm transfers all its profit to the low tax region.

Overall net profit of the firm located in region i is:

$$\pi_i = (1 - t_i)\pi_{ii}(g_i) + (1 - t_j)\pi_{ij}(g_i) - MC$$

where t_i is the tax rate in region i, $\pi_{ii}(g_i)$ is production plant declared profit and $\pi_{ij}(g_i)$ is the sales office declared profit.

$$\pi_{ii}(g_i) = (p_{ii} - w)q_{ii} + (g_i - w)q_{ij} = \frac{A^2 + 2A\tau(g_i - w)}{4\beta}$$
$$\pi_{ij}(g_i) = (p_{ij} - g_i - \tau)q_{ij} = \frac{A\tau^2 - 2A\tau(g_i - w)}{4\beta}$$

Total profit before taxes is therefore (we shall refer to it simply as *total profit*):

$$\frac{A^2 + A\tau^2}{4\beta}$$

where $A^2/4\beta$ is the profit effectively generated by the production plant (which we refer to as *production plant profit*) and $A\tau^2/4\beta$ is the equivalent one in the sales office (which we refer to as the *sales office profit*).

When the effective policy is strict, the firm does not manipulate g_i and therefore:

$$g_i = w \tag{1}$$

On the other hand, with a loose effective policy the firm transfers all its profits to the low tax region, therefore if $t_i > t_j$, g_i solves $\pi_{ii}(g_i) = 0$ and we obtain:

$$g_i = \underline{g} = w - \frac{A^2}{2A\tau} \tag{2}$$

Conversely, if $t_i < t_j$, g_i solves $\pi_{ij}(g_i) = 0$ and hence:

$$g_i = \overline{g} = w + \frac{A\tau}{2} \tag{3}$$

Whatever the effective control policy, the firm never wants to manipulate the transfer price if $t_i = t_j$.

The firm locates where its overall profit π_i is higher. We assume that when it is indifferent, it chooses region 2. This preference for region 2 may be due to any (however small) advantage from region 2 (or conversely cost in region 1), as e.g. a marginal difference in the population size.[3]

[3]This assumption plays an important role.We use it to avoid having our results based on the perfect symmetry of the regions, as they would be non-robust to a slight change in the model. In the extensions section, we solve the model without this assumption and discuss its implications.

2.2 The government

Each government faces a trade-off between the advantage of hosting the firm and the fiscal cost of attracting it. The advantage is the increase in the consumer surplus due to the absence of transport costs. The fiscal cost stems from the marginal cost of public funds. Implicitly, we have in mind a general equilibrium model where governments tax different sources besides company profits (like labor and consumption) to balance a fixed budget and when decreasing profit taxation they have to increase other forms of taxation.

The trade-off we want to model is captured by the following reduced form of the government's objective function, where we have normalized the marginal cost of public funds to unity:

$$\frac{(\alpha - p_i)^2}{2\beta} + T_i \tag{4}$$

where T_i is the profit tax revenue of region i.

Policy makers have two policy instruments: the profit tax (t_i) and the control policy (δ_i). We make the simplifying hypothesis that the cost of implementing a monitoring policy is zero.[4] The same profit tax must apply to both the multinational firm and domestic firms. We shall refer to the profits of the latter as the *domestic tax base*.

The government of region i may either control ($\delta_i = S$, for strict) or not ($\delta_i = L$, for loose) the firm's transfer price. If it does not and $t_i > t_j$ then the firm declares all its profit in j. Therefore, by being loose, a region commits to forego all fiscal receipt from the multinational if its tax is above that of its neighbor. Region j may then tax the firms' total profit even if does not host it. Whatever the location of the firm, under a strict effective policy its profits do not move whereas under a loose one they do.

The size of domestic tax base determines the cost to attract the firm. The larger it is, the more funds have to be collected from other sources when the profit tax decreases. We keep things simple by considering an inelastic domestic tax base but we allow it to differ across regions.[5] We let R_i denote the domestic tax base of region i with $R_1 = R$ and $R_2 = \gamma R$, and $\gamma \in [0, \infty]$. Governments may subsidize as much as they want but they cannot tax more

[4]We show that even under this hypothesis, it may happen that the government will make no effort at equilibrium. Extending the model to allow for a cost of monitoring would, if anything, make the loose policy more likely.

[5]The alternative of an elastic tax base is discussed in the extensions section, where we conclude that our results would be qualitatively similar under this alternative assumption.

than the existing tax base: $t_i \leq 1$ for $i = 1, 2$. We constrain the government to use the same tax on both the multinational profit and the domestic tax base.

The profit tax revenue of each government is given by:

$$\begin{aligned} T_i &= t_i(R_i + \pi_{ii}(g_i)) \text{ if the firm locates in } i \\ T_i &= t_i(R_i + \pi_{ji}(g_j)) \text{ if the firm locates in } j \end{aligned}$$

Substituting T_i into (4) one gets the payoff of each regional government. We denote the payoff of region 1 by u_i and the payoff of region 2 by U_i, where the i index denotes the location of the firm. Letting t denote the tax vector (t_1, t_2) and δ the control vector (δ_1, δ_2), we have, for region 1:

$$\begin{aligned} u_1(t, \delta) &= \frac{A^2}{8\beta} + t_1 \left(R + \frac{A^2 + 2A\tau(g_1 - w)}{4\beta} \right) \qquad (5) \\ u_2(t, \delta) &= \frac{A\tau^2}{8\beta} + t_1 \left(R\gamma + \frac{A\tau^2 - 2A\tau(g_2 - w)}{4\beta} \right) \end{aligned}$$

Similarly, for region 2:

$$\begin{aligned} U_2(t, \delta) &= \frac{A^2}{8\beta} + t_2 \left(R\gamma + \frac{A^2 + 2A\tau(g_2 - w)}{4\beta} \right) \qquad (6) \\ U_1(t, \delta) &= \frac{A\tau^2}{8\beta} + t_2 \left(R\gamma + \frac{A\tau^2 - 2A\tau(g_1 - w)}{4\beta} \right) \end{aligned}$$

Note from (5) and (6) that the consumer surplus advantage of hosting the firm is equal to

$$\frac{A^2 - A\tau^2}{8\beta}$$

This is increasing in τ, since the consumer price is higher in the foreign region because consumers bear a part of the transport cost.

On the other hand, regions compete for the taxation of the multinational. The amount each of them is able to tax may take one of four values: 0, the sales office profit $A\tau^2/4\beta$,the production plant profit $A^2/4\beta$ or the total profit $\left(A^2 + A\tau^2\right)/4\beta$. To see this, take for instance region 1 and the firm locating in 1. If $g_1 = w$ then region 1 taxes the production plant profit. If $g_1 = \bar{g}$ then region 1 taxes the total profit whereas if $g_1 = \underline{g}$ then it does not tax the multinational at all. The total profit of the multinational is decreasing with the transport cost. Economic integration due to a decrease in the transport cost makes it less and less profitable for regions to fight for the location of the firm, while it makes it all the more interesting to compete for the taxation of its total profit.

3 Equilibrium

The game is played in three stages. In the first stage, governments choose δ_1 and δ_2. Then they set t_1 and t_2.[6] Governments decide simultaneously. In the third stage the firm decides where to locate and finally production and consumption take place.

3.1 Location decision

We define the profit differential $\Delta\pi = \pi_2 - \pi_1$. If $\Delta\pi \geq 0$ the firm locates in region 2 otherwise in region 1. The firm decides location knowing both t and δ.

When $t_i = t_j$ or $t_i \neq t_j$ and the effective policy is S one has:

$$\pi_{ii}\left(g_i\right) = \frac{A^2}{4\beta} \quad \text{and} \quad \pi_{ij}\left(g_i\right) = \frac{A\tau^2}{4\beta}, \text{ for any } i = 1,2 \text{ and } i \neq j$$

whereas when $t_i > t_j$ and the effective policy is L:

$$\pi_{ii}\left(g_i\right) = 0 \quad \text{and} \quad \pi_{ij}\left(g_i\right) = \frac{A^2 + A\tau^2}{4\beta} \quad \text{for any } i = 1,2 \text{ and } i \neq j$$

From which one obtains

$$\begin{aligned}
\Delta\pi &= \frac{A^2 - A\tau^2}{4\beta}\left(t_1 - t_2\right) \quad \text{under effective policy } S \\
\Delta\pi &= 0 \quad \text{under effective policy } L
\end{aligned}$$

The following table summarizes the location equilibrium:

$\delta_2 = L$	Firm in 2	
$\delta_2 = S$	$t_1 < t_2$	Firm in 1
	$t_1 \geq t_2$	Firm in 2

Profit shifting takes place in two cases. Region 1 taxes the total profit if $\delta_2 = L$ and $t_2 > t_1$ $\left(g_2 = \underline{g} \text{ hence } \pi_{22}\left(g_2\right) = 0\right)$. Region 2 taxes the total

[6] By choosing this sequence of decisions, we implicitly assume that δ is essentially determined by reputation and therefore is a long-term policy variable.

It can be shown that both regions obtain the same payoffs at equilibrium under the alternative timing, except for region 1 in a small part of the parameter space. Moreover, the control policy choice of region 1. The only substantial difference is that, for intermediate values of τ, the control policy of region 2 will depend on the particular realization of the mixed strategy on taxes of the first stage. (calculations available upon request).

profit in case $t_1 > t_2$ and $\delta_1 = L$ ($g_2 = \bar{g}$ hence $\pi_{21}(g_2) = 0$). Apart from these situations, the firm chooses $g = w$.

Note that δ_1 does not enter the picture. This is due to the hypothesis of the firm's marginal preference for region 2. To see that, note that whenever $\delta_2 = L$ the firm will always set up its production plant in 2: it may locate in its favorite region and if the tax is higher there, it may shift its profit to the low tax region. When $\delta_2 = S$, region 1 may decide to be loose. This turns out to be the effective policy only if $t_1 > t_2$; then the firm chooses to locate in 2. Region 1 could otherwise decide to be strict. In that case, it captures the firm if $t_1 < t_2$. In both cases it is the tax difference that determines location and not the value of δ_1.

Not surprisingly, the region marginally preferred by the firm is better armed to compete for it, as it may use taxes as much as a loose policy while the other region may only compete through taxes. Once $\delta_2 = L$, governments may only compete for the taxation of the multinational's profit. Region 1 may either choose $t_1 < t_2$ and tax the total profit or it may set $t_1 > t_2$ in which case it will only be capable of taxing the sales office profit by choosing a strict control policy.

3.2 Tax game

In this stage of the game governments simultaneously choose tax rates, given their previous choice of a control policy and anticipating the firm's location decision.

3.2.1 The nature of the tax game

Whatever the combination of monitoring policies chosen in the first stage of the game, a region i is always better off if t_j is bigger than t_i. This gives it at least a higher tax base than when $t_i > t_j$: some profit from the multi-national adds up to the domestic tax base.

