

Disaggregated Cost Pass-Through Based
Econometric Inflation-Forecasting Model for
Hungary¹

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April 2003

¹I am indebted to Zsolt Darvas, whose help was indispensable to construct the model. Also, I am grateful for every comment, recommendation and professional assistance that I received from the Economics Department staff of the MNB and from participants of the 2002 conference of the Economic Modelling Society. The remaining errors are those of the author.

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Abstract

This paper presents one of the inflation forecasting models used by the Magyar Nemzeti Bank in its recent inflation forecasts.

The model attempts to integrate all the properties of the former models considered by the author as being advantageous and desirable into a unified framework. Thus, this model is based on disaggregated econometric estimates, complemented by expert assumptions. The model explains the prices of marketed goods using their cost factors, capturing an assumed process whereby costs gradually pass through into consumer prices. It is the empirical estimation of this slow cost-price pass-through that provides the uniqueness of the model in terms of economic and econometric theory.

1 Introduction

The Magyar Nemzeti Bank adopted the inflation targeting monetary system in June 2001. According to the textbook epitome of this regime, the Bank declares an inflation target for the future which it compares with its own forecast for the consumer price index (CPI), based on currently available information, and it revises monetary conditions, in order to eliminate any potential discrepancies between the announced target and its forecast for the CPI. With the adoption of the inflation targeting system, it became especially important for the Bank to produce reliable forecasts for the consumer price index. One of the possible ways to enhance the reliability of forecasts is to build new models, in addition to the forecasting techniques currently employed. This will allow the Bank to reduce uncertainties inherent in the forecasts. The model presented in this paper, which has by now become a member of the group of other inflation forecasting models¹ built by the Bank used to make official forecasts for the Bank's Inflation Report, was constructed as part of these efforts.

The model attempts to integrate all properties, considered by the author as advantageous and desirable, into a unified framework which the earlier models have already possessed individually.² Thus, *this model is based on disaggregated econometric estimates which are complemented by expert assumptions.*

The intention of the model is to forecast movements in the prices of goods included in the consumer price index which are determined by the market. On the forecast horizon, it explains movements in the prices of those goods with changes in their cost factors. More precisely, *the model attempts to capture an assumed process whereby costs gradually pass-through to consumer prices.* In our case, cost pass-through also means tracing the *spillover* of costs, as certain consumer prices themselves, which costs pass through to, constitute the costs of other goods.³ It is the empirical estimation of this

¹For a full documentation of the inflation models constructed and used by the Bank, see Hornok–Jakab–Reppa–Villányi [2002]. The paper by Hornok–Jakab [2002] provides an overview of the inflation forecasting techniques and the procedures applied in the central banks of Central Eastern Europe.

²Naturally, from this it does not follow that this model performs better in terms of forecasts than the earlier models.

³Chart 1 on page 7. illustrates this.

slow cost-price pass-through process that provides the interesting feature of the model in terms of economics and econometrics.⁴

Performing calculations with cost pass-through means that, in its current state, the model posits cost factors (for example, the exchange rate and wages) as being given for the forecast horizon which are actually in a natural interaction with prices. For this reason, some of the cost factors of the model are exogenous, the exchange rate and wages accounting for the largest weights. In other words, different tools are required to forecast them. For the time being, in the model we are only able to simulate the impact of demand and supply shocks as well as of monetary and fiscal policies on the consumer price index by changing these exogenous cost factors. Naturally, this way we eliminate those simultaneous mechanisms which ensure co-movements in the major cost factors, the exchange rate and wages over the long term whose ultimate common determinant is monetary policy. Consequently, the model plays a role in answering two questions: (1) How do prices change as an effect of changes in the prices of their cost factors? (2) What happens during the transitory period when changes in costs and prices separate (the difference between the two is the profit, in which changes are reflected first)? This period is called the period of cost 'pass-through'. However, the model does not answer why and to what extent prices and costs separate as an effect of supply and demand shocks (and of monetary policy).

The remainder of the paper is structured as follows. The main section of the paper presents the model framework as well as the considerations which justified the use of Shiller's method to estimate the pass-through profiles. Then, the ex-ante model forecasts are presented in a nutshell. Finally, the possible directions of future development are discussed. The Appendix describes a procedure used for estimating the pass-through profiles.

⁴After having developed the estimation technique, László Hunyadi drew my attention to the fact that, starting from similar assumptions, Shiller [1973] had basically developed the same technique.

2 Model Framework

As was referred to in the introduction, the model has been constructed to forecast the CPI. The source data on which the model relies are the monthly consumer price indices released by the Central Statistical Office (CSO) that are composed of 160 sub-groups/components.

The model explains variations in consumer prices by changes in costs and their pass-through. In other words, it treats prices as being ultimately determined by costs. This assumption is defensible within the range of goods of the consumer basket, on the market of which numerous sellers compete with each other, as this way one can reasonably assume that sellers must adjust their prices to costs in the contest for customers. Explanation for this is that, if they set prices 'too high'⁵ relative to costs, then other, new producers could potentially gain a share of the market by offering lower prices. Conversely, if costs were not covered by the selling price, then all producers would sooner or later abandon producing or dealing in a given piece of goods. In other words, there is a close relationship between prices and costs over both the short and long term. However, this very reasoning, and the completely different way of determining prices (the discretionary decision taken by the central government or local authorities), rules out the cost-based modelling of non-market-priced goods and public goods (regulated goods).⁶ Due to this problem, the model attempts to forecast the price indices of market-priced goods and services.⁷

However, the large number of primary goods included in the potentially applicable market-priced consumer price index, the difference between the weights represented by the individual goods within the consumer price index as well as the aim of maintaining the model within a manageable framework justified it to apply some aggregation. For

⁵Naturally, selling prices must provide cover for traders' profit. Thus, a 'too high' price denotes a situation in which higher-than-average profit can be earned dealing in particular a piece of goods.

⁶I treat 17 groups of goods within the consumer price index of the original 160 groups as regulated. These are the following: sewage disposal, meals at kindergartens and schools, natural and manufactured gas, pharmaceutical products, local transport, rent, travel to work and school, postal services, refuse disposal, gambling, purchased heat, other travels, telephone, TV fee, electricity and water charges.

⁷Of the CSO's consumer price index composed of 160 groups, I categorised 143 groups into market-priced goods, whose total weight was 81.2% within the 2002 consumer basket.

this reason, I attempted to create homogenous, aggregate groups which are similar in terms of usage and which can be assumed to have nearly identical cost structures. As a result, I aggregated the 143 individual groups into 43 sub-groups, as a result of which the weight ratios of sub-groups became more homogenous.⁸

In performing the cost-based modelling task, I applied the error-correction approach. Accordingly, I separated the problem of identifying the long-term equilibrium cost weights from the issue of identifying the dynamic (short-term) cost pass-through adjustment path leading to equilibrium.

