The Numeraire Problem in General Equilibrium Models with Market Power: Much Ado About Nothing?

Dirk Willenbockel
Middlesex University Business School - London
Enfield EN3 4SF - UK
d.willenbockel@mdx.ac.uk

Abstract:
In general equilibrium models with oligopolistic firms, equilibrium outcomes may critically depend on the choice of numeraire. When firms have the power to influence prices strategically, different price normalisations entail profit functions which are generally not monotone transformations of each other. Hence, under the assumption of profit maximization an arbitrary change in the price normalisation rule amounts effectively to a change in the objective pursued by firms. Despite Ginsburgh's (1994) provocative numerical example, applied general equilibrium analysts using models with imperfectly competitive firm conduct have largely ignored the price normalisation problem. In several recent contributions to the literature, applied policy modellers are explicitly criticized for their neglect to address the numeraire issue. The purpose of this paper is to assess the validity and practical relevance of these criticisms from a practical modelling perspective. The analysis suggests that the key issue is the formulation of firms' perceptions of the general equilibrium repercussions of their own strategic choices. It is argued that under "reasonable" restrictions of oligopolistic firms' information set, the numeraire choice problem can safely be neclected. Since pure theorists are probably not persuaded by this line of reasoning, a number of simulation exercises within models allowing for numeraire dependency of results are presented, in order to assess the quantitative significance of the price normalisation problem under empirically plausible parameter choices.

JEL Classification: D58, D43, L21

Keywords: Applied general equilibrium analysis; Imperfect competition; Firm objectives; Price normalisation problem.

1. Motivation

Economic theorists have long been aware of the fact that in general equilibrium models with oligopolistic firms, equilibrium outcomes may critically depend on the choice of numeraire. When firms have the power to influence prices strategically, different price normalizations entail profit functions which are generally not monotone transformations of each other. Hence, under the assumption of profit maximization an arbitrary change in the price normalization rule amounts effectively to a change in the objective pursued by firms.
Despite Ginsburgh's (1994) provocative numerical example, applied general equilibrium analysts using models with imperfectly competitive firm conduct have largely ignored the

price normalization problem.[1] In several recent contributions to the literature, applied policy modellers are explicitly criticized for their neglect to address the numeraire issue. Kletzer / Srinivasan (1999) argue that

> " the dependence of equilibria on the choice of a numeraire is an important problem for theoretical models of international trade under imperfect competition and their empirical implementation. (...) Once it is established that equilibria are sensitive to the specification of the numeraire, it is a straightforward conclusion that estimates of the effects on welfare and resource allocation of changes in indirect or direct tax rates, tariff rates or quantitative restraints on international or national trade from computable general equilibrium models incorporating imperfect competition should be treated with suspicion. (...) The analyses of trade reforms using computable general equilibrium with monopolistically competitive or oligopolistic industries by Harris [1984..], Cox and Harris [1985], de Melo and Roland-Holst [*1991*] and Devarajan and Rodrik [*1991*], among others, are all subject to the criticism that the results depend upon the arbitrary choice of price normalization made."[2]

In a similar vein, Cordella (1998) suggests that

> " far from being a theoretical curiosity, the normalization problem ... has far-reaching implications in applied models".

The purpose of this paper is to address the price normalization problem from an applied modelling perspective. It is shown that under most of the specifications of imperfectly competitive firm conduct actually employed in applied general equilibrium studies, a problem of numeraire dependency of results does not arise in the first place, and therefore the above-mentioned criticisms are formally invalid. Hence, an effort is made to pinpoint precisely under which assumptions about firm conduct the normalization problem raises its ugly head. The analysis suggests that the key issue is the formulation of firms' perceptions of the general equilibrium repercussions of their own strategic choices. It is argued that under "reasonable" assumptions about oligopolistic firms' information set, the numeraire choice problem can safely be neclected. Since pure theorists are probably not pursuaded by this line of reasoning, a number of simulation exercises within models allowing for numeraire dependency of results are presented, in order to assess the quantitative relevance, or other, of the price normalisation problem under empirically plausible parameter choices.