The nature of the strategic interaction stems from the tax base being inelastic as long as $t_i \neq t_j$ and infinitely elastic when $t_i = t_j$. We denote $H_i^\delta(t_i)$ the payoff of region i in subgame δ defined by the pair (δ_1, δ_2) if its tax rate is *higher* than that of the other region and $L_i^\delta(t_i)$ if it is *lower*. That is, given t_j, region i is either on its H_i^δ or L_i^δ branch of the utility, depending on whether $t_i > t_j$ or $t_i < t_j$.

Region 1, by moving its tax rate from above to below that of region 2 changes its payoff from $H_1^\delta(t_1)$ to $L_1^\delta(t_1)$. We always have $L_i^\delta(t_i) > H_i^\delta(t_i)$

hence for a given tax rate, both regions prefer to be the low tax one.[7] Note also that both functions are increasing in t_i. Therefore, along the $L_i^{\delta}(t_i)$ branch the region will fix its tax as close to t_j as possible and along the $H_i^{\delta}(t_i)$ it will set $t_i = 1$.[8] Put otherwise, each region faces a choice between taxing at 1 and being sure to be in its $H_i^{\delta}(t_i)$ branch or undercutting the other region and attaining $L_i^{\delta}(t_i)$. Notice that $H_i^{\delta}(1)$ is the utility the region may obtain whatever the other region's decision. We shall refer to this as **region i's security utility level**. One sees clearly that, given region j's strategy, there are two local maxima in region i's utility.

Given this payoff structure, a pure strategy equilibrium cannot exist. Take, for instance, subgame (L, L) where the firm will be in 2 anyway and regions may only compete for taxing the firm's total profit. Suppose $t_1 = t_2 = t$ strictly positive. There is no profit shifting and both regions may deviate to $t - \varepsilon$ thus capturing the profit made by the multinational in the other region, having a discrete gain at a marginal cost. Therefore no symmetric equilibrium exists with positive tax rates. A candidate equilibrium would be $t_1 = t_2 = 0$, with no tax revenue at all in both regions. This is not an equilibrium as both regions would gain by increasing their tax to unity in order to tax the domestic base. Consider now asymmetric equilibrium candidates such that $t_i < t_j$. Since region j does not collect any profits from the multi-national, the pair (t_i, t_j) is an equilibrium if and only if $t_j = 1$, i.e. j maximizing tax revenue on its domestic base. Region i optimally reacts by increasing its tax as its fiscal base is inelastic as long as it does not overbid region j. We are left with $t_i = 1 - \varepsilon$ and $t_j = 1$ as the only asymmetric candidate. This is not an equilibrium since j may profitably deviate to $1 - 2\varepsilon$ thus taxing the total profit of the firm.

The payoff structure responsible for the non-existence of a pure strategy equilibrium is common to the four subgames. Each of them differs in what a region gains by undercutting the other. For instance, the subgame (S, S) is such that there is no profit shifting and therefore regions can only compete for the firm's location.

Suppose that region i passes from $t_i = 1$ to undercutting j with a tax $t_i = t_j - \varepsilon$. Gains from undercutting may be of three types:

consumer surplus advantage: $\dfrac{A^2 - A\tau^2}{8\beta}$, which we call *location gain*

[7] This is true only for tax rates higher than t_i^{int} defined by $H_i^{\delta}\left(t_i^{int}\right) = L_i^{\delta}\left(t_i^{int}\right)$. However, it is straightforward to show that all $t_i < t_i^{int}$ are dominated by t_i^{int} (see Appendix).

[8] This full taxation comes obviously from the inelasticity of the local tax base, whose implication is discussed in the extensions.

$$\text{total profit taxation: } t_i \frac{A^2 + A\tau^2}{4\beta}, \text{ which we call } \textit{total profit gain}$$

$$\text{production plant profit taxation: } t_i \frac{A^2}{4\beta}, \text{ which we call } \textit{local profit gain}$$

The following table describes what each region gains when undercutting the other in each subgame.

	Region 1	Region 2
(L, L)	total profit gain	total profit gain
(S, L)	total profit gain	local profit gain
(S, S)	location gain and local profit gain	location gain and local profit gain
(L, S)	location gain and local profit gain	location gain and total profit gain

These gains come at a cost: fiscal revenue from the domestic tax base decreases. This cost is equal to $(1 - t_i) R_i$. There is a further cost from undercutting for region 1 in subgame (S, L), for region 2 in subgame (L, S) and for both of them in subgame (S, S). When the sales office region is strict, it has a fiscal revenue equal to the sales office profit $A\tau^2/4\beta$ which is lost when undercutting.

The balance between the gain to undercut and its cost will determine the region's *aggressiveness*, that is, how far is it ready to go in undercutting the other region. In each case there is a tax rate t_i that makes gains and losses from undercutting cancel out, i.e., such that higher (lower) taxes are such that the gain is higher (lower) than the cost. Region i never wants to play a tax rate lower than that one, as it would enjoy a lower utility than what it can guarantee itself by playing $t_i = 1$. We thus denote this tax rate $t_i^{\min\delta}$, formally defined as $t_i^{\min\delta} = \left\{ t_i : H_i^\delta(1) = L_i^\delta(t_i^{\min}) \right\}$. It depends negatively on the gain and positively on the cost from undercutting.[9] The smaller is $t_i^{\min\delta}$, the higher is the aggressiveness of the region: it is ready to play lower tax rates without preferring to rely on its security utility level.

The location gain is increasing in the transport cost whereas the total taxation gain is decreasing in τ. When the gains at stake stem from location, aggressiveness is increasing in the transport cost and the reverse is true when the gains concern taxation.[10]

[9] Fiscal gains from undercutting increase aggressiveness if and only if the region undercuts with a positive tax.

[10] This remains true if one takes into consideration the $A\tau^2/4\beta$ cost in the cases in which it is relevant.

3.2.2 Mixing over tax rates

Given that a pure strategy equilibrium fails to exist, we rely on mixed strategies to compute the tax game equilibrium. In the Appendix, we prove by construction the existence of an equilibrium and the uniqueness of equilibrium utilities. Our approach follows closely that of Kreps and Scheinkman (1983) to compute the mixed strategy equilibrium of the capacity constrained Bertrand competitors.[11]

With this type of equilibrium concept, strategies are lotteries in tax levels: regions play probability distributions $F_i(t_i)$ on the support $t_i \in [\underline{t_i}, \overline{t_i}]$. Moreover, $0 \leq F_i(t_i) \leq 1$ and $F_i'(t_i) \geq 0$ for $t_i \in [\underline{t_i}, \overline{t_i}]$ and $i = 1, 2$. We further denote $f_i(t_i)$ the probability density corresponding to $F_i(t_i)$. For instance, for the (L, L) subgame, payoffs for a given realization of the lottery are as follows:

$$u^{(L,L)}(t) = \begin{cases} H_1^{(L,L)}(t_1) = \frac{A\tau^2}{8\beta} + t_1 R & \text{if } t_1 > t_2 \\ S_1^{(L,L)}(t_1) = \frac{A\tau^2}{8\beta} + t_1 \left(R + \frac{A\tau^2}{4\beta}\right) & \text{if } \quad t_1 = t_2 \\ L_1^{(L,L)}(t_1) = \frac{A\tau^2}{8\beta} + t_1 \left(R + \frac{A^2 + A\tau^2}{4\beta}\right) & \text{if } t_1 < t_2 \end{cases}$$

$$U^{(L,L)}(t) = \begin{cases} H_2^{(L,L)}(t_2) = \frac{A^2}{8\beta} + t_2 R\gamma & \text{if } t_2 > t_1 \\ S_2^{(L,L)}(t_2) = \frac{A^2}{8\beta} + t_2 \left(R\gamma + \frac{A^2}{4\beta}\right) & \text{if } \quad t_1 = t_2 \\ L_2^{(L,L)}(t_2) = \frac{A^2}{8\beta} + t_2 \left(R\gamma + \frac{A^2 + A\tau^2}{4\beta}\right) & \text{if } t_2 < t_1 \end{cases}$$

Proposition 1 *Each subgame δ has a unique equilibrium.*

Proof The proof is given in the Appendix.□

Let us be more explicit about the mixed strategy equilibrium. Given that the other region is mixing over tax rates, when region i plays t_i it has a probability $1 - F_j(t_i)$ of getting its utility from the good branch $L_i^{\delta}(t_i)$ and a probability $F_j(t_i)$ of getting it from the bad branch $H_i^{\delta}(t_i)$. The mixing behavior of region j is a way to make i indifferent among all taxes on the support of its mixed strategy: higher taxes give higher utility in both branches of the utility function *but* they decrease the probability of falling into the good branch L_i^{δ}.

The following proposition characterizes equilibrium utilities and strategies.

[11]Although we do not need to appeal to it in our paper as existence is proven by construction, an existence theorem in Dasgupta and Maskin (1986) applies to our game (Theorem 5B).

Proposition 2 *In each subgame δ, equilibrium utilities are such that*

(i) $U^{\delta *}=H_2^\delta(1)$ *and* $u^{\delta *}=L_1^\delta\left(t_2^{\min\delta}\right)$ *if and only if* $t_1^{\min\delta}<t_2^{\min\delta}$

(ii) $u^{\delta *}=H_1^\delta(1)$ *and* $U^{\delta *}=L_2^\delta\left(t_1^{\min\delta}\right)$ *if and only if* $t_2^{\min\delta}<t_1^{\min\delta}$

Moreover, whenever $t_i^{\min\delta}\geq t_j^{\min\delta}$*, equilibrium strategies are played on the support* $\left[t_i^{\min\delta},1\right]$ *and such that* $F_j(t_j)\geq F_i(t_i)$ *and no tax level is played with positive probability except* $t=1$ *by region* i*.*

Proof The proof is given in the Appendix.□

Equilibrium utilities are quite intuitive to interpret. The less aggressive region, say region i, is the one getting an equilibrium utility as if it is undercut by the other region, say j. Given that i's equilibrium strategy (first degree) stochastically dominates j's one, on average, i *is undercut* more often by j than the reverse. Moreover, none of the regions has a profitable deviation. Since $H_i^\delta(1)=L_i^\delta(t_i^{\min\delta})$, i has no interest to deviate since in order to undercut j (by playing taxes lower than $t_i^{\min\delta}$) it looses utility. Region j could conceivably play higher taxes and get a higher utility. But this would give i a profitable deviation, namely, a positive probability of undercutting j at taxes higher than $t_i^{\min\delta}$.

The relative aggressiveness of regions (that is, which is more aggressive) depends on the size of the domestic base: for each subgame, there is a threshold γ above which region 1 is more aggressive than region 2. If $R_1=R_2$ (i.e. $\gamma=1$) and one region is strict while the other is loose, the additional cost of losing full taxation of the sales-office profit makes the strict region less aggressive. On the other hand, if regions are both strict or both loose, they are equally aggressive.