2.1 Identifying Cost Weights (Long Run)

Prices of the groups of market goods represented in the consumer basket and aggregated according to the method noted above were assumed to be composed of various cost elements, such as labour costs, energy, basic materials, farm crops, imports as well as other costs which themselves are goods included in the consumer basket as well, such as flour in the case of bread, textiles in connection with clothing, etc. It was furthermore assumed that cost elasticities are constant and, therefore, over the long term prices are determined on the basis of the Cobb-Douglas cost function below:

$$P_{i,t} = \left(A_i e^{\lambda_i t} C_{1,t}^{\gamma_1} C_{2,t}^{\gamma_2} \cdot \dots \cdot C_{n-1,t}^{\gamma_{n-1}} C_{n,t}^{(1-\gamma_1-\gamma_2-\dots-\gamma_{n-1})} \right) H_i, \quad (1)$$

where $P_{i,t}$ is the consumer price index of the i th good in t , $C_{j,t}$ is the price index of the j th cost element in t , A_i is the factor normalising costs to price, λ_i is the rate of change of productivity and H_i is the profit margin assumed to be proportional to total costs. It is important to note that the price index of the cost factors is not projected to a unit of a piece of goods or services sold. Instead, I took its price index expressed as a 'natural unit' (index of monthly average wages, price index of 1 kWh of electricity,

⁸For a detailed description of the sub-groups generated, see point B. of the Appendix.

price index of flour, etc.). That is why the term $A_i e^{\lambda_i t}$ is included in the function, in which A_i is intended to transform the cost-price indices, expressed in various units, to a consumer price index, while parameter λ_i of the term $e^{\lambda_i t}$ captures all changes in productivity. The rate of change of productivity (λ_i) a priori is expected to be a negative parameter, as, if there is an improvement in productivity, then the consumer price index can be lower than the cost-price indices. However, the positivity of λ_i is not unexplainable either - in this case, we are talking about a good, on the market of which one can earn (temporarily) an increasing profit margin. The sum of cost elasticities (γ_i) was restricted to 1, in order that, with an unchanged mark-up, the price of goods in question also rises by 1%, provided that the prices of all their correspondent cost factors rise by 1%. In addition, a requirement against the parameters of cost elasticity is that all of them should have a positive value. For the econometric estimation of the long-term cost weights and other parameters, the logarithmised form of the cost function (1) above was used:

$$p_{i,t} = a_i + \lambda_i t + \gamma_1 c_{1,t} + \gamma_2 c_{2,t} + \dots + \gamma_{n-1} c_{n-1,t} + (1 - \gamma_1 - \gamma_2 \dots - \gamma_{n-1}) c_{n,t} + h_i + \varepsilon_{i,t}, \quad (2)$$

where small case letters denote the logarithm of variables and price indexes and $\varepsilon_{i,t}$ is the error term.⁹

In order that parameters could be provided for the above (2) equation, the main cost factors constituting the respective prices of the individual groups of goods had to be first identified. As shown in a detailed manner in A.2. of the Appendix, nearly each group of goods is assumed to include transportation, electricity, natural gas and wage costs.¹⁰

⁹These long-term cointegration-type estimates were made with the method of ordinary least squares. The prices of fuel, transport and vehicles are determined simultaneously in the model. Though such prices would have required the application of a different estimation method, the standard OLS was adopted, as, eventually, each cost weight item in this simultaneous sub-system was calibrated.

¹⁰Transport, electricity, and natural and manufactured gas are also components of CPI themselves. Actually, it is their producer prices that would be needed. As they are unavailable, it is assumed that both producer and consumer prices *change* identically.

The inclusion of these cost factors was deemed as self-evident: goods must be delivered to shops, shops need heating and lighting and wages must be paid to shop assistants or service providers. It also seemed obvious that the forint equivalent (multiplication of the prevailing exchange rate and the relevant price in foreign currency) of the respective prices of similar foreign goods be considered as cost factors in the case of such groups of goods that are mainly imported or manufactured from imported raw materials. The weighted price indices of euro area countries, in particular, the price indexes of the groups of goods similar or identical to the groups of corresponding domestic goods were selected as relevant foreign prices.¹¹ In addition, further cost factors were also included in each group of goods when such inclusion was deemed as obvious and when data on them were available. In the case of food, such additional components included relevant agricultural purchase prices. Also, the input of other goods included in the consumer basket was also contemplated. e.g. flour in connection with bread, sugar within the category of sweets, fresh fruit and vegetables within the category of preserved food, raw meat, bakery products and preserved food within the category of meals outside the home and clothing materials within the category of clothing, etc.¹² Keeping track of a few effect mechanisms of the changes in crude oil prices, the chart below provides some insight into a diversity of pass-through and some further spillover effects.¹³

¹¹Except for, in the case of motor fuel, the price of the Brent crude oil serving as raw material. It was its London-listed price that was taken into consideration. For additional details on motor fuel, see further sections of this study.

¹²In fact, producer prices should have been had for these goods presented as cost factors.

¹³Not the fullest diversity, of course, as price changes in transport pass-through to nearly every group of goods, triggering newer and newer spillover of price changes.

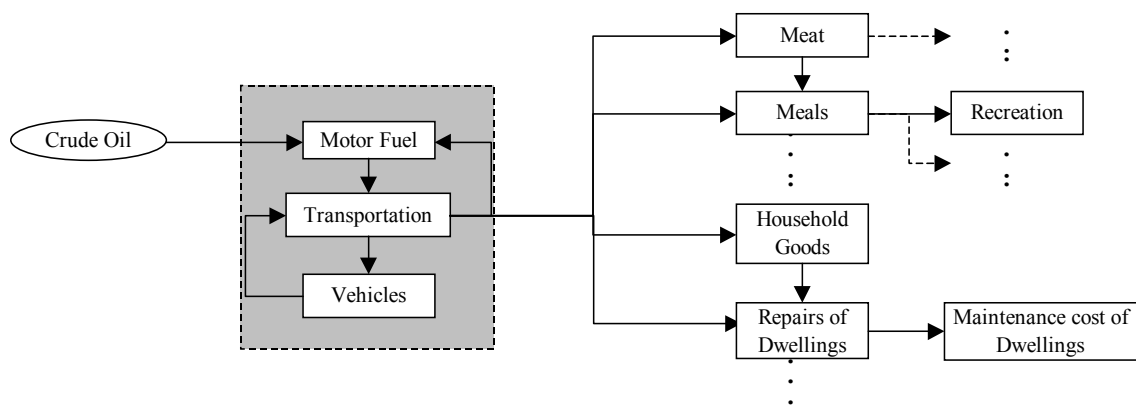


Chart 1

The grey area represents the unit of goods whose respective prices are determined simultaneously. The arrows denote the directions of pass-through.