2. The Price Normalisation Problem I: Monopoly in General Equilibrium

Consider a closed economy which produces two consumption goods C_1 and C_2 using a single intersectorally mobile primary factor with linear production technologies. The

[1] A recent exception is Hoffmann (2003). Burniaux / Waelbroeck (1993:xx), Mercenier (1995:169n), Kehoe / Prescott (1995:4) and Willenbockel (2002:6) mention the price normalisation problem *en passant*.

[2] References to working papers in the original text have been updated to refer to more accessible final published versions where appropriate.

economy is populated by numerous price-taking households with identical homothetic preferences represented by a CES utility function

(1) $$U = [dC_1^{(s-1)/s} + (1-d)C_2^{(s-1)/s}]^{s/(s-1)},$$

where s is the elasticity of substitution between the two goods. The factor market equilibrium condition is

(2) $$L^s = C_1 + C_2,$$

where L^s denotes the aggregate exogenous factor endowment, which is evenly spread across households[3]. Good 2 is produced by perfectly competitive firms so that

(3) $$p_2 = w,$$

where p_i, $i \in \{1,2\}$, and w denote output prices and factor price respectively. In contrast, good 1 is supplied by a profit-maximizing monopolist. We assume initially that ownership titles to monopoly profits are evenly distributed. The demand function facing the monopolist is

(4) $$C_1 = d^s q^{s-1} p_1^{-s} Y$$

where

(5) $$q = [d^s p_1^{1-s} + (1-d)^s p_2^{1-s}]^{1/(1-s)}$$

is the true price index dual to U and

(6) $$Y = wL^s + p$$

is aggregate household income including monopoly profits $p = (p_1-w)C_1$.
As long as the monopolist is assumed to neglect the indirect general equilibrium repercussions of variations in its own decision variable on p_2, w and Y – i.e. as long as the firm is taken to act like a standard textbook partial equilibrium monopolist – no price normalization problem arises. In this case, subjectively optimal pricing behaviour is unambiguously characterised by the familiar Lerner condition

(7) $$p_1(1-1/e(.)) = w \quad ,$$

where

[3] Despite the homotheticity of preferences the distribution of the endowment matters in the present setting.

(8) $$e = -\frac{\partial \ln C_1}{\partial \ln p_1} = s + (1-s)S \quad , \quad S \equiv d^{s}\left[\frac{p_1}{q}\right]^{1-s} = \frac{p_1 C_1}{Y}$$

is the perceived elasticity of demand. Since e is homogeneous of degree zero in prices, the optimal mark-up, and hence the general equilibrium of the two-sector economy is independent of any price normalization rule a modeller may adopt to determine nominal variables.

The situation changes once the assumption of limited cognition is dropped and the monopolist is assumed to recognise his influence on prices in other markets and thus on aggregate income via factor price and profit feedback effects. With full recognition of general equilibrium interdependence, monopoly profits can be expressed in the form

(9) $$p = \frac{wL^{s}}{(p_1 - w)h(p_1, p_2) - 1} \quad , \quad h(p_1, p_2) \equiv d^{s} q^{s-1} p_1^{-s} = \frac{C_1}{Y} .$$

Without a nominal anchor, the maximisation of nominal profits is obviously an ill-defined problem. The choice of a numeraire or, more generally, a price normalisation rule is required *before* the optimal equilibrium mark-up can be characterised. Figure 1 shows the profit profile as a function of monopoly output for the three normalisations p_1=1, p_2(=w)=1, and ?=1, thus measuring profits respectively in units of the monopoly good, in units of the competitive good (or in factor units), and in utility units, i.e. in units of the consumption index U. Evidently the profit-maximizing output level does not remain invariant to a change in the numeraire - the general equilibrium profit functions under different price normalizations are not monotone transformations of each other. Since the first-order conditions for utility-maximising consumer behaviour in combination with the resource constraint (1) entails that the relative price $P=p_1/p_2$ varies with the choice of monopoly output according to

(10) $$P(C_1) = \frac{d}{1-d}\left(\frac{L^{s}}{C_1} - 1\right)^{1/s} ,$$

the maximisation of profits in terms of good 2 ($p^{(2)}$) and in terms of good 1 ($p^{(1)}$) are different objectives. Formally,