In subgames (L,L) and (S,S), regions' aggressivenesses differ only because of the different R_i. Therefore, we will have $t_2^{\min\delta}\geq t_1^{\min\delta}$ for $\delta\in\{(L,L),(S,S)\}$ if and only if $\gamma\geq 1$. Under (L,S), R_2 has to be sufficiently smaller than R_1 for 2 to be more aggressive than 1. Sufficiently smaller means just enough to cover the additional cost of $A\tau^2/4\beta$. Therefore on the (L,S) subgame we have $t_2^{\min(L,S)}\leq t_1^{\min(L,S)}$ if and only if $\gamma\leq\hat{\gamma}=1-A\tau^2/(4R\beta)$. Conversely, under (S,L) we have $t_1^{\min(S,L)}\leq t_2^{\min(S,L)}$ if and only if $\gamma\geq\hat{\hat{\gamma}}=1+A\tau^2/(4R\beta)$.[12] The three thresholds $\hat{\gamma},1$ and $\hat{\hat{\gamma}}$ determine four regions in the (γ,τ) parameter space (note that $\hat{\gamma}$ and $\hat{\hat{\gamma}}$ depend on τ).

We now turn to the regional choice of the control policy.

[12]See Appendix for exact derivation of $\hat{\gamma}$ and $\hat{\hat{\gamma}}$.

3.3 Choice of monitoring policy

Depending on the value of γ, the choice of a control policy by regions 1 and 2 will yield different (expected) firm location and profit shifting behavior, as summarized in Figure 1. For each zone in the (γ, τ) space, the figure shows a table with four entries. Each of this entries corresponds to the firm location and profit shifting in a subgame δ. The upper left hand side entry of the table corresponds to $\delta = (L, L)$ and then clockwise one has successively $\delta = (L, S)$, (S, L) and (S, S) . For instance, for $1 < \gamma < \widehat{\hat{\gamma}}$ one reads "if both regions are loose, the firm locates in 2 and all its profit is declared in 1", "if both regions are strict, the firm locates in 1 and does not shift profits" and so forth.

Before fully characterizing the equilibrium, one may wonder under which circumstances will the countries use a loose monitoring policy. Our model predicts that when a marginal advantage for one region exists, the other region is always strict.

Proposition 3 *Region 1 will always be strict in equilibrium.*

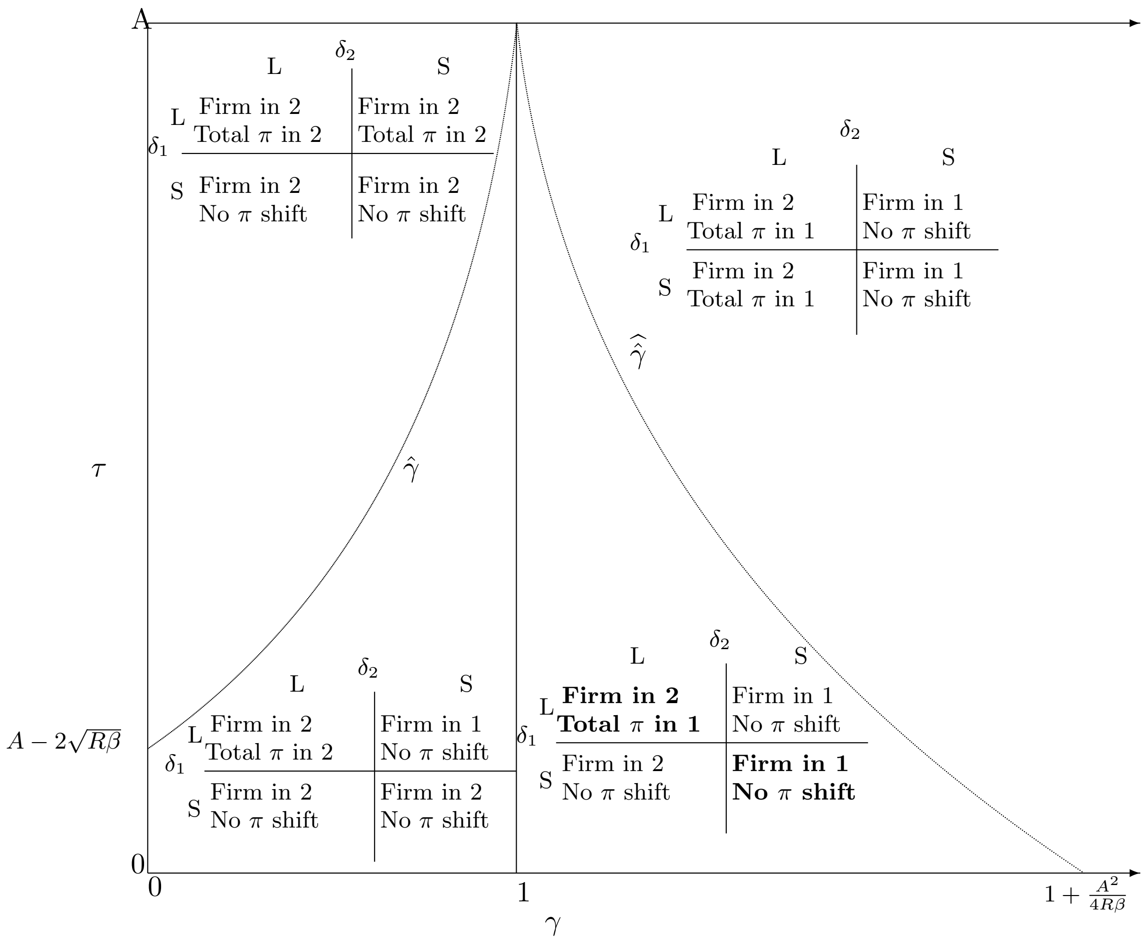
Proof The proof is given in the Appendix.□

Note that this result does not stem from a simple dominant strategy argument, i.e., there are situations under which region 1's best reply is to be loose.

The fact that (L, L) is never an equilibrium is a direct consequence of our assumption of a marginal preference for region 2. By fixing $\delta_2 = L$ region 2 is sure to host the firm. Given that regions play mixed strategies, region 1 may either have the lowest tax (in which case its control policy is not effective, thus irrelevant for its payoff) or it may have the highest tax. In this last case, choosing $\delta_1 = S$ allows it to tax the sales-office profit whereas by choosing $\delta_1 = L$ it does not tax the multi-national at all.

As regards (L, S), the intuition is easy to grasp if one keeps in mind Figure 1, in order to understand each region's motivation to change its control policy. When $\gamma \leq \hat{\gamma}$, region 1 is undercut more often *at equilibrium* irrespective of the monitoring policy it chooses and the same reasoning as for (L, L) above applies. With $\gamma \geq \hat{\gamma}$, there are parameter constellations under which region 1 prefers to be loose, but these are such that $U^{(L,S)*} < U^{(L,L)*}$, that is, if 1 deviates from (S, S) to (L, S) then 2 further deviates to (L, L). To see why this is the case, note that there are two relevant cases. For $\gamma < 1$, region 1 has both the location and the local profit gain when deviating from (S, S) to (L, S); when region 2 further deviates to (L, L) it has both the location and the total profit gain. Hence if 1 wants to deviate so does 2.

Figure 1: Equilibrium Utilities of the Tax Game



For $\gamma > 1$, it is easier to begin by looking at region 2's aggressiveness in both subgames (S, S) and (L, S). In the former it competes for location and local profits whereas in the latter it competes for location and total profits. Therefore, as long as $t_2^{\min}$ is positive (negative), it is more (less) aggressive in the second case. Region 1 hosts the firm and there is no profit shifting in both (S, S) and (L, S) and the only reason why it would deviate is to decrease region 2's aggressiveness. It will only be interested in so doing if $t_2^{\min}$ is negative, which may happen if the location gain is so high that it pays to subsidize the firm. Now region 2, when deviating from (L, S) to (L, L), does so precisely to get the firm within its borders. Therefore, whenever $t_2^{\min}$ is negative, 2 wants to further deviate to (L, L).

The following proposition addresses region 2's choice of a monitoring policy: contrary to what happens with region 1, the favored region will under some circumstances be loose. Interestingly, it does not do so only as a means to attract the firm. Even if it plays lower taxes (on average) than region 1 on subgame (S, S) it may be interested to switch to (S, L) because by doing so it may undercut 1 at higher tax rates. Our setting enables us to identify another effect of the monitoring policy: strategic manipulation of the other region's aggressiveness in the tax game.

Proposition 4 *Region 2 chooses $\delta_2 = L$ for high and $\delta_2 = S$ for low transport costs.*

Proof The proof is given in the Appendix.□

To understand this result, recall that region 1 is strict. When $\gamma < 1$, Figure 1 shows that region 2 is more aggressive than region 1 in both subgames (S, S) and (S, L) (it enjoys $U^{\delta *} = L_2^{\delta}(t_1^{\min \delta})$ in both subgames). It undercuts 1 at equilibrium and is able to do so with higher taxes the less 1 is aggressive. It therefore acts as to minimize region 1's aggressiveness: the monitoring policy entails an *aggressiveness effect*. As region 1 competes for the location gain if region 2 is strict and for the total profit gain if 2 is loose, we have that 1's aggressiveness is higher (lower) in the (S, S) subgame than in the (S, L) for high (low) τ.

When $\gamma > 1$, region 2 is no longer capable of hosting the firm in subgame (S, S) and the choice of the control policy corresponds to deciding whether it is profitable to attract the firm by being loose: *the location effect*. Region 2 prefers to host the firm for high τ as loosing it would entail a high cost in consumer surplus and to let go of it for low τ.

However, attracting the firm entails a fiscal cost as by being strict region 2 taxes both the sales-office profit (decreasing with τ) and the domestic base with a unit tax. This fiscal cost may be attenuated if by getting the firm region 2 is also able to tax its profits, which is made possible if the switch to the loose control policy entails an aggressiveness effect. When $1 < \gamma < \widehat{\hat{\gamma}}$, it can do so as the relative aggressiveness of region 1 is lessened in the (S, L) subgame as compared to the (S, S) one. This explains why the threshold τ (see Figure 2) is increasing with γ: the fiscal cost of being loose is both increasing with γ and decreasing with τ. When $\gamma > \widehat{\hat{\gamma}}$, being loose does not have any effect on relative aggressiveness thus the threshold τ remains constant (the choice of δ_2 is in this case motivated by a pure location effect).

Note that paradoxically region 1 is able to host the firm only when it does not have that much of an impact in utility. The firm's infinitesimal preference for region 2 gives it the power to host the firm when it is more profitable to do so.

An interesting question to ask is if profit shifting does arise at equilibrium, that is, whether the firm is able to take advantage from the announcement of a loose policy by the government of region 2. The following proposition addresses this issue.

Proposition 5 *Equilibria with profit shifting are such that the multinational locates in 2 and on average declares its total profit in 1. This type of equilibrium exists for $\tau \geq \tilde{\tau}$ and $\gamma \geq \widehat{\hat{\gamma}}$:*

(i) τ is high enough such that region 2 is loose and hosts the firm and

(ii) γ is high enough such that region 1 undercuts region 2 on average.