In the case of consumer prices, both identifying cost factors and the appropriate handling of taxes pose problems. On the one hand, the price of each product and service in the CPI basket includes VAT. (Naturally, VAT rates differ from product/service to product/service.) Fortunately, VAT rates have remained unchanged in the past years.¹⁴ No major shifts between VAT classifications have occurred, either. Thus, if this type of tax is considered as a fixed cost ratio, it only means a straightforward issue of price scaling, which is appropriately tackled with the A_i term of modelling. On the other hand, some excise goods (e.g. tobacco, coffee, motor fuel and oils) whose excise content has changed several times over the past years are also included in CPI. As excise duty accounts for a large portion of the respective consumer prices of such goods,¹⁵ any change in it can and does in effect influence consumer prices.¹⁶ Therefore, taxes were removed from the final consumer price of motor fuel, and the resulting series was considered as determined by its cost factors.¹⁷

¹⁴The horizon that is relevant from our point of view starts in January 1996 as this is the very point of time since when all data have been available.

¹⁵In the case of motor fuels, for instance, excise content accounted for approximately 46% of the final consumer price in August 2002.

¹⁶As is well-known in macroeconomics, the extent to which changes in excise duty can be 'passed through' to consumers depends on the price elasticity of the demand for and supply of the product in question. It follows that if demand almost completely lacks price elasticity (as it is supposed to be the case in, for example, the motor fuel market), changes in taxes in their entirety are (may be) reflected in prices.

¹⁷A similar method should be adopted also in the case of (alcoholic) beverages, tobacco and coffee

The cost factors of the individual group of goods thus having been a given, the task to be carried out was the estimation and parameterisation of the (2) model equation. (It follows from the (2) mode of modelling that a_i could not be separated from h_i ; only their sum could be assessed.) Seeing that, from a theoretical point of view, only positive cost weights, not necessarily guaranteed by the standard econometric estimation of the (2)-type equation, are acceptable, iterative expert considerations were made for both incorrect signs and cost weights deemed as extremely high/low, and equations were re-estimated. The respective sums of parameters having been continuously stipulated, this approach was followed until each parameter was either positive or acceptable. When both econometric estimates and other expert considerations failed to provide for a reliable basis, it was also a point of consideration that the cost factors which had been included in each group of goods (e.g. transportation, electricity and gas) be identically weighted everywhere. Accordingly, the majority of cost weights were calibrated and only a few estimated.¹⁸ While selecting the sample period, periods deemed as 'quiet' from the point of view of pricing, were selected. The reason for doing so was that long-term equilibrium cost weights were justifiably expected to manifest themselves in prices during such periods. Thus, the period between the January 1995 and December 1995 (or even later when the need arose) was excluded. Behind this reason stands the distortional effect of the austerity package introduced in March 1995. Sub-groups including foreign prices (exchange rates) were investigated separately so that it could be ascertained whether the period following the widening of the band in May 2001 should be excluded. Having examined the data from the period that has elapsed since then, however, this idea was dismissed. For the final parameters calculated for long-term equations, see the tables in Section A.2. of the Appendix.

owing to the excise content of these goods. However, what prevents such treatment is that the market for such goods is price-sensitive, which means that changes in excise duty cannot be passed through consumers in their entirety.

¹⁸Only 18 of the 268 cost factors included in the model were estimated. Conversely, both constant and trend parameters were estimated in each equation.

2.2 Identifying Cost Pass-Through (Short-Run Dynamics)

In accordance with the error correction approach, once long-run equilibrium has been parameterised, the task to be completed is defining short-run dynamics. As is customary, short-run dynamics consistent with long-run dynamics can be written as follows:

$$\begin{aligned} \Delta p_{i,t} = & \lambda_i + \gamma_1 B_1(L) \Delta c_{1,t} + \gamma_2 B_2(L) \Delta c_{2,t} + \dots + \gamma_{n-1} B_{n-1}(L) \Delta c_{n-1,t} + \\ & + (1 - \gamma_1 - \gamma_2 \dots - \gamma_{n-1}) B_n(L) \Delta c_{n,t} - \phi_i \varepsilon_{i,t-1} + \xi_{i,t}, \end{aligned} \quad (3)$$

where Δ is difference, L is the lag operator, $\varepsilon_{i,t}$ is the residual of long-term equation i , ξ_i is the error term of the short-term equation and γ_i is the estimated/calibrated cost weights of the long-term equation. $B_j(L)$ polynomes are of $B_j(L) = b_{j,0} + b_{j,1}L + b_{j,2}L^2 + \dots + b_{j,q_i}L^{q_i}$ shape, where the degree of the polynome (length of lag) is q_i , and where the sum of parameters is 1. ($\sum_{k=0}^{q_i} b_{j,k} = 1$). $B_j(L)$ polynomes represent the dynamics of the relevant cost pass-through.¹⁹ However, the (3) approach applied to estimate cost pass-through processes cannot be adapted mechanically. The reason for this is that cost pass-through processes have (at least) four characteristics that render the usual econometric estimability of equation (3) impossible and dubious. Such characteristics are as follows:

1. Costs take a long time to pass-through into prices.
2. The pass-through speed of the individual cost factors is different.
3. The lagged coefficients of cost changes can only be non-negative.
4. The lagged coefficients of cost changes are interdependent.

¹⁹Though no separate i index has been specified, the pass-through profile of a given cost factor may vary from group of goods to group of goods.

Ad (1) Cost pass-through is a slow gradual process, market competition being unable to enforce immediate price adjustments. The reason for this being the case is that producers, vendors or service providers and customers must first recognise and identify the relevant changes in prices (and expect such changes in prices to last long enough) to be willing to change their respective prices and consumer behaviour accordingly. In addition, owing to the existence of long-term contracts as well as the random nature of price revisions, such willingness and constraints manifest themselves even more slowly. Another factor that puts a brake on this process is that in the case of certain products and services that themselves serve as raw material or input for other products and services, a series of transmissions occur before changes appear in final consumer prices. Therefore, it is safe to assume that cost pass-through may well be several-year-long process, in the first phase of which no price effect is discernible.

Ad (2) We have, however, every reason to believe that price changes in the individual cost factors are manifest themselves in consumer prices relatively more rapidly than in other prices. For example, changes in the price of crude oil manifest themselves as quickly as a week or two; by contrast, the price of transport, of which motor fuel is a cost factor, is very likely not to respond to changes in the price of crude oil instantly. Differing market structures are likely to be responsible for differences in the speed of the pass-through effect: any rise in costs is likely to be quicker in manifesting itself in prices in a monopolistic market; also, the frequency of price revision in the underlying markets may affect the speed of the pass-through effect.