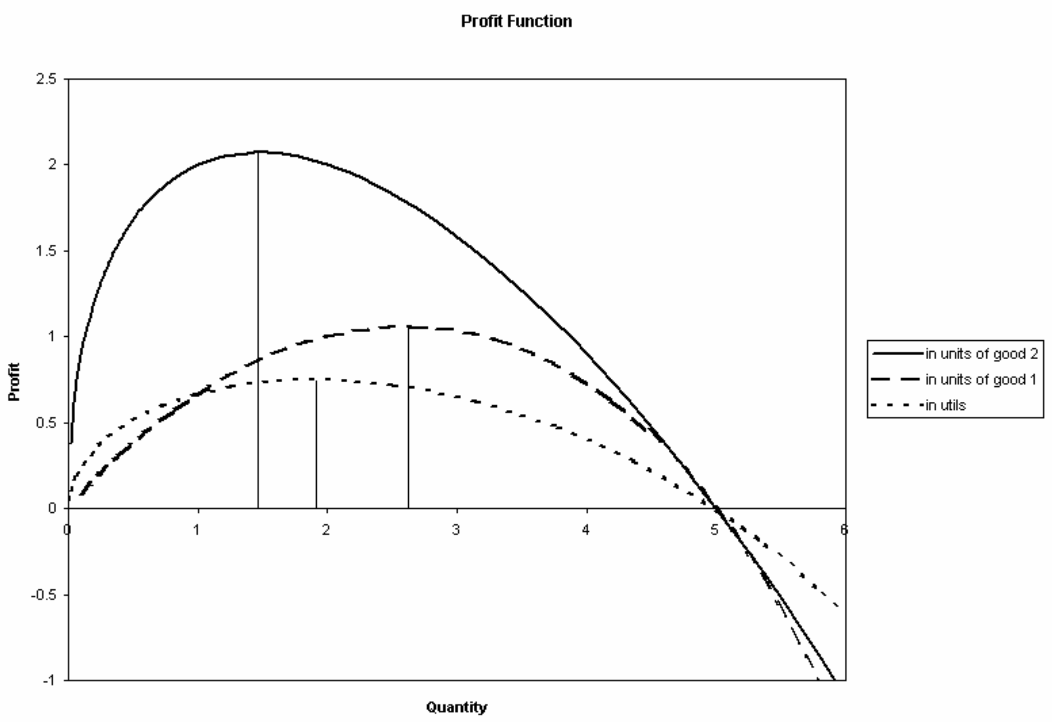
(10) $$p^{(2)}(C_1) = p^{(1)}(C_1) \cdot P(C_1)$$

and hence the first-order condition for a maximum of $p^{(2)}$,

(11) $$\frac{dp^{(2)}}{dC_1} = \frac{dp^{(1)}}{dC_1} P + p^{(1)} \frac{dP}{dC_1} = 0 ,$$

differs from the first-order condition for a maximum of $p^{(1)}$ unless equilibrium profits are zero.
The fact that the choice of price normalisation rule affects the equilibrium levels of real variables illustrated by this example is a generic feature of general equilibrium models with imperfectly competitive profit-maximising firms, given that these firms fully recognise their influence on the price system. Indeed Böhm (1994) and Grodal (1996) present oligopoly examples in which virtually every feasible production plan is an equilibrium for some normalisation rule. Other oligopoly examples in the literature demonstrate that an equilibium in pure strategies may exist for some price normalisation rules while other normalisations entail non-existence.[4]
In short, contrary to the case of competitive Arrow-Debreu economies, in which no agent can influence the price system strategically, an *arbitrary* choice of price normalisation in the present setting yields necessarily arbitrary results. Or as Dierker / Grodal (1998:153-4) put it, “if price normalization rules and hence firms’ objectives fail to be based on economic considerations, only ill-founded, arbitrary conclusions can be drawn from such models”.