Proof The proof is given in the Appendix.□

As regards expected utility, three equilibrium types arise: *(i)* the firm is expected to locate or locates for sure in 2 and pays or is expected to pay all taxes locally; *(ii)* the firm is expected to locate in 1 and pays all taxes locally; *(iii)* the firm locates for sure in 2 and is expected to pay all its taxes in 1. Profit shifting arises in the third type of equilibrium, which is characterized by both region 1 being very aggressive and region 2 being very much interested in hosting the firm.[13]

Note that we must have at least $R_2 \geq R_1$ $(\gamma \geq 1)$. Region 1 can only capture the firm's total profit if its aggressiveness is sufficiently high. $\widehat{\hat{\gamma}}$ is decreasing in τ as region 1's cost to undercut amounting to the loss of the full taxation of the sales office profit is decreasing in τ. The lower is τ, the

[13]Note that the threshold τ is the one that leaves 2 indifferent between hosting the firm and not taxing it at all $\frac{A^2}{8\beta} + R\gamma$ or hosting the sales office and taxing it fully $\frac{A\tau^2}{8\beta} + \frac{A\tau^2}{4\beta} + R\gamma$.

smaller has to be R_1 relative to R_2 (the higher γ), for 1 to be more aggressive than 2.

When τ falls below a certain threshold, it is no longer the constraint on 1's aggressiveness but the one on 2 wanting the firm which is active. Region 2 then lets go of the firm (not to bad for low τ) and gets in return the sales office and its profits to tax (high since τ is low) by being strict.

Figure 2 summarizes the findings about the equilibrium choice of the control policy and the (expected) firm location and profit shifting behavior arising under such a choice. The thick line separates different types of equilibrium in terms of monitoring policy.[14] Thin lines separate different types of equilibrium in terms of location and taxation.

3.4 How does equilibrium vary with R?

Figure 2 clearly shows that the relative size of domestic tax bases has an impact on equilibrium. The absolute size of the tax base also influences regional behavior. The following proposition shows that R and γ actually have similar effects.

Proposition 6 *As R increases, the parameter space under which*

(i) region 2 is loose diminishes

(ii) profit shifting arises at equilibrium expands.

Proof The proof is given in the Appendix.□

Figure 3 illustrates how equilibrium changes as R varies.

The intuition is straightforward if one thinks about how R influences both effects of the control policy. For small values of R, regions are very aggressive in the tax game as they do not have a lot to loose. Given that they are ready to fight through taxes, the location effect of the control policy is less important. As R increases, the location effect progressively replaces the aggressiveness one, the same effect of an increase in γ.

The loose policy gains importance as domestic tax bases decrease. Take the limit case of $R = 0$, when the profit tax only concerns the multi-national. Regions are then extremely aggressive when competing for location (in the (S, S) subgame) ultimately transferring the location gain to the firm.[15] Region 2 therefore has an additional interest to be loose and attract the firm

[14] Analytical expressions for $\check{\tau}$, $\hat{\tau}$ and $\tilde{\tau}$ are given in Appendix 2.

[15] Formally, both regions are driven down to their respective security utility level.

Figure 2: Monitoring Equilibrium in the (γ, τ) space

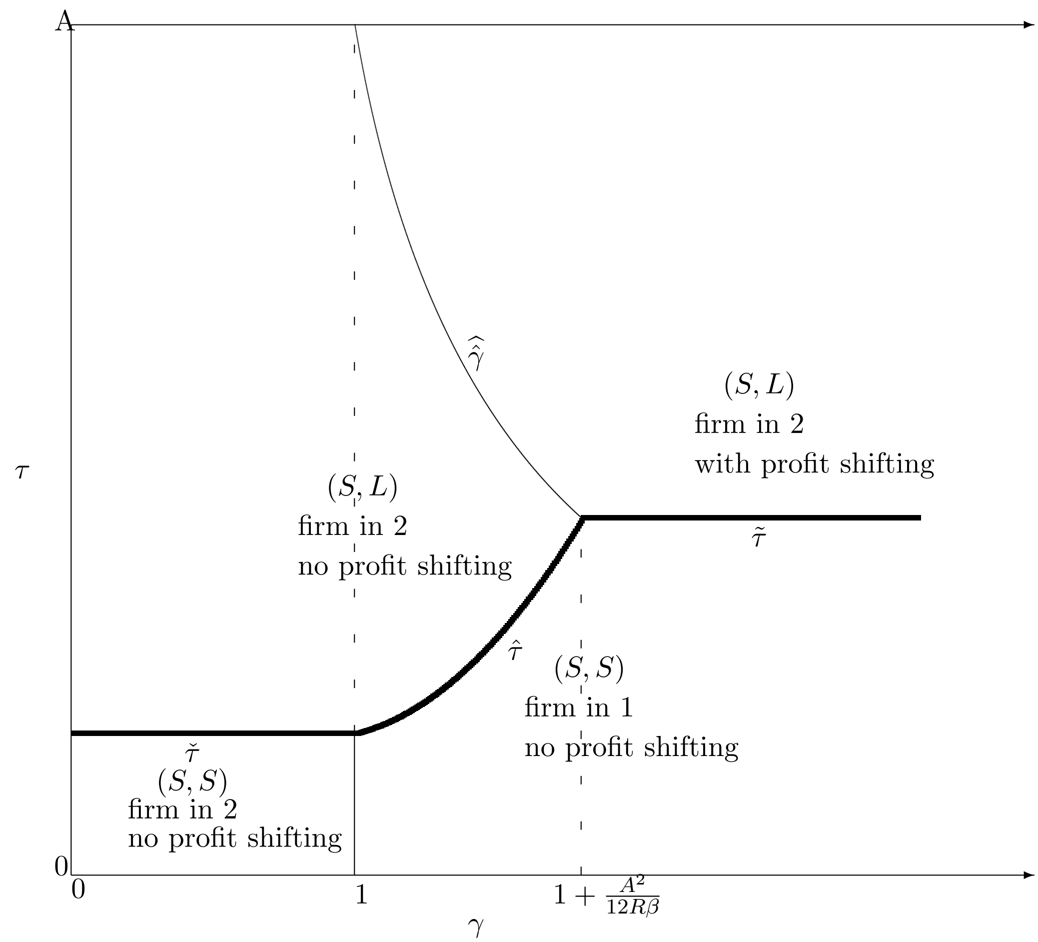
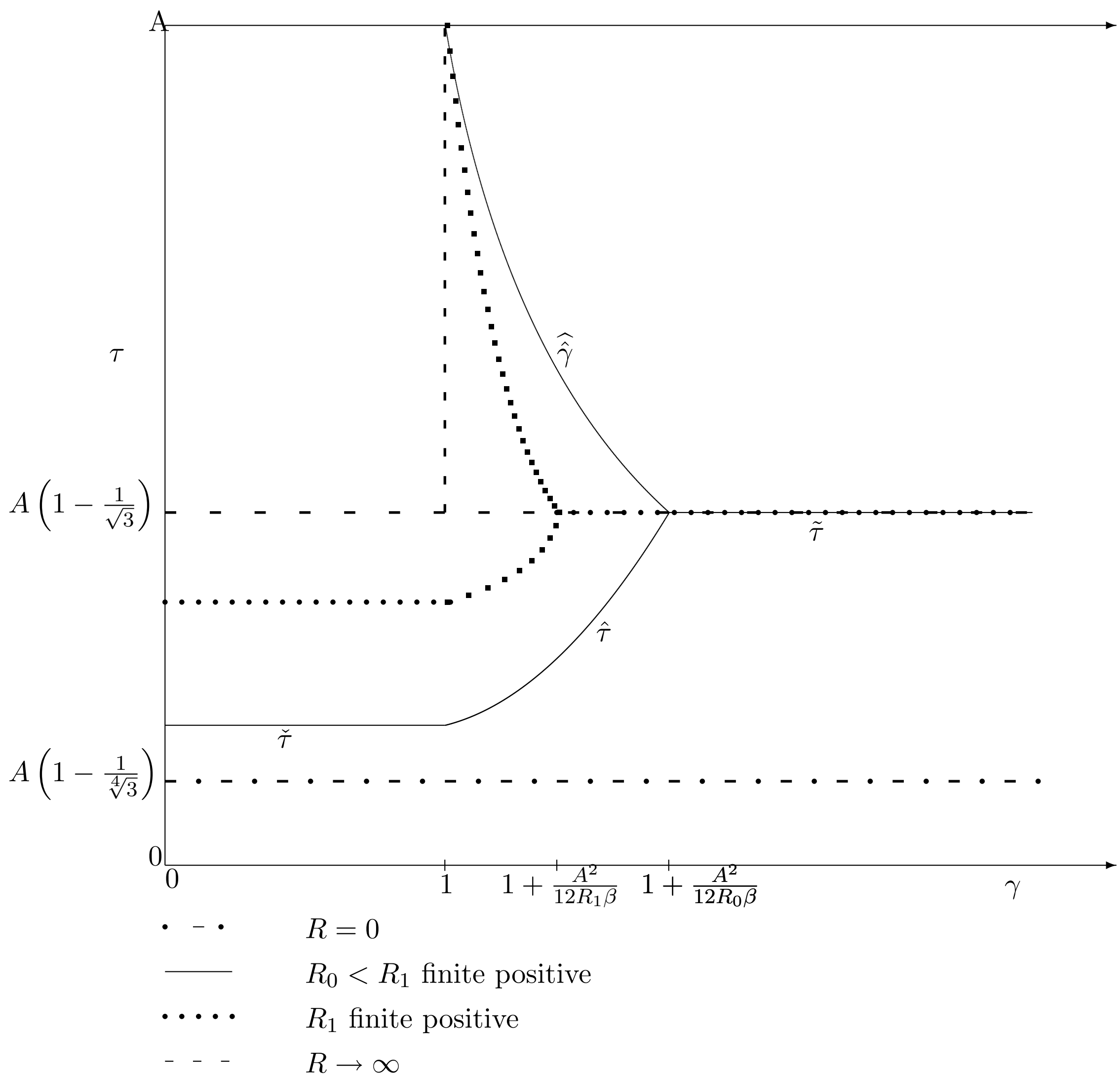


Figure 3: Effect of changing R



independently of taxes as this avoids this type of destructive tax competition. Therefore the loose policy is good starting from a lower threshold transport cost than when $R > 0$. Moreover, the location effect is never present independently of the aggressiveness one (put otherwise, profit shifting never arises: region 2 never chooses to be loose in order to attract the firm with a unit tax).

An increase in R makes it all the more interesting for region 2 to be strict. On the one hand, due to the aggressiveness effect: an increase in R causes a higher rise in $t_1^{\min(S,S)}$ than in $t_1^{\min(S,L)}$. This makes the strict policy more attractive for 2. The reason is that 1 undercuts in (S, L) for the total profits gain which depends on the tax rate it sets. On the contrary, in (S, S) the undercutting gain has a part which is invariant to tax rate used (the location gain). Aggressiveness is more responsive to R when the gains are non fiscal. Say if R decreases, the region becomes more aggressive and it is obviously ready to quote a lower tax if its gain is invariant with the tax rate chosen. On the other hand, the location effect becomes more and more important as it is increasingly costly to attract the firm through taxation. The parameter space under which this effect plays a role independently of the aggressiveness one becomes wider ($\widehat{\gamma}$ decreases or equivalently profit shifting becomes more common at equilibrium).