Ad (3) The non-negativity of the coefficients of the $B_i(L)$ polynomes can be derived from the phenomenon of cost pass-through. The process of cost pass-through in its purest form is hypothesised as a path of price changes which would be discernible in the case of a one-off change in the price of a cost factor, with the respective prices of other cost factors remaining unchanged. This path is assumed to mean that one-off increases/decreases in costs lead to gradual increase/decrease in prices, without this gradual process being interrupted by potential increases/decreases in prices. Rephrasing the same with the $b_{i,j}$ coefficients of the $B_i(L)$ polynome would mean that any increase in costs in period t , for instance would result in decrease in prices in the

$t + j$ period. Though a few such negative parameters can certainly be attributed to overshooting, their appearance at any place and with any frequency rules out this explanation.

Ad (4) As was mentioned several times, *cost pass-through is a gradual process*. This means that, if the change in prices induced by a change in costs in a period is small, there cannot be a sudden large change in the subsequent period either. In other words, the successive coefficients of the polynome $B_i(L)$ are not independent of each other - if, for example, the speed of pass-through is only slow in one period, it will also be slow in the subsequent period; if it is relatively fast in a given period, it will also remain fast in the subsequent period. Consequently, the coefficients of the pass-through polynome change only gradually. Plotting them on a chart, we would obtain a smooth curve. Due to these four characteristics, we are faced with the following problems when making the estimates. First, the slowness of the pass-through would make it necessary to estimate extremely many lagged parameters.²⁰ Second, it would be difficult to interpret both the large number of negative parameters appearing in estimating the long lags and the hectic changes in lag coefficients from one period to the other. The idea may arise that the parameter ϕ_i of the error correction is able to partially handle these problems, as, provided that it is small enough, the pass-through will be gradual and slow in the model in the case of every cost factor (actually, with a geometric distributed lag structure), and it is enough to use the first few terms with non-negative parameters of acceptable size of the long lags. For us, however, this is not an acceptable compromise, as, due to what has been said, the speed of cost pass-through may differ, and the geometric distributed lag implied by the error-correction model implies exactly that the change in costs shows the greatest effect in the first periods. The problem to be solved, therefore, is represented by the requirement to estimate the pass-through profiles in a way that, first, the pass-through parameters should be non-negative and adequately smooth, i.e. they should not exhibit 'jig-saw shape' and, second, the varied forms of

²⁰ Assuming a two-year pass-through horizon, this means estimating $5 \cdot 24 - 5 = 115$ lag parameters in the case of 5 cost factors, aside from the rest of the parameters. Consequently, data for several decades would be required even in the case where the data are available at a monthly frequency, in order to make reliable estimates.

profiles should be captured easily, without the need to estimate too many parameters. I gave expression to this wide-ranging requirement by the idea below.²¹

2.2.1 Non-Parametric Distributed Lag

As the pass-through profiles may have a very diverse shape, an attempt to describe them using a function that can be flexibly changed by parameters, it would require a high number of parameters. Therefore, we have rejected this option and applied the following non-parametric technique in order to estimate flexible pass-through profiles. Equation (3) above has been estimated by defining a smoothness criterion for the estimated $b_{i,j}$ parameters, using w_i weights, in the form of:

$$\sum_{i=1}^n w_i \sum_{j=1}^{q_i-1} ((b_{i,j} - b_{i,j-1}) - (b_{i,j} - b_{i,j+1}))^2 \quad (4)$$

which is a 'punishment' for the variability of parameters $b_{i,j}$.

This smoothness criterion and the constraints for the non-negativity and appropriate sum of the parameters have been used to modify the standard ordinary least square estimation. As a result, the pass-through profiles (and the other parameters) have been estimated via solving the following quadratic programming problem:

$$\begin{aligned} & \min_{\boldsymbol{\beta}} (\mathbf{y} - \mathbf{X}\boldsymbol{\beta})'(\mathbf{y} - \mathbf{X}\boldsymbol{\beta}) + \boldsymbol{\beta}'\mathbf{S}\boldsymbol{\beta} & (5) \\ \text{s.t.} & \quad \mathbf{A}\boldsymbol{\beta} = \mathbf{d} \\ & \quad \boldsymbol{\beta}_{low} \leq \boldsymbol{\beta} \leq \boldsymbol{\beta}_{up}, \end{aligned}$$

where vector \mathbf{y} denotes the prices changes in the group of goods, \mathbf{X} the matrix of the explanatory (cost) variables, $\boldsymbol{\beta}$ the coefficients to be estimated, including the vector for the parameters of the pass-through profiles, \mathbf{S} the matrix version of the smoothness criterion, under the restriction $\mathbf{A}\boldsymbol{\beta} = \mathbf{d}$, where \mathbf{A} and \mathbf{d} denote the matrix

²¹It was noted in the introduction that Shiller [1973] had also implemented a similar idea.

and vector of the restrictions made for the sum of the parameters of the pass-through profiles respectively, and finally β_{low} and β_{up} the lower and upper constraints of the parameters to be estimated. For the exact definition of the notations and the solution of the above problem, see the Appendix.

It may seem at first sight that this approach does not reduce the number of the parameters to be estimated. However, as demonstrated in the Appendix, changing the weights w_i will also change the system's degree of freedom. If every $w_i = 0$, then the problem is reduced to the principle of estimating least squares (ignoring other constraints). If, however, $w_i \rightarrow \infty$, then the distributed lag pass-through profile of that particular cost factor will be linear, which means that only the trend parameter of each pass-through profiles needs to be estimated. (Only the trend parameter, because the constant variable of the linear trend can be derived from the constraint imposed on the sum of the pass-through parameters.)

In the course of the estimations, we have found that the calibration of the weights w_i produces sufficiently smooth cost pass-through profiles that also satisfy the prior assumptions. Although this method is suitable for reducing the number of parameters to be estimated (and increasing the degree of freedom), the number of observations that can be actually used for the estimation is extremely low. In December 2002, there were only 60 observations, due to the two or even three-year long pass-through profiles of some cost factors. At the same time, our tests indicate that changing the length of the sample will cause hardly any change in the profiles, except for the profiles for imported goods and the exchange rate, as their shape is more significantly affected by sample periods used, due to the exchange rate appreciation seen in the period after May 2001. We have enclosed as an illustration a cost pass-through profile system of 'repairs of dwellings' obtained after the appropriate calibration of parameters w_i . The chart below shows the pass-through of the costs²² of repairs of dwellings, showing lag structures $B_i(L)$.

²²In this case, the costs included articles for dwelling (maintenance), transport, electricity, construction industry wages and market services wages.