Figure 1: The Objective Profit Function under Alternative Normalisations



The crucial point is that here the goal of profit maximisation in combination with a normalisation rule that takes no account of firm owner’s actual interests is generally not consistent with the aims of shareholders in the imperfectly competitive setting, and is

[4] See Dierker / Grodal (1986).

thus not a rational objective. In perfectly competitive production economies, on the other hand, the goal of profit maximisation is unambiguously in the interest of shareholders irrespective of the choice of price normalisation

The present example serves to elaborate the point. Maintaining the assumption of an even spread of monopoly shares for a moment, it is immediately evident that the goal of monopoly profit maximisation is irrational or indeed schizophrenic under any normalisation rule. With full recognition of his control over the price system via (10) the monopolist as agent of shareholders is in effect in the position of an omniscient central planner and must mimic the perfectly competitive outcome by setting P equal to the marginal rate of transformation (MRT=1) in order to maximise shareholder welfare. The optimum is of course associated with zero profits under any normalisation. The selection of a relative price P>MRT along the general equilibrium price schedule (10) would generate positive profit income but would at the same reduce the purchasing power of factor income in terms of good 1 and entail a net welfare loss. In other words, the maximisation of "producer surplus" without regard to the consequences for shareholders' "consumer surplus" is generally not in the interest of firm owners. The example may appear trivial, since as a matter of course there is no room for strategic behaviour in what is effectly a single-representative-agent framework. Yet the key message that a rational, not self-defeating strategy for an imperfect competitor must take shareholders' preferences and endowments into account, as highlighted by this extreme example, carries over to settings with real scope for strategic behaviour.

Thus let us disaggregate the household by distinguishing a representative monopoly shareholder with income Y_s=wL_s+p and a representative non-shareholder with income Y_n=wL_n. Both household types have identical CES preferences as before, so that the aggregate demand function for the monopoly good (4) and the general equilibrium price schedule (10) still apply.

The rational objective of the monopolist is to maximise

(12) $$U(C_1^s, C_2^s) = V(p_1, p_2, Y_s) = q(p_1, p_2)^{-1} Y_s \, .$$

Since the indirect utility function on the RHS of (12) is homogeneous of degree zero in its arguments, the optimal supply strategy is of course independent of the choice of price normalization. Without loss of generality, we can normalise the true consumer price index ? at unity. Thus the rational objective of the monopolist can equivalently be expressed as maximisation of shareholders' *total* real income (in units of the consumption index),

(13) $$Y_s = p_2 L_s + (p_1 - p_2) C_1 = \frac{p_2 L_s + (p_1 - p_2) h(.) p_2 L_n}{1 - (p_1 - p_2) h(.)} \ \ with q = 1.$$

Note that Y_s, which can be expressed as a function of C_1 by using (10) in (13) – is synonymous with the general equilibrium profit function (9) for the normalisation ?=1 in Figure 1, if L_s=0. Thus only if shareholders' only income source is monopoly dividends, is the maximisation of profits in combination with the specific class of normalisation rules ?=constant>0 a fully rational objective, i.e. an objective that is in complete agreement with the interests of shareholders.[5]

Profit maximisation together with a specific normalisation rule - namely p_2=1 - would also be totally consistent with shareholder preferences if these preferences take the form $U_s=u(C_2^s)$, u'>0, so that shareholders don't consume the output of their own firm.[6]

This extreme case suggests the conjecture, that the practical relevance of the numeraire problem may be negligible if the share of monopoly output in agents' total consumer expenditure is sufficiently small. But how small is sufficiently small? Table 1 provides a tentative answer.

Table 1: Deviations of Limited Cognition Model from Full Cognition Model

d	*Monopoly Share* S_{Full} *%*	*Monopoly Share* $S_{Limited}$ *%*	*Price* *?P %*	*Monopoly Output* *?*C_1 *%*	*Welfare* *?U %*
0.1	0.6	0.6	+0.3	-0.0	-0.00
0.2	3.0	3.0	+1.5	-3.2	-0.00
0.3	8.4	8.1	+4.2	-8.0	-0.06
0.4	18.1	16.8	+9.1	-14.6	-1.45
0.5	32.7	29.3	+17.3	-23.4	-4.43
0.6	50.9	44.5	+29.8	-31.5	-10.13
0.7	69.5	60.6	+48.0	-38.9	-18.36
0.8	84.7	75.7	+77.7	-42.4	-27.97
0.9	95.1	89.0	+138.4	-45.7	-38.06

Model parameter values: s =2, L_s=0, L_n=10.