4 Extensions

We have put a lot of structure in our model in order to establish our results. Accordingly, it is only fair to wonder to what extent our conclusions depend on the details of our model. We discuss hereafter a few of its key aspects.

(i) Discriminatory taxation

Call t_i^R the tax rate on the domestic base and t_i the tax on multinational profits. It is clear that $t_i^R = 1$, $i = 1, 2$. The game on t_i is then formally equivalent to the non-discriminatory one with $R = 0$.[16] Therefore the threshold τ above which region 2 is loose corresponds to the down-most line in Figure 3 above. As for $R = 0$ above, the aggressiveness effect is clearly the most important one determining such a threshold.

(ii)Marginal preference for region 2

Changing the hypothesis of a marginal preference for region 2 modifies the payoffs in a non-trivial way. If we let, say, the firm toss a fair coin when $\Delta\pi = 0$, then region 2 no longer has the possibility of guaranteeing itself

[16]Bear in mind that it is not equivalent in terms of utilities as in the discriminatory case one has to add $t_i^R R_i = R_i$.

the firm by playing $\delta_2 = L$. Indeed, if 1 responds by playing $\delta_1 = L$, they are bound to share the firm. This being the case, it is no longer true that 1 will never be loose in equilibrium, since it is as likely as region 2 to use its control policy to attract (half of) the firm. Two new types of equilibrium are possible: (L, L) and (L, S).

To see that, take high values of τ such that not hosting the firm entails a huge cost in consumer surplus. In the original analysis, for all such values region 1 never has the firm (although it taxes its total profit for γ high enough). This can no longer be an equilibrium in this modified game, as 1 may host half of the firm by being loose. By an analogous argument for region 2, we have that equilibrium for high values of τ is (L, L) and profit shifting arises at equilibrium for all γ (all profit taxed in 1(2) for γ bigger(smaller) than 1).[17] As τ decreases, competition for the firm is less fierce and as before we will have the region with the smallest γ hosting the firm and all taxes being paid locally. For small values of τ, the prevailing equilibrium will be (S, S) and for intermediate values we have (S, L) for small γ and (L, S) for high γ. The switch from (S, S) to (S, L) is, as in the original analysis, motivated solely by the aggressiveness effect. The same behavior by region 1 explains why equilibrium switches from (S, S) to (L, S) for high τ. However, (L, S) arises only at γ big enough and (S, L) only at γ small enough such that there no profit shifting arises.

Given the symmetric strategic position of regions, there is a region of intermediate τ in which both (S, L) and (L, S) coexist. To see why, it is instructive to begin by looking at the fully symmetric case $\gamma = 1$. As τ increases and region 1 switches from (S, S) to (L, S) due to the aggressiveness effect, by the same token 2 will do the same. Now take equilibrium (S, L) in which 2 hosts the firm and there is no profit shifting and let γ increase. One of two things may happen. Either region 1 prefers to change to $\delta_1 = L$ as it undercuts region 2 in subgame (L, L) and it is able to do so with higher taxes the higher γ or it becomes too expensive for the high domestic base region 2 to keep up with this type of equilibrium where it undercuts region 1. A parallel argument holds for when one lets γ decrease. The multiple equilibrium region arises only for γ close to 1.

(iii)Number of multinational firms

Our model builds on the assumption that there exists only one mobile multinational firm. There is no doubt that reality is different. In partic-

[17] The threshold value of τ is in this case $A\left(1 - 1/\sqrt{5}\right)$: the one that leaves the region indifferent between hosting the firm on average and not taxing at all $\frac{A^2+A\tau^2}{16\beta} + R_i$ or hosting the sales office and taxing it fully $\frac{A\tau^2}{8\beta} + \frac{A\tau^2}{4\beta} + R_i$.

ular, a model with a continuum of mobile firms characterized by different degrees of mobility may look more appealing. As should be obvious by now, the fact that we consider only one mobile firm automatically induces payoff discontinuity in the tax game. By contrast, a continuum of heterogeneously mobile firms would clearly restore continuity. Let us stress however that discontinuity is not crucial for our argument to be developed. What matters is the trade-off between high taxes on the immobile tax base and lower taxes which attract firms (and possibly increase the tax base). This trade-off being formally expressed by the non-concavity of regions' payoffs. Clearly enough, this non-concavity would be preserved should there be a continuum of multinational firms, under most reasonable assumptions regarding mobility. In this respect the qualitative features of Nash equilibria in tax subgames would be preserved, i.e. regions could be ranked by their aggressiveness with the less aggressive region enjoying its security utility level. In other words, the trade-off at work when choosing the $\delta = L$ is still present. Clearly, assuming that there is only one mobile firm allows for more clear-cut results and in this respect eases the analysis of the tax competition subgames.

(iv)Rigid domestic tax base

One may wonder what would happen if instead of an inelastic domestic tax base we let it vary with the tax rate, e.g. be a concave function of it. The substance of our results would not change. The security utility level would then be given by the optimum of the concave high-tax branch. One would compute $t_i^{\min\delta}$, $i = 1, 2$ as before and Propositions 1 and 2 would follow. The crucial thing for the whole construction to work is that at the security level tax rate the L-branch of the utility function is above its H-branch. It is straightforward to check that any reasonable domestic tax revenue function (that is, one that attains a maximum at a positive tax level) does the job.[18]

(v)Market structure

Our results are derived under the hypothesis that the multi-national is a monopoly in each market and moreover it faces a linear demand. This assumptions are useful to keep the model tractable. A more general model featuring both an advantage to host the firm and a fiscal cost to attract it through taxes would yield similar results. Both the attraction and the aggressiveness effects of the control policy would still play a role. However,

[18]Capacity constrained Bertrand competitors do have concave payoff functions (Cfr. Kreps and Scheikman, 1983). In their case some pairs of capacity levels are such that a pure strategy equilibrium exists, either because the L branch coincides with the H one or because the security price is to the left of the intersection of the two branches (meaning that at the security price the L branch is below the H one).

determining whether countries are to be loose or strict at equilibrium then depends on which parameters of the model influence the advantage to host the firm, on the one hand, and its total profit on the other. E.g., if we take the case of an oligopolistic market then the transport cost will play the same role as in our case, as soon as the negative impact of hosting the multi-national on local firms' profits is outweighed by the positive one on the consumer surplus.

5 Conclusion

This paper analyzes how leniency of government in the control of multinational's profit shifting behavior can be used as an instrument along with the tax rate to compete for mobile firms.

It provides a detailed analysis of the nature of tax competition when fiscal authorities cannot discriminate among tax bases with different elasticities. On the one hand, it enriches the strategy set of regional governments in allowing them to choose the control policy. In a simple tax competition game, the analysis boils down to the (S, S) subgame. For $\gamma \leq 1$, the firm is expected to be in 2 and pay its taxes locally; for $\gamma \geq 1$, the firm is expected to be in 1 and pay its taxes locally: as region 2 looses its additional tool to capture the firm when it cannot do it by fighting in taxes, it is only the difference in domestic bases that matters for equilibrium. Location of the firm becomes independent of τ and not surprisingly profit shifting never arises. On the other hand, it identifies two effects of the monitoring policy: the location and the aggressiveness effects. The first one allows the region to attract the firm by being loose, irrespectively of its tax rate. Through the second one, the monitoring policy is a device to control the other region's aggressiveness. This latter effect is more important the smaller is γ (and/or R) whereas the former one, quite intuitively, gains importance as the relative and/or the absolute size of the tax base increase.

Our model has clear empirical predictions. Countries with a relatively large domestic tax base are less likely to be loose. Moreover, a decrease in the transport cost makes countries more willing to control profit shifting. Whether these predictions are verified or not is an empirical question that is beyond the scope of this paper. Table 1, shown in the introduction, allows us however to make some (albeit loose) comments. The relatively new emergence of profit shifting laws doesn't contradict the second prediction. The fact that almost all small open economies (at the exception of Denmark) are lenient may be taken as an indication that the first prediction is not contradicted. The precursor role of the US in the implementation of profit shifting rules as pointed out by The Economist could also be interpreted in that direction. Nevertheless there are countries like Germany or Italy that could be seen as counter examples.

6 Appendices

A1 : The tax game

To prove propositions 1 and 2 we proceed as follows. We begin by defining the regional payoffs in each subgame; then we take subgame (L, L) and prove by construction that it has an equilibrium with the features described in the propositions. Equilibria for the three remaining subgames may be constructed analogously.

Regional payoffs

Recalling (1), (2) (3), (5) and (6) it is straightforward to obtain the payoffs of region 1 in each subgame:[19]

$$\begin{aligned}
H_1^{(L,L)}(t_1) &= H_1^{(L,S)}(t_1) = \frac{A\tau^2}{8\beta} + t_1 R \\
L_1^{(L,L)}(t_1) &= L_1^{(S,L)}(t_1) = \frac{A\tau^2}{8\beta} + t_1 \left(R + \frac{A^2 + A\tau^2}{4\beta} \right) \\
H_1^{(S,S)}(t_1) &= H_1^{(S,L)}(t_1) = \frac{A\tau^2}{8\beta} + t_1 \left(R + \frac{A\tau^2}{4\beta} \right) \\
L_1^{(S,S)}(t_1) &= L_1^{(L,S)}(t_1) = \frac{A^2}{8\beta} + t_1 \left(R + \frac{A^2}{4\beta} \right)
\end{aligned}$$

and those of region 2:

[19] For the sake of completeness, we shall define S_i^δ the utility of the region when its tax rate is the same as that of region j. Though these functions are not essential for the understanding of the paper, they are used in the existence proof. For a strict effective control policy when $t_2 = t_1$ the firm chooses region 2. When the effective policy is loose, $t_2 = t_1$ implies that firms choose the true transfer price $g = w$ and therefore everything works as under a strict effective policy.