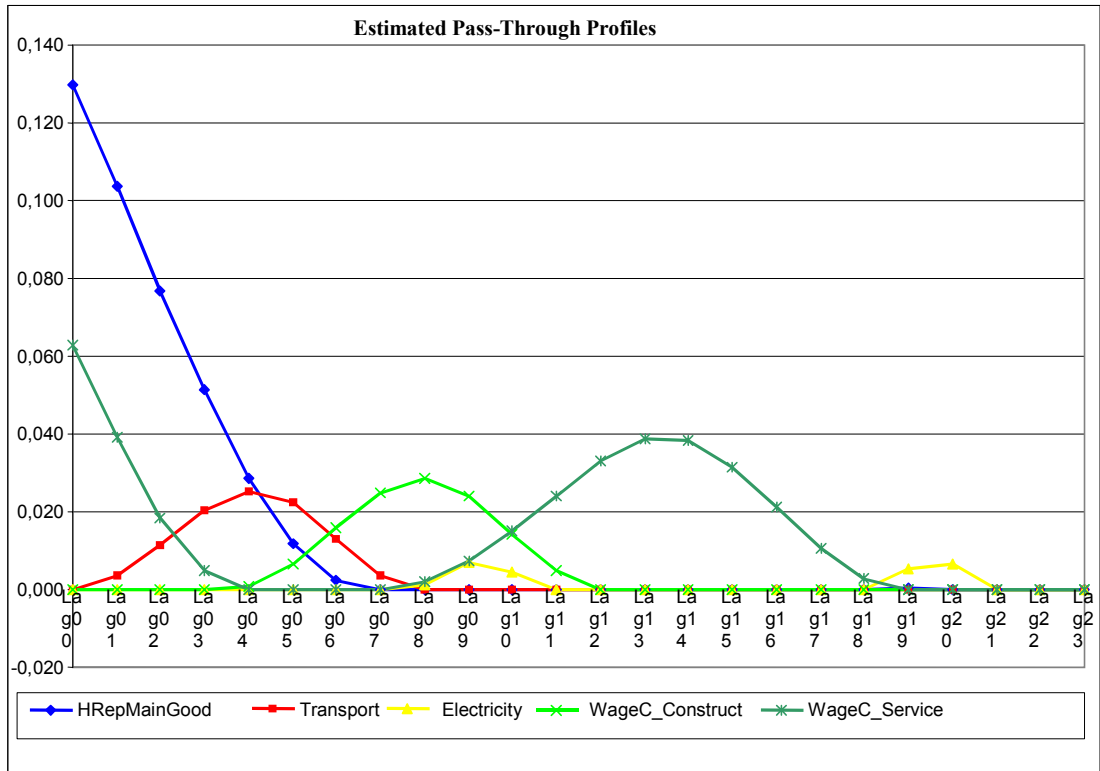


Chart 2

It is clear from the chart that the speed of pass-through of the various cost factors are different. In this case, dwelling maintenance articles exhibited the fastest pass-through, in contrast to wages, which passed through after a six-month (construction) and one-year lag (market services).

3 Ex-Ante Forecasting Ability of the Model

The performance of a model constructed primarily for the purpose of forecasting can be tested by applying it to past data. We have therefore made two ex-ante simulations, one for the period from December 1999 and the other from December 2000. The long-run and short-run parameters of the models were estimated using only these shortened sample periods, and two dynamic simulations have been run taking the exogenous variables, such as wages, agricultural purchase prices, exchange rates, import prices, the crude oil price and regulated prices as a given.

We believe that the ex-ante dynamic simulations obtained with the model give good approximation of actual developments in the consumer price index.²³

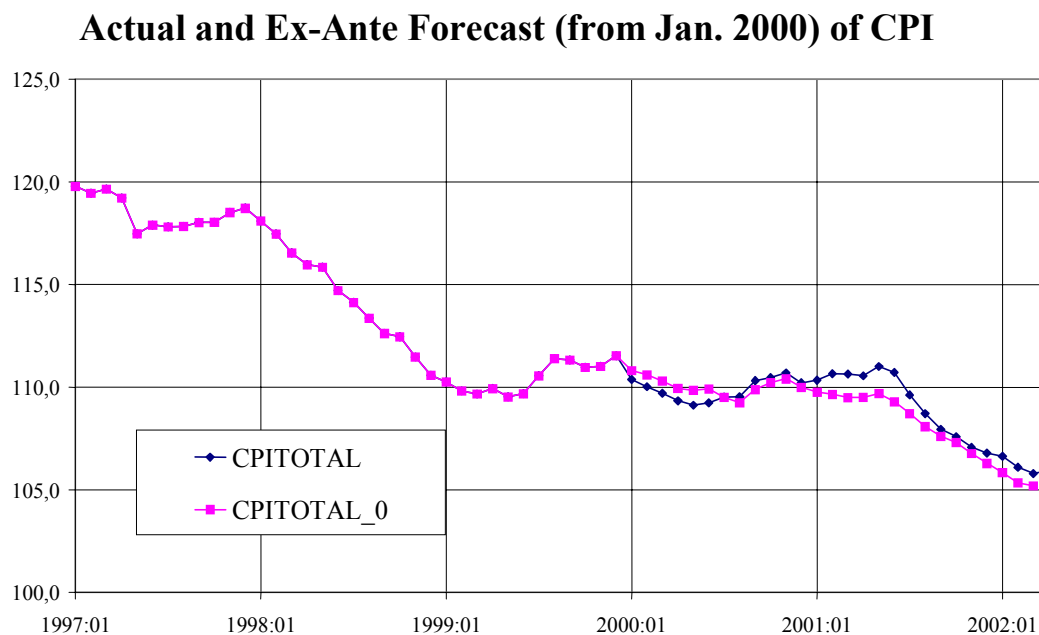


Chart 3

In the chart, CPITOTAL denotes the actual, and CPITOTAL_0 the simulated year-on-year consumer price indices

²³Of course, knowledge of the crude oil price, agricultural purchase prices, and especially regulated prices, which account for 18.88% of the consumer goods basket, improves the forecasts by itself.

Actual and Ex-Ante Forecast (from Jan. 1999) of CPI

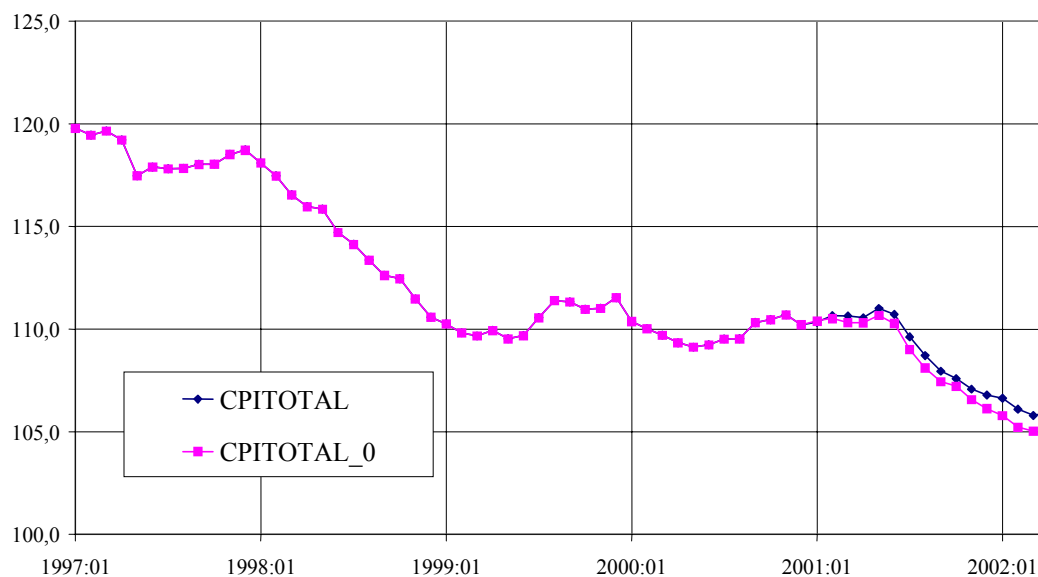


Chart 4

In the chart, CPITOTAL denotes the actual, and CPITOTAL_0 the simulated year-on-year consumer price indices