The Table compares the general equilibria of the two-sector model when the monopolist has respectively limited and full cognition of the equilibrium consequences of his price-setting behaviour for alternative values of the preference intensity parameter d which governs the market share of the monopolistic sector in total consumer expenditure. In the limited cognition model, the monopoly mark-up is determined in partial-analytical fashion via (7) and (8), i.e. the monopolist ignores his influence on Y and pays no attention to the true interests of shareholders as consumers in his price setting decision. Not surprisingly, when the monopoly sector is small in the economy, the actual general equilibrium income feedback effect is indeed negligible, so that the limited cognition model provides an almost perfect approximation of the equilibrium with an omniscient

[5] However, once heterogeneity among shareholders is introduced, the very notion of shareholders' preferences becomes an elusive concept due to Arrow's impossibility theorem. See however Dierker / Grodal (1998,1999)'s approach to the formulation of a rational firm objective in the presence of heterogeneous shareholders.

[6] The island model of Hart(1985) can be seen as an extension of this observation to a multi-sector multi-agent setting.

rational monopolist. More interestingly, the deviation remains moderate even under empirically unreasonable values for the share of a single firm in GDP.
Since pricing behaviour in applied general equilibrium models with imperfect competition in the tradition of Harris (1984) is indeed typically derived under the assumption that firms neglect general equilibrium effects – a reasonable assumption given the undeniable empirical fact that firms may be large in their own market but are generally small within the economy as a whole – Table 1 may be seen to provide a first indication of the practical irrelevance of the price normalisation problem for quantitative policy analysis. However, since imperfectly competitive sectors in computable general equuilibrium models are typically oligopolies rather than monopolies, the next section extends the analysis to a setting with strategic interaction among firms.

3. The Price Normalisation Problem II: Oligopoly in General Equilibrium

We now assume that sector 1 is populated by n symmetric firms and characterised by horizontal product differentiation a la Dixit / Stiglitz (1977). Consumer preferences over the composite output of sector 1 and the competitive good C_2 are Cobb-Douglas with share parameter a, where

(14) $$C_1 = \left[\sum_{v}^{n} x_v^{(s-1)/s} \right]^{s/(s-1)},$$

x is output per firm, and s >1 the elasticity of substitution between firm-specific varieties. Thus the demand function facing an individual oligopolist takes the form

(15) $$x = ap^{-s} P_1^{s-1} Y ,$$

where p is the price of an individual variety and

(16) $$P_1 = \left[\sum_{v=1}^{n} p_v^{1-s} \right]^{1/(1-s)}$$

is the consistent price index dual to C_1.

On the production side, we maintain the assumption of linear single-factor technologies but add a recurrent fixed factor requirement per firm in sector 1 to introduce increasing returns to scale. This setting is a stylised two-sector closed-economy version of typical multi-sectoral open-economy computable general equilibrium models as employed in the studies cited by Kletzer / Srinivasan (1999) above.[7]

Supply behaviour in sector 1 depends on the assumed form of strategic interaction among firms. Most applied studies assume either Bertrand or Cournot competition and the individual firm perceives to have no influence on Y, factor prices and prices in other sectors. Under Bertrand competition, the perceived elasticity of demand, which determines the equilibrium mark-up via (7) is then

$$(17) \qquad e = -\frac{\partial \ln x}{\partial \ln p} = s + (1 - s)/n$$

while Cournot competition entails

$$(18) \qquad \frac{1}{e} = \frac{1}{s} + \left(1 - \frac{1}{s}\right)\frac{1}{n}.$$

In both cases the equilibrium mark-up is independent of price normalisation, or stated differently, the problem is evaded through the implicit introduction of bounded rationality.