We therefore have: $S_1^\delta = H_1^{(\infty,\infty)}$ and $S_2^\delta = L_2^{(\infty,\infty)}$ for the four subgames δ.

$$
\begin{aligned}
H_2^{(L,L)}(t_2) &= H_2^{(S,L)}(t_2) = \frac{A^2}{8\beta} + t_2 R\gamma \\
L_2^{(L,L)}(t_2) &= L_2^{(L,S)}(t_2) = \frac{A^2}{8\beta} + t_2\left(R\gamma + \frac{A^2 + A\tau^2}{4\beta}\right) \\
H_2^{(S,S)}(t_2) &= H_2^{(L,S)}(t_2) = \frac{A\tau^2}{8\beta} + t_2\left(R\gamma + \frac{A\tau^2}{4\beta}\right) \\
L_2^{(S,S)}(t_2) &= L_2^{(S,L)}(t_2) = \frac{A^2}{8\beta} + t_2\left(R\gamma + \frac{A^2}{4\beta}\right)
\end{aligned}
$$

Equilibrium Construction

We are looking for equilibria with the following characteristics. Strategies are lotteries in tax levels: regions play probability distributions $F_i(t_i)$ on the support $t_i \in [\underline{t_i}, \overline{t_i}]$. Moreover, $0 \leq F_i(t_i) \leq 1$ and $F_i'(t_i) \geq 0$ for $t_i \in [\underline{t_i}, \overline{t_i}]$ and $i = 1, 2$. We further denote $f_i(t_i)$ the probability density corresponding to $F_i(t_i)$. For instance, for the (L, L) subgame, payoffs for a given realization of the lottery $t = (t_1, t_2)$ are as follows:

$$
u^{(L,L)}(t) = \begin{cases}
H_1^{(L,L)}(t_1) = \frac{A\tau^2}{8\beta} + t_1 R & \text{if } t_1 > t_2 \\
S_1^{(L,L)}(t_1) = \frac{A\tau^2}{8\beta} + t_1\left(R + \frac{A\tau^2}{4\beta}\right) & \text{if } \quad t_1 = t_2 \\
L_1^{(L,L)}(t_1) = \frac{A\tau^2}{8\beta} + t_1\left(R + \frac{A^2+A\tau^2}{4\beta}\right) & \text{if } t_1 < t_2
\end{cases}
$$

$$
U^{(L,L)}(t) = \begin{cases}
H_2^{(L,L)}(t_2) = \frac{A^2}{8\beta} + t_2 R\gamma & \text{if } t_2 > t_1 \\
S_2^{(L,L)}(t_2) = \frac{A^2}{8\beta} + t_2\left(R\gamma + \frac{A^2}{4\beta}\right) & \text{if } \quad t_1 = t_2 \\
L_2^{(L,L)}(t_2) = \frac{A^2}{8\beta} + t_2\left(R\gamma + \frac{A^2+A\tau^2}{4\beta}\right) & \text{if } t_2 < t_1
\end{cases}
$$

We begin by assuming that *(a.1)* no tax t_i on $\left]\underline{t_i}, \overline{t_i}\right[$, $i = 1, 2$, is played with positive probability *(a.2)* $\overline{t_1}$ and $\overline{t_2}$ are not played both with positive probability and *(a.3)* $\underline{t_1}$ and $\underline{t_2}$ are not played both with positive probability. We later prove that this is indeed the case. This means that the event $t_1 = t_2$ has zero probability and allows us to write the expected utility for region i of playing t_i as:

$$
EU_i(t_i) = F_j(t_i)H_i^{(L,L)}(t_i) + (1 - F_j(t_i))\, L_i^{(L,L)}(t_i), \quad j \neq i, \quad i = 1, 2 \quad (7)
$$

For players to be ready to randomize, the expected utility of playing each of the actions in the support of the mixed strategy must be the same. Denoting the equilibrium utilities as $u^{(L,L)*}$ and $U^{(L,L)*}$ for regions 1 and 2 respectively, we have that $EU_1(t_1) = u^{(L,L)*}$ and $EU_2(t_2) = U^{(L,L)*}$ for $t_i \in [\underline{t_i}, \overline{t_i}]$ and $i = 1, 2$.

Recall the definition of $t_i^{\min(L,L)} = \left\{ t_i : H_i^{(L,L)}(1) = L_i^{(L,L)}(t_i) \right\}$. We further denote the intersection of the two branches as $t_i^{int(L,L)} = \left\{ t_i : H_i^{(L,L)}(t_i) = L_i^{(L,L)}(t_i) \right\}$.

The following is easily established.

1. Region i will never play $t_i < t_i^{int(L,L)}$, or equivalently, all possible actions by region i are such that $H_i^{(L,L)}(t_i) \le L_i^{(L,L)}(t_i)$

   Take any $t_j < t_i^{int(L,L)}$. The best region i can do by playing $t_i < t_i^{int(L,L)}$ is $H_i^{(L,L)}(t_j + \varepsilon)$ which is dominated by what it gets when playing $t_i^{int(L,L)}$. Take any $t_j \ge t_i^{int(L,L)}$. Region i can have a payoff of at least $H_i^{(L,L)}(t_j + \varepsilon)$ by playing $t_i > t_i^{int(L,L)}$ which dominates $L_i^{(L,L)}(t_i)$ that it obtains by playing $t_i < t_i^{int(L,L)}$.

2. $u^{(L,L)*} \ge H_1^{(L,L)}(1)$ and $U^{(L,L)*} \ge H_2^{(L,L)}(1)$.

   If $U^{(L,L)*} < H_2^{(L,L)}(1)$ then given $F_1(t_1)$ region 2 may play $t_2 = 1$ with probability one and get at least $H_2^{(L,L)}(1)$: since region 1 never plays $t_1 > 1$, with $t_2 = 1$, 2 gets at least $H_2^{(L,L)}(1)$ if $t_1 < 1$ and at most $L_2^{(S,S)}(1)$ if $t_1 = 1$.

   A similar argument holds for region 1: since region 2 never plays $t_2 > 1$, with $t_1 = 1$, 1 gets at least $H_1^{(L,L)}(1)$ if $t_2 < 1$ and at most $H_1^{(S,S)}(1)$ if $t_2 = 1$.

3. $\underline{t}_1 = \underline{t}_2 = \underline{t}$ and none is played with positive probability and therefore $u^{(L,L)*} = L_1^{(L,L)}(\underline{t})$ and $U^{(L,L)*} = L_2^{(L,L)}(\underline{t})$.

   - Suppose $\underline{t}_i < \underline{t}_j$ then $EU_i(\underline{t}_i) = L_i^{(L,L)}(\underline{t}_i)$. But any other $t_i \in \left]\underline{t}_i, \underline{t}_j\right[$ gives i an utility of $L_i^{(L,L)}(t_i) \ge L_i^{(L,L)}(\underline{t}_i)$. Therefore $\underline{t}_i < \underline{t}_j$ may not be an equilibrium and we must have $\underline{t}_i \ge \underline{t}_j$. But by the same token, we must also have $\underline{t}_j \ge \underline{t}_i$ and therefore $\underline{t}_i = \underline{t}_j = \underline{t}$.
   - Suppose $f_2(\underline{t}) \ne 0$. Then $EU_1(\underline{t}) = (1 - f_2(\underline{t}))\, L_1^{(L,L)}(\underline{t}) + f_2(\underline{t}) H_1^{(S,S)}(\underline{t})$ and 1 can increase its utility by choosing $\underline{t}_1 = \underline{t} - \varepsilon$ for ε arbitrarily small thus getting $EU_1(\underline{t}_1) = L_1^{(L,L)}(\underline{t} - \varepsilon) \ge EU_1(\underline{t})$.

A similar argument is valid if instead $f_1(\underline{t}) \neq 0$ then $EU_2(\underline{t}) = (1 - f_1(\underline{t}))\, L_2^{(L,L)}(\underline{t}) + f_1(\underline{t}) L_2^{(S,S)}(\underline{t})$ and 2 can do better by choosing $\underline{t}_2 = \underline{t} - \varepsilon$.

4. $\underline{t} \geq Max\{t_1^{\min(L,L)}, t_2^{\min(L,L)}\}$

   Suppose $\underline{t} < t_i^{\min(L,L)}$ then i is getting $L_i^{(L,L)}(\underline{t}) < L_i^{(L,L)}(t_i^{\min(L,L)}) = H_i^{(L,L)}(1)$ which contradicts (1).

5. $\bar{t}_1 = 1$ or $\bar{t}_2 = 1$ or both and therefore either $u^{(L,L)*} = H_1^{(L,L)}(1)$ or $U^{(L,L)*} = H_2^{(L,L)}(1)$ or both.

   Under *(a.2)* one of the two following situations may happen:

   - $\bar{t}_2 > \bar{t}_1$ or $\bar{t}_2 = \bar{t}_1$ and $\bar{t}_1$ is not played with positive probability Then $EU_2(\bar{t}_2) = H_2^{(L,L)}(\bar{t}_2)$ and therefore the optimal $\bar{t}_2$ is 1 and $U^{(L,L)*} = H_2^{(L,L)}(1)$.
   - $\bar{t}_1 > \bar{t}_2$ or $\bar{t}_1 = \bar{t}_2$ and $\bar{t}_2$ is not played with positive probability Then $EU_1(\bar{t}_1) = H_1^{(L,L)}(\bar{t}_1)$ and therefore the optimal $\bar{t}_1$ is 1 and $u^{(L,L)*} = H_1^{(L,L)}(1)$.

6. $\underline{t} = Max\{t_1^{\min(L,L)}, t_2^{\min(L,L)}\}$

   From (4), we have $\underline{t} \geq Max\{t_1^{\min(L,L)}, t_2^{\min(L,L)}\}$. Suppose $\underline{t} > Max\{t_1^{\min(L,L)}, t_2^{\min(L,L)}\}$ then $U^{(L,L)*} > H_2^{(L,L)}(1)$ and $u^{(L,L)*} > H_1^{(L,L)}(1)$ which contradicts (5).

Computing $t_i^{\min(L,L)}$, $i = 1, 2$ and equilibrium strategies

Solving $H_1^{(L,L)}(1) = L_1^{(L,L)}(t_1)$ for $t_1^{\min(L,L)}$ and $H_2^{(L,L)}(1) = L_2^{(L,L)}(t_2)$ for $t_2^{\min(L,L)}$ we get

$$t_1^{\min(L,L)} = \frac{4R\beta}{A^2 + A\tau^2 + 4R\beta} \qquad \text{and} \qquad t_2^{\min(L,L)} = \frac{4R\beta\gamma}{A^2 + A\tau^2 + 4R\beta\gamma}$$

and there are two relevant cases:

(a) either $\gamma \leq 1$ and $\underline{t} = t_1^{\min(L,L)}$, $u^{(L,L)*} = H_1^{(L,L)}(1)$ and $U^{(L,L)*} = L_2^{(L,L)}(\underline{t})$ or

(b) $\gamma \geq 1$ and $\underline{t} = t_2^{\min(L,L)}$, $u^{(L,L)*} = L_1^{(L,L)}(\underline{t})$ and $U^{(L,L)*} = H_2^{(L,L)}(1)$.

We now compute the equilibrium distributions $F_1(t_1)$ and $F_2(t_2)$. To do so, take e.g. $\gamma \leq 1$ and solve (7) $= H_1^{(L,L)}(1)$ in $F_2(t_2)$ to get

$$F_2(t_2) = 1 - \frac{4R\beta}{A^2 + A\tau^2}\frac{1-t_2}{t_2}$$

and (7) $= L_2^{(L,L)}\left(t_1^{\min(L,L)}\right)$ in $F_1(t_1)$ to get

$$F_1(t_1) = \frac{A^2 + A\tau^2 + 4R\beta\gamma}{A^2 + A\tau^2 + 4R\beta} - \frac{A^2 + A\tau^2 + 4R\beta\gamma}{A^2 + A\tau^2 + 4R\beta}\frac{4R\beta}{A^2 + A\tau^2}\frac{1-t_1}{t_1}$$

Clearly,

$$\begin{aligned} F_2'(t_2) &> 0 \text{ and } F_1'(t_1) > 0 \\ F_1\left(t_1^{\min(L,L)}\right) &= F_2\left(t_1^{\min(L,L)}\right) = 0 \end{aligned}$$

$F_2(1) = 1$ and $F_1(1) \leq 1$ therefore 1 plays $t_1 = 1$ with positive probability whereas 2 does not. For $\gamma \geq 1$ the procedure is analogous.