4 Further Lessons to Be Drawn and Future Directions of Development

Estimating cost pass-through offers a number of interesting conclusions to be drawn. Wage costs have been generally found to begin to pass through into prices after a relatively long lag of between at least six months to one year. Another typical feature of pass-through profiles is that, without exception, foreign price changes pass through into prices much sooner than exchange rate changes, which is completely consistent with the general opinion found in the literature on exchange rate pass-through.

In the context of the possible directions of model development, the first that should be mentioned is the lessons to be drawn from the model's 'real-life' forecasting performance in the future, which will perhaps show whether the model monolithically performs well or it has elements that result systematically in poor forecasts. However, we can

suggest a few directions for future development even on the basis of the results obtained so far.

1. Improving the applicability of source data. Some of the detailed consumer price index data reported by the Central Statistical Office are burdened with outliers (the index of other services, for instance). This is presumably due to the effects of goods removed from or included in the observed group. The elimination of these breaks and outliers would seem to be warranted, as they might introduce a bias into the estimation of cost pass-through.
2. Eliminating excise duties and tax changes from consumer prices. We are contemplating a switch to modelling prices without taxes in respect of a few goods, such as tobacco and alcohol, just as in the case of fuels. The treatment of the problems arising from potential changes in VAT rates and the reclassification of certain products and services will be among future challenges.
3. Incorporating further cost components. In the estimation of the long-run equations of the model, prices for a few groups of goods behaved very differently from their costs we assumed. This may be because we have used inadequate cost components to account for the prices of the goods categories concerned. An example for this is 'vegetable fats', in respect of which the commodity exchange price of sunflowers may be the right cost factor.
4. Incorporating a demand variable. We have found in the estimation of long-run cost equations that the residuals of the equations, which can also be interpreted as differences from long-run profit margins, are cyclical. Therefore, it may be fruitful to examine whether these cycles are in correlation with consumption cycles.

To sum up, we are confident that this model is suitable for forecasting inflation even at this stage of development. We hope to improve the reliability of the forecasts by making further developments in the model.

5 References

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- HORNOK C. – JAKAB Z. – REPPA Z. – VILLÁNYI K. [2002]: Inflation Forecasting at the National Bank of Hungary. NBH mimeo
- SHILLER, R. [1973]: A Distributed Lag Estimator Derived from Smoothness Priors. *Econometrica*, 41. pp. 775-788.

A Appendix: Detailed Model Description

A.1 Aggregation of Consumer Price Indices

In this section we expand on how we have aggregated the original 160 components of the consumer price index. Here, we first enumerate the equations of market price goods, then those of price regulated products and services. As we have mentioned in the main part of this paper, we have tried to aggregate the different goods into homogeneous groups regarding their cost structures. First, we have seasonally adjusted the price indices of market price goods. But we have not adjusted the price indices of regulated goods, as the changes in their price are not regular e.g. they occurred in different months in the last decade. After the adjustment, the aggregation process was carried out on 'previous December = 100' type indices. We used this index type, as the Central Statistic Office aggregates it in the same way.

We used the weights of year 2002 for aggregation. As the price index of the 'Cost of owner occupied dwellings', which has a 5.869% weight in the CPI index, is a 50%-50% composite index of 'Household repairing and maintenance goods' and 'Repairs and maintenance of dwellings', we have decomposed the price index of the 'Cost of owner occupied dwellings' into its original part, adding a 2.9345%-2.9345% extra weight to 'Household repairing and maintenance goods' and 'Repairs and maintenance of dwellings' in our model.

As one can see, among regulated price goods we made aggregation only on 'Meals at schools' and 'Meals at kindergartens, nurseries'.

In the equations below, the labels relate to 'previous December = 100' type indices and the weights are measured in percentages.

A.1.1 Equations for Aggregating Market Price Goods

1. $UNPROCMEAT = 1.320 * \text{pork} + 0.162 * \text{beef and veal} + 0.031 * \text{mutton, rabbit and other meat} + 0.090 * \text{edible offals} + 1.100 * \text{poultry meat}$

2. PROCMEAT = 0.765*salami, sausages, ham + 1.071*other meat preparations + 0.112*canned meat + 0.211*pork fat + 0.261*bacon
3. FISH = 0.106*fish + 0.053*canned fish
4. EGG = 0.416*egg
5. MILK = 1.619*milk
6. MILKPROD = 0.431*cheese + 1.334*milk products (excl. cheese) + 0.086*but-
ter
7. VEGETFAT = 0.412*edible oil + 0.290*margarine
8. FLOUR = 0.303*flour, groats
9. BREADROLLS = 1.497*bread + 0.392*rolls
10. SUGAR = 0.603*sugar
11. SWEETS = 0.230*other confectionery products + 0.191*candies, honey
12. OTHERCEREAL = 0.148*rice, other cereals + 0.294*pasta products + 0.612*choco-
late, cocoa + 0.453*confectionery and ice-cream
13. FRESHVEGETAB = 0.384*potatoes + 0.822*fresh vegetables + 0.992*fresh do-
mestic and tropical fruit + 0.045*dried vegetables + 0.073*nuts, poppy-seed
14. PRESERVFOOD = 0.573*fruit and vegetable juice + 0.205*preserved and frozen
vegetables + 0.085*preserved and frozen fruit + 0.134*preserved meat products
+ 0.325*preserved meals without meat + 0.745*spices
15. MEALS = 0.606*meals at restaurants not by subscription + 1.339*meals at can-
teens and meals by subscription + 0.526*buffet products + 0.132*cup of coffee
in catering
16. COFFETEA = 0.846*coffee at shops + 0.093*tea