In order to determine firm behaviour under full cognition of general equilibrium feedbacks, the price normalisation problem re-appears, since the elasticity of Y with respect to p is indeterminate without a normalisation rule. In analogy to the previous section, the appropriate normalisation rule is to normalise the true consumer price index dual to U, i.e.

$$(19) \qquad q = \prod_{i=1}^{2} \left(\frac{P_i}{a_i}\right)^{a_i}$$

at unity, provided that shareholders receive only profit income. Under this assumption, the perceived elasticity with full cognition must obey[8]

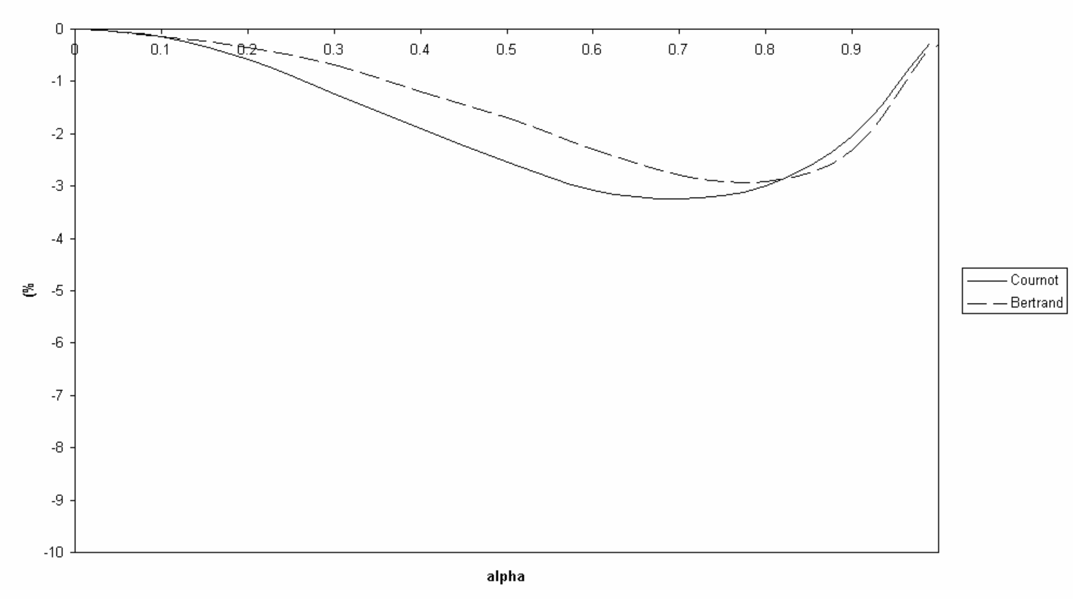
$$(20) \qquad e = s + (1 - s)/n + a/n - a(1 - 1/n)(s - e)/e.$$

Note that the limit of (20) for a? 0, i.e. for a shrinking market share of the oligopoly in the economy, the perceived elasticity under full cognition converges to the limited cognition elasticity (17). The algebra of the Cournot case under full cognition is slightly more tedious and is skipped here for brevity’s sake, yet the same limit result applies in this case.

[7] See Willenbockel (1994, 2002) for further references to applied policy studies of this type.
[8] d’Aspremont et al. (1996) derive the corresponding perceived elasticity expression for the normalisation w=1 but do not address the dependency of the result on this arbitrary numeraire choice.

Figure 2, which plots the percentage deviation of equilibrium welfare levels between the limited and complete cognition models for varying market shares of an oligopolistic sector in the economy reconfirms the results of Table 1 in the foregoing section. For empirically relevant ranges of the relative size of an individual oligopolistic sector, in which individual firms produce similar products, within the economy as a whole (which are of course to be found in the top left corner of the graph well below a=0.1), the simulated deviations remain barely noticeable.We do not claim that this result is particularly surprising.

Figure 2: Equilibrium Welfare Deviation between Limited and Full Cognition Oligopoly Model (fixed n)



4. Conclusion

The message of this paper is simple and straightforward: Applied general equilibrium models with imperfect competition studies may suffer from numerous conceptual and practical problems, yet the price normalisation problem is certainly not one of these.

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