Lemma 1 *The subgame $\delta = (L, L)$ has a unique equilibrium. Equilibrium utilities are given by:*

(i) if $\gamma \leq 1$, $u^{(L,L)*} = H_1^{(L,L)}(1) = \frac{A\tau^2}{8\beta} + R$ and $U^{(L,L)*} = L_2^{(L,L)}\left(t_1^{\min(L,L)}\right) = \frac{A^2}{8\beta} + R\frac{A^2+A\tau^2+4R\beta\gamma}{A^2+A\tau^2+4R\beta}$.

Both regions play a mixed strategy on the support $t \in \left[t_1^{\min(L,L)}, 1\right]$ and no tax is played with positive probability by any region except $t = 1$ by region 1.

(ii) if $\gamma \geq 1$, $u^{(L,L)*} = L_1^{(L,L)}\left(t_2^{\min(L,L)}\right) = \frac{A\tau^2}{8\beta} + R\gamma\frac{A^2+A\tau^2+4R\beta}{A^2+A\tau^2+4R\beta\gamma}$ and $U^{(L,L)*} = H_2^{(L,L)}(1) = \frac{A^2}{8\beta} + R\gamma$.

Both regions play a mixed strategy on the support $t \in \left[t_2^{\min(L,L)}, 1\right]$ and no tax is played with positive probability by any region except $t = 1$ by region 2.

Proof The proof is obvious from steps (1) to (6) and the computation of equilibrium distributions. e.g., for case (i), regions are by construction indifferent among all taxes in the support of their mixed strategies; they are not allowed to name taxes bigger than 1; region 1 does not want to use taxes smaller than $t_1^{\min(L,L)}$ and region 2 is granted a level of utility higher than its security level and therefore will not get anything better by deviating. Moreover, the equilibrium strategies respect all the properties of a continuous probability distribution.

To show that the equilibrium is unique, we already have from steps (1)-(6) that equilibrium utilities are unique and all that is left to demonstrate is that uniqueness of equilibrium strategies, that is that no tax in the open interval $]\underline{t}_i, 1[$ is played with positive probability. Suppose this were not the case and, say, region 1 places positive probability on some tax $\hat{t} \in]\underline{t}_i, 1[$, then it is clear that region 2 wants to transfer probability from an ε neighborhood above $\hat{t}$ to some μ neighborhood below it. Then region 1 would not be playing optimally by putting positive probability on $\hat{t}$. This argument is adapted from Baye et al. (1992).□

As the four subgames have the same payoff structure, we may apply the same reasoning throughout. For each subgame, one computes $t_i^{\min\delta}$, $i = 1, 2$ and then checks which is higher and equilibrium utilities and distributions follow.

As for subgame (L, S), we have:

$$
\begin{aligned}
t_1^{\min(L,S)} &= \frac{8R\beta + A\tau^2 - A^2}{2\left(A^2 + 4R\beta\right)} \\
t_2^{\min(L,S)} &= \frac{8R\beta\gamma + 3A\tau^2 - A^2}{2\left(A^2 + A\tau^2 + 4R\beta\gamma\right)}
\end{aligned}
$$

yielding the Lemma:

Lemma 2 *The subgame $\delta = (L, S)$ has a unique equilibrium. Equilibrium utilities are given by:*

(i) if $\gamma \leq \hat{\gamma}$, $u^{(L,S)*} = H_1^{(L,S)}(1) = \frac{A\tau^2}{8\beta} + R$ and $U^{(L,S)*} = L_2^{(L,S)}\left(t_1^{\min(L,S)}\right) = \frac{A^2}{8\beta} + \frac{\left(A^2 + A\tau^2 + 4R\beta\gamma\right)\left(-A^2 + A\tau^2 + 8R\beta\right)}{8\beta\left(A^2 + 4R\beta\right)}$.

Both regions play a mixed strategy on the support $t \in \left[t_1^{\min(L,S)}, 1\right]$ and no tax is played with positive probability by any region except $t = 1$ by region 1.

(ii) if $\gamma \geq \hat{\gamma}$, $u^{(L,S)*} = L_1^{(L,S)}\left(t_2^{\min(L,S)}\right) = \frac{A^2}{8\beta} + \frac{\left(A^2 + 4R\beta\right)\left(-A^2 + 3A\tau^2 + 8R\beta\gamma\right)}{8\beta\left(A^2 + A\tau^2 + 4R\beta\gamma\right)}$ and $U^{(L,S)*} = H_2^{(L,S)}(1) = \frac{3A\tau^2}{8\beta} + R\gamma$.

Both regions play a mixed strategy on the support $t \in \left[t_2^{\min(L,S)}, 1\right]$ and no tax is played with positive probability by any region except $t = 1$ by region 2.

Proof As in Lemma 1.□

Regarding subgame (S, L) one has:

$$\begin{aligned} t_1^{\min(S,L)} &= \frac{A\tau^2 + 4R\beta}{A^2 + A\tau^2 + 4R\beta} \\ t_2^{\min(S,L)} &= \frac{4R\beta\gamma}{A^2 + 4R\beta\gamma} \end{aligned}$$

and consequently:

Lemma 3 *The subgame $\delta = (S, L)$ has a unique equilibrium. Equilibrium utilities are given by:*

(i) if $\gamma \leq \widehat{\hat{\gamma}}$, $u^{(S,L)*} = H_1^{(S,L)}(1) = \frac{3A\tau^2}{8\beta} + R$ and $U^{(S,L)*} = L_2^{(S,L)}\left(t_1^{\min(S,L)}\right) = \frac{A^2}{8\beta} + \frac{\left(A\tau^2+4R\beta\right)\left(A^2+4R\beta\gamma\right)}{4\beta(A^2+A\tau^2+4R\beta)}$.

Both regions play a mixed strategy on the support $t \in \left[t_1^{\min(S,L)}, 1\right]$ and no tax is played with positive probability by any region except $t = 1$ by region 1.

(ii) if $\gamma \geq \widehat{\hat{\gamma}}$, $u^{(S,L)*} = L_1^{(S,L)}\left(t_2^{\min(S,L)}\right) = \frac{A\tau^2}{8\beta} + R\gamma\frac{A^2+A\tau^2+4R\beta}{A^2+4R\beta\gamma}$ and $U^{(S,L)*} = H_2^{(S,L)}(1) = \frac{A^2}{8\beta} + R\gamma$.

Both regions play a mixed strategy on the support $t \in \left[t_2^{\min(S,L)}, 1\right]$ and no tax is played with positive probability by any region except $t = 1$ by region 2.

Proof As in Lemma 1.□

Finally, for subgame (S, S):

$$\begin{aligned} t_1^{\min(S,S)} &= \frac{-A^2 + 3A\tau^2 + 8R\beta}{2\left(A^2 + 4R\beta\right)} \\ t_2^{\min(S,S)} &= \frac{-A^2 + 3A\tau^2 + 8R\beta\gamma}{2\left(A^2 + 4R\beta\gamma\right)} \end{aligned}$$

leading to:

Lemma 4 *The subgame $\delta = (S, S)$ has a unique equilibrium. Equilibrium utilities are given by:*

(i) *if* $\gamma \leq 1$, $u^{(S,S)*} = H_1^{(S,S)}(1) = \frac{3A\tau^2}{8\beta} + R$ *and* $U^{(S,S)*} = L_2^{(S,S)}\left(t_1^{\min(S,S)}\right) = \frac{A^2}{8\beta} + \frac{\left(A^2+4R\beta\gamma\right)\left(-A^2+3A\tau^2+8R\beta\right)}{8\beta(A^2+4R\beta)}$.

Both regions play a mixed strategy on the support $t \in \left[t_1^{\min(S,S)}, 1\right]$ *and no tax is played with positive probability by any region except* $t = 1$ *by region 1.*

(ii) *if* $\gamma \geq 1$, $u^{(S,S)*} = L_1^{(S,S)}\left(t_2^{\min(S,S)}\right) = \frac{A^2}{8\beta} + \frac{\left(-A^2+3A\tau^2+8R\beta\gamma\right)\left(A^2+4R\beta\right)}{8\beta(A^2+4R\beta\gamma)}$ *and* $U^{(S,S)*} = H_2^{(S,S)}(1) = \frac{3A\tau^2}{8\beta} + R\gamma$.

Both regions play a mixed strategy on the support $t \in \left[t_2^{\min(S,S)}, 1\right]$ *and no tax is played with positive probability by any region except* $t = 1$ *by region 2.*

Proof As in Lemma 1.□

Proposition 1 *Each subgame δ has a unique equilibrium.*

Proof Follows from Lemmas 1 to 4. □

Proposition 2 *In each subgame δ, equilibrium utilities are such that*

(i) $U^{\delta*} = H_2^{\delta}(1)$ *and* $u^{\delta*} = L_1^{\delta}\left(t_2^{\min\delta}\right)$ *if and only if* $t_1^{\min\delta} < t_2^{\min\delta}$

(ii) $u^{\delta*} = H_1^{\delta}(1)$ *and* $U^{\delta*} = L_2^{\delta}\left(t_1^{\min\delta}\right)$ *if and only if* $t_2^{\min\delta} < t_1^{\min\delta}$

Moreover, whenever $t_i^{\min\delta} \geq t_j^{\min\delta}$, *equilibrium strategies are played on the support* $\left[t_i^{\min\delta}, 1\right]$ *and such that* $F_j(t_j) \geq F_i(t_i)$ *and no tax level is played with positive probability except* $t = 1$ *by region* i.

Proof Follows from Lemmas 1 to 4. □

A2 : Monitoring game

Note that the choice of δ_1 and δ_2 is a simple normal form two players game:

	$\delta_2 = L$	$\delta_2 = S$
$\delta_1 = L$	$u^{(L,L)*}, U^{(L,L)*}$	$u^{(L,S)*}, U^{(L,S)*}$
$\delta_1 = S$	$u^{(S,L)*}, U^{(S,L)*}$	$u^{(S,S)*}, U^{(S,S)*}$

We prove Propositions 3, 4 and 5 using two lemmas.

Lemma 5 *When $\gamma \leq 1$, equilibrium in the first stage of the game is as follows. Region 1 always chooses $\delta_1 = S$ and region 2 chooses $\delta_2 = S$ if $\tau < \check{\tau} = A - \sqrt{\sqrt{\frac{(A^2+2R\beta)^2+8R^2\beta^2}{3}} - 2R\beta}$ and $\delta_2 = L$ otherwise.*

Proof

- (L, L) is never an equilibrium since $u^{(L,L)*} < u^{(S,L)*}$.
- (S, L) is an equilibrium as soon as $U^{(S,L)*} > U^{(S,S)*}$. We have

$$U^{(S,L)*} - U^{(S,S)*} = \left(A^2 + 4R\beta\gamma\right) \frac{A^4 - 3A\tau^4 + 4R\beta\left(A^2 - 3A\tau^2\right)}{8\beta\left(A^2 + 4R\beta\right)\left(A^2 + A\tau^2 + 4R\beta\right)} \tag{8}$$

  and all we have to do is to check the sign of $\left(A^4 - 3A\tau^4 + 4R\beta\left(A^2 - 3A\tau^2\right)\right)$ which is positive for $\tau > \check{\tau}$.