17. NONALCBEVER = 1.409*non-alcoholic beverages
18. ALCBEVER = 1.288*wine + 2.662*beer + 2.449*spirits
19. TOBACCO = 2.655*tobacco
20. CLOTHMAT = 0.150*clothing materials made of cotton and cotton type + 0.129*clothing materials made of wool and woolen type + 0.046*other clothing materials
21. SHOES = 0.445*men's footwear + 0.544*women's footwear + 0.238*children's footwear
22. CLOTHING = 0.198*men's coats + 0.140*men's suits + 0.431*men's slacks and jackets + 0.182*men's pullovers, cardigans + 0.238*women's coats + 0.148*women's dress, costume + 0.331*women's skirts and trousers + 0.184*women's pullovers, cardigans + 0.119*children's coats + 0.283*children's overwear + 0.162*children's pullovers, cardigans
23. UNDERWEAR = 0.310*men's underwear incl. shirts + 0.177*men's socks + 0.210*women's underwear + 0.237*women's hose, socks + 0.053*children's underwear + 0.110*children's socks + 0.219*infant's clothing + 0.235*clothing accessories + 0.148*haberdashery + 0.123*suitcases, leather goods
24. FURNITURE = 0.799*living, dining- room furniture + 0.314*kitchen and other furniture
25. DURHOUSGOOD = 0.381*refrigerators, freezers + 0.263*washing-machines, spin-dryers + 0.589*heating and cooking appliances + 0.266*vacuum cleaners, sewing machines + 0.150*bicycle + 0.292*jewellery
26. VEHICLES = 2.065*passenger cars, new + 0.685*passenger cars, second-hand + 0.060*motorcycle + 0.759*tyres, parts and accessories for vehicles
27. DURRECREAGOOD = 0.033*radio sets + 0.405*tv sets + 0.397*videos, tape recorders + 0.341*cameras, watches etc.

28. COALWOOD = 0.258*coal + 0.157*briquettes, coke + 0.430*firewood
29. BPGAS = 0.613*butane and propane gas
30. HREPMAINGOOD = 3.539*household repairing and maintenance goods
31. HOUSEGOOD = 0.405*furnishing fabrics, carpets, curtains + 0.263*bed and table linen + 0.278*cooking utensils, cutlery + 0.413*parts and accessories of housing + 0.090*parts and accessories of 'do it yourself' + 0.627*household paper and other products
32. DETERGOODS = 1.154*detergents + 1.397*toilet articles
33. FUEL = 5,227*motor fuels and oils
34. NEWSPBOOK = 1.187*newspapers, periodicals + 0.560*books + 0.317*school-books
35. RECREGFLOW = 0.316*school and stationery supplies + 0.327*sport and camping articles, toys + 0.215*records, tapes, cassettes + 0.153*photographic supplies + 0.326*video cassettes ect. + 0.357*flowers, ornamental plants + 0.223*pets
36. MAINTCOST = 1.617*maintenance cost at private houses
37. REPAIRDWELL = 4.498*repairs and maintenance of dwellings
38. TRANSPORT = 0.160*transport of goods
39. RECREINLAND = 1.073*recreation in the country
40. RECREABROAD = 0.965*recreation abroad
41. REPAIR = 0.227*repairs and make clothing and footwear etc. + 0.265*rent, services for dwellings + 1.138*repairs, maintenance of vehicles + 0.180*repairs of recreational goods
42. CULTSERVICE = 1.004*educational services + 0.066*theatres, concerts + 0.120*cinemas + 0.556*other public entertainment + 0.352*photographic services

43. OTHERSERVICE = 0.156*cleaning, washing + 0.647*personal care services + 0.673*health services + 0.157*rent a car, garage services + 0.225*taxi + 1.253*services n.e.c.

A.1.2 Equations for Aggregating Regulated Price Goods

1. REGMEALS = 0.386*meals at schools + 0.122*meals at kindergartens, nurseries
2. PURCHHEAT = 1.720*purchased heat
3. ELECTRICITY = 3.148*electricity
4. GAS = 2.001*natural and manufactured gas
5. DRUGS = 2.137*pharmaceutical products
6. RENT = 0.117*rent
7. DISPOSAL = 0.754*refuse disposal, etc.
8. WATER = 1.114*water charges
9. SEWER = 0.584*sewage disposal
10. LOCTRANSP = 0.902*local transport excluding taxi
11. TRAVELWORK = 0.456*travel to work, school
12. OTHERTRAVEL = 0.489*other travels
13. TELEPHONE = 3.832*telephone
14. POSTALSER = 0.151*postal services
15. TV = 0.860*tv fee
16. GAMBLING = 0.493*gambling

A.2 Determining the Long-Run Cost Factor Structures

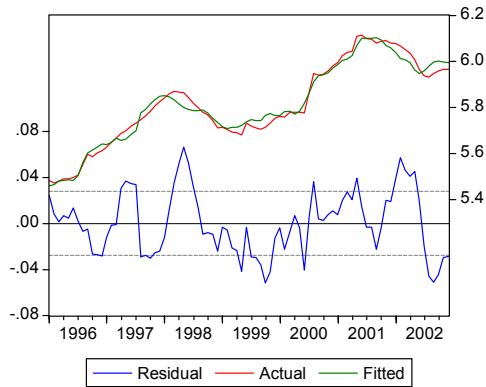
The vast majority of cost factors were calibrated based on expert information and a few of them were estimated. This can be seen in the next four tables presented below that show the long-run cost factor structures of our model. From the tables below one can easily identify which cost factors were used for each group of goods and their weights. In the tables, each equation has three rows. The names of cost factors are presented in the first row. In the second row, the mark 'E' means that the given parameter was estimated, the mark 'R' appears when the parameter was estimated under restriction, in order to ensure that the sum of cost factor parameters equals to 1. Lastly, the final parameters used by our model can be found in the third row. The sample period of estimation is reported in the second column of the table. To make our tables more transparent, the estimated parameters are marked with a grey background. The trend and constant parameters were estimated in each equation.

Names of the CPI subgroups	Sample period	Cost factors									
		1	2	3	4	5	6	7	8	9	
1 UnProcMeat	From: 1996:01	_Const	Electricity	Gas	Labc_Sale	Aggr_Meat					
	To: 2002:12	E	0,050	0,025	0,150	0,750					
2 ProcMeat	From: 1996:01	_Const	Electricity	Gas	Labc_Sale	Labc_Food	Aggr_Meat(-4)				
	To: 2002:12	E	0,050	0,035	0,150	0,100	0,630				
3 Fish	From: 1996:01	_Const	Electricity	Gas	Labc_Sale	Labc_Food	EU_Fish	huf_EUR			
	To: 2002:12	E	0,050	0,035	0,150	0,100	0,630	0,600			
4 Egg	From: 1996:01	_Const	Electricity	Gas	Labc_Sale	Aggr_Egg					
	To: 2002:12	E	0,050	0,025	0,100	0,800					
5 Milk	From: 1996:01	_Const	Electricity	Gas	Labc_Sale	Aggr_Milk					
	To: 2002:12	E	0,050	0,025	0,100	0,800					
6 MilkProd	From: 1996:01	_Const	Electricity	Gas	Labc_Sale	Labc_Food	Aggr_Milk				
	To: 2002:12	E	0,050	0,025	0,100	0,100	0,700				
7 VegetFat	From: 1996:01	_Const	Electricity	Gas	Labc_Sale	Labc_Food	EU_Fat	huf_EUR			
	To: 2002:12	E	0,050	0,025	0,100	0,400	0,400	0,400			
8 Flour	From: 1996:01	_Const	Electricity	Gas	Labc_Sale	Labc_Food	Aggr_Wheat				
	To: 2002:12	E	0,050	0,025	0,100	R	R				
9 BreadRolls	From: 1996:01	_Const	TransPort	Electricity	Gas	Labc_Sale	Labc_Food				
	To: 2002:12	E	0,470	0,050	0,100	0,100	0,240				
10 Sugar	From: 2001:07	_Const	Electricity	Gas	Labc_Sale	Labc_Food	EU_Sugar	huf_EUR			
	To: 2002:12	E	0,050	0,050	0,150	0,400	0,325	0,325			
11 Sweets	From: 2001:01	_Const	TransPort	Electricity	Gas	Labc_Sale	Labc_Food	EU_Sugar	huf_EUR		
	To: 2002:12	E	0,250	0,050	0,050	0,250	0,250	0,050	0,050		

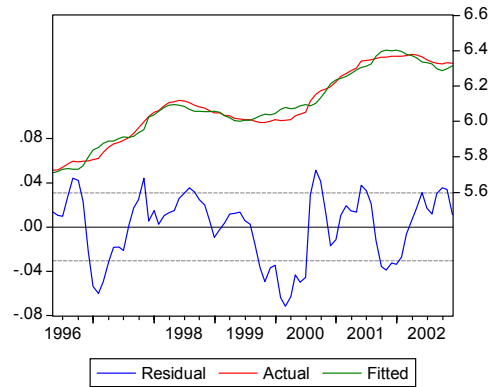
Names of the CPI subgroups	Sample period	Cost factors												
		1	2	3	4	5	6	7	8	9				
34 NewsBook	From: 1996:01		TransPort	Electricity	Gas	Labc_Sale	Labc_Wood	PPI_WoodPap						
	To: 2002:12	-2,9870	0,100	0,050	0,050	0,150	R	R	0,517	0,133				
35 RecreGFlow	From: 1996:01		TransPort	Electricity	Gas	Labc_Sale	EU_OthFlower	huf_EUR						
	To: 2002:12	-4,7304	0,050	0,025	0,025	R	R	R	0,282	0,282				
36 MaintCost	From: 1996:01		RepairDwell	Disposal	Water	Labc_Service	Sewer							
	To: 2002:12	-1,0430	0,125	0,162	0,189	0,200	0,324							
37 RepairDwell	From: 1996:01		HouseGood	TransPort	Electricity	Labc_Constr	Labc_Service							
	To: 2002:12	-2,6909	0,445	0,100	0,025	0,150	R	0,280						
38 TransPort	From: 1996:01		Vehicles	Fuel	Labc_Service									
	To: 2002:12	-3,1019	0,200	0,300	0,500									
39 RecreInland	From: 1996:01		Meals	Electricity	Gas	Labc_Hotel	Labc_Service	EU_Recreation	huf_EUR					
	To: 2002:12	-4,4837	0,100	0,050	0,050	0,300	0,300	0,200	0,200					
40 RecreAbroad	From: 1998:01		Labc_Service	EU_Recreation	huf_EUR									
	To: 2002:12	-5,6635	0,750	0,250	0,250									
41 Repair	From: 1996:01		DurHousGood	DurRecreaGood	TransPort	Electricity	Gas	Labc_Service						
	To: 2002:12	-2,3972	0,393	0,100	0,050	0,025	0,025	R	0,407					
42 CultService	From: 1996:01		Electricity	Gas	Labc_Service									
	To: 2002:12	-5,3491	0,050	0,050	0,900									
43 OtherService	From: 1998:07		ChemGoods	Fuel	Electricity	Gas	Labc_Service							
	To: 2002:12	-2,8634	0,300	0,100	0,050	0,050	0,500							

The next graphs depict the fit of our estimated long run equations. As we can see there is a strong co-movement among the final prices of services and their presumed cost factors. Moreover, there are groups where the fits of equations are relatively poor (e.g. Fish, Vegetable Fat). In the graphs the variable are logarithmised, thus the residuals can be interpreted as a relative (percentage) errors.

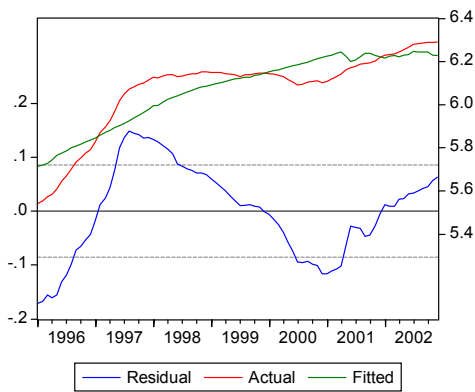
Unprocessed Meat



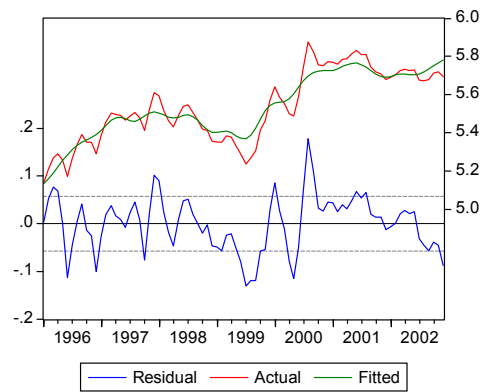
Processed Meat



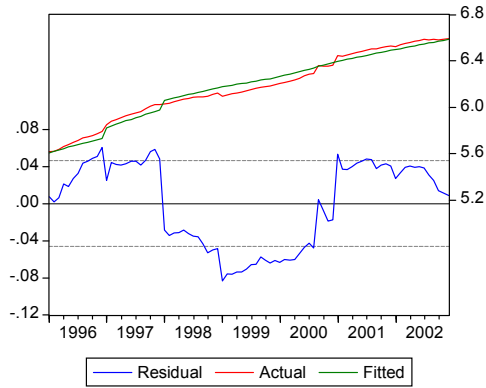
Fish



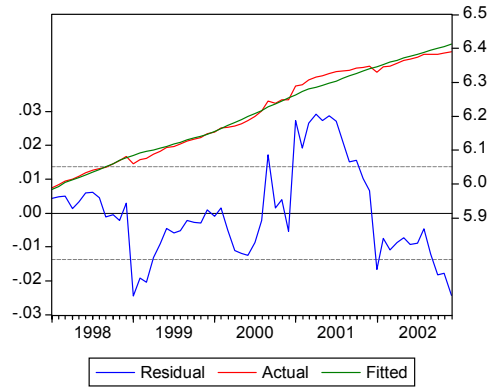
Egg