- (L, S) is not an equilibrium. When $\gamma \leq \hat{\gamma}$, one obtains $u^{(L,S)*} < u^{(S,S)*}$ straightforwardly.

  When $\gamma \geq \hat{\gamma}$ then

$$\mathbf{??} u^{(L,S)*} - u^{(S,S)*} = \frac{A\tau^2\left(A^2 - 3A\tau^2\right) + 4R\beta\left(\left(A\tau^2 - 3A^2\right) + 3\gamma\left(A^2 - A\tau^2\right)\right)}{8\beta\left(A^2 + A\tau^2 + 4R\beta\gamma\right)} \tag{9}$$

  Note that $\left(A\tau^2 - 3A^2\right) + 3\gamma\left(A^2 - A\tau^2\right) < 0$ for any $\hat{\gamma} \leq \gamma \leq 1$. And for Region 2:

$$U^{(L,S)*} - U^{(L,L)*} = \frac{8\left(A^2 + A\tau^2\right)R\beta\gamma - A^4 + 3A\tau^4 + 4A\tau^2 R\beta + 2A^2(A\tau^2 - 6R\beta)}{8\beta\left(A^2 + A\tau^2 + 4R\beta\right)}$$

  is increasing in γ and its numerator has a value of

$$-\left(A^2 - 3A\tau^2\right)\left(A^2 + A\tau^2 + 4R\beta\right)$$

  at $\gamma = 1$. For $U^{(L,S)*} - U^{(L,L)*}$ to be positive at some point we need $\left(A^2 - 3A\tau^2\right) < 0$ but then $u^{(L,S)*} - u^{(S,S)*}$ is always negative.

- (S, S) is an equilibrium for $\tau < \check{\tau}$. Note that if $\left(A^2 - 3A\tau^2\right) > 0$ then (8) is positive and (S, S) is not an equilibrium. If $\left(A^2 - 3A\tau^2\right) < 0$ then by (**??**) $u^{(S,S)*} > u^{(L,S)*}$and moreover $U^{(S,S)*} > U^{(S,L)*}$ for $\tau < \check{\tau}$.

□

Lemma 6 *When $\gamma \geq 1$, the Nash equilibrium in the first stage of the game is as follows. Region 1 always chooses $\delta_1 = S$ and when:*

(i) $\gamma \leq 1 + \frac{A^2}{12R\beta}$, *region 2 chooses* $\delta_2 = S$ *if* $\tau < \hat{\tau} = A - \sqrt{\sqrt{\frac{(A^2+2R\beta)^2+8R^2\beta^2+8A^2R\beta(1-\gamma)}{3}} - 2R\beta}$ *and* $\delta_2 = L$ *otherwise.*

(ii) $\gamma \geq 1 + \frac{A^2}{12R\beta}$, *region 2 chooses* $\delta_2 = S$ *if* $\tau < \tilde{\tau} = A - \frac{A}{\sqrt{3}}$ *and* $\delta_2 = L$ *otherwise.*

Proof

- (L, L) is never an equilibrium because $u^{(S,L)*} > u^{(L,L)*}$. With $\gamma \geq \widehat{\hat{\gamma}}$, it is straightforward. With $\gamma \leq \widehat{\hat{\gamma}}$ one has:

$$u^{(S,L)*} - u^{(L,L)*} = \frac{A\tau^2}{4\beta} - R(\gamma - 1)\frac{A^2 + A\tau^2}{A^2 + A\tau^2 + 4R\beta\gamma}$$

  which is positive since it is decreasing in γ and positive at the highest relevant $\gamma = \widehat{\hat{\gamma}}$.

- (L, S) is never an equilibrium. Recalling the expressions for $u^{(S,S)*}$, $u^{(L,S)*}, U^{(L,S)*}$ and $U^{(L,L)*}$ it is obvious that $U^{(L,S)*} > U^{(L,L)*}$ if and only if $3A\tau^2 - A^2 > 0$ in which case $u^{(S,S)*} > u^{(L,S)*}$.

- (S, L) is an equilibrium as long as $U^{(S,L)*} > U^{(S,S)*}$. Either $\gamma \leq \widehat{\hat{\gamma}}$ and

$$U^{(S,S)*} - U^{(S,L)*} = \frac{-A^4 + 3A\tau^4 - 12R\beta\left(A^2 - A\tau^2\right) + 8A^2R\beta\gamma}{8\beta\left(A^2 + A\tau^2 + 4R\beta\right)} \quad (10)$$

  obtaining that $U^{(S,L)*} > U^{(S,S)*}$ when $\tau > \hat{\tau}$.

  Or $\gamma \geq \widehat{\hat{\gamma}}$ and

$$U^{(S,S)*} - U^{(S,L)*} = \frac{3A\tau^2 - A^2}{8\beta}$$

  and we get $U^{(S,L)*} > U^{(S,S)*}$ when $\tau > \tilde{\tau}$.

  The critical $\gamma = 1 + \frac{A^2}{12R\beta} = 1 + \frac{(A-\tilde{\tau})^2}{4R\beta}$ is the value of γ for which $\hat{\tau} = \tilde{\tau}$ (see Figure 2).

- (S,S) is an equilibrium for $\tau < \hat{\tau}$ when $\gamma \leq 1 + \frac{A^2}{12R\beta}$ and $\tau < \tilde{\tau}$ when $\gamma \geq 1 + \frac{A^2}{12R\beta}$. As for region 2, it is obvious from (10). In the discussion of (L,S) above we noted that whenever $-A^2 + 3A\tau^2 > 0$ or equivalently $\tau < \tilde{\tau}$, $u^{(S,S)*} > u^{(L,S)*}$ and this is always true when $U^{(S,S)*} > U^{(S,L)*}$, taking into account that $\hat{\tau} < \tilde{\tau}$ for $\gamma \leq 1 + \frac{A^2}{12R\beta}$ (see Figure 2).

□

Proposition 3 *Region 1 will always be strict in equilibrium.*

Proof Follows from Lemmas 5 and 6.□

Proposition 4 *Region 2 chooses $\delta_2 = L$ for high and $\delta_2 = S$ for low τ.*

Proof Follows from Lemmas 5 and 6.□

Proposition 5 *Equilibria with profit shifting are such that the multinational locates in 2 and on average declares its total profit in 1. This type of equilibrium exists for $\tau \geq \tilde{\tau}$ and $\gamma \geq \widehat{\hat{\gamma}}$:*

(i) τ is high enough such that region 2 is loose and hosts the firm and

(ii) γ is high enough such that region 1 undercuts region 2 on average.

Proof From Lemmas 5 and 6, 3 different regions emerge on the (γ, τ) space regarding equilibrium utilities:

1. $\left\{(\gamma,\tau) \in \,]0,\infty[\, \times \,]0,A[\, : \gamma \leq 1 \cup \left(\tau > \hat{\tau} \cap \gamma \leq \widehat{\hat{\gamma}} = 1 + \frac{(A-\tau)^2}{4R\beta}\right)\right\}$ in which equilibrium expected utilities correspond to the firm locating in 2 and paying taxes locally:
$$u^* = \frac{3A\tau^2}{8\beta} + R$$
$$U^* = \frac{A^2}{8\beta} + t_1^{\min\delta}\left(\frac{A^2}{4\beta} + R\gamma\right)$$

2. $\{(\gamma,\tau) \in \,]0,\infty[\, \times \,]0,A[\, : \gamma \geq 1 \cap \tau < \tilde{\tau} \cap \tau < \hat{\tau}\}$ in which equilibrium expected utilities correspond to the firm locating in 1 and paying taxes locally:
$$u^* = \frac{A^2}{8\beta} + t_2^{\min(S,S)}\left(\frac{A^2}{4\beta} + R\right)$$
$$U^* = \frac{A\tau^2}{8\beta} + \frac{A\tau^2}{4\beta} + R\gamma$$

3. $\left\{(\gamma,\tau) \in]0,\infty[\times]0,A[: \gamma \geq \widehat{\hat{\gamma}} = 1 + \frac{(A-\tau)^2}{4R\beta} \cap \tau > \tilde{\tau}\right\}$ in which equilibrium expected utilities correspond to the firm locating in 2 and paying all its taxes in 1:

$$u^* = \frac{A\tau^2}{8\beta} + t_2^{\min(S,L)}\left(\frac{A^2}{4\beta} + \frac{A\tau^2}{4\beta} + R\right)$$
$$U^* = \frac{A^2}{8\beta} + R\gamma$$

□

Comparative statics on R

Proposition 6 *As R increases, the parameter space under which*

(i) region 2 is loose diminishes

(ii) profit shifting arises at equilibrium expands.

Proof In order to show how equilibrium changes with R, we exactly compute it when $R = 0$ and when $R \to \infty$. These two limit cases clearly suggest how R affects equilibrium. We therefore do not directly present the derivatives of $\check{\tau}$ and $\hat{\tau}$ with respect to R (a straightforward exercise).

Take $R = 0$. Quite clearly, $\hat{\tau} = \check{\tau} = A\left(1 - 1/\sqrt[4]{3}\right)$ and $1 + \frac{A^2}{12R\beta} \to \infty$ (or equivalently $\widehat{\hat{\gamma}} \to \infty$): a flat line divides the parameter space and profit shifting never arises.

Now take $R = \infty$. First of all, $1 + \frac{A^2}{12R\beta} \to 1$ (or equivalently $\widehat{\hat{\gamma}} \to 1$) and therefore $\hat{\tau}$ disappears from the picture. Moreover, $\check{\tau} \to \tilde{\tau}$; to see that, note that

$$\sqrt{\frac{\left(A^2 + 2R\beta\right)^2 + 8R^2\beta^2}{3}} - 2R\beta \to \frac{A^2}{3}$$

in fact:

$$\begin{aligned}
\sqrt{\frac{\left(A^2 + 2R\beta\right)^2 + 8R^2\beta^2}{3}} - 2R\beta &= \frac{1}{3}\frac{4A^2\beta R + A^4}{\sqrt{4\beta^2R^2 + \frac{4}{3}A^2\beta R + \frac{A^4}{3}} + 2\beta R} \\
&= \frac{1}{3}\frac{4A^2\beta + A^4/R}{\sqrt{4\beta^2 + \frac{4}{3}A^2\frac{\beta}{R} + \frac{A^4}{3R}} + 2\beta}
\end{aligned}$$

taking limits one gets the desired result. A flat line divides the parameter space and there is no profit shifting as $t_i^{\min\delta} \to 1$ for all δ and for $i = 1, 2$.

We thus have that as R increases:

1. $\breve{\tau}$ moves upwards from $A\left(1 - 1/\sqrt[4]{3}\right)$ to $A\left(1 - 1/\sqrt{3}\right)$;

2. the intersection between $\hat{\tau}$, $\tilde{\tau}$ and $\widehat{\hat{\gamma}}$ gets closer to 1;

3. $\widehat{\hat{\gamma}}$ becomes steeper: the parameter space under which there is profit shifting becomes larger.

$\square$

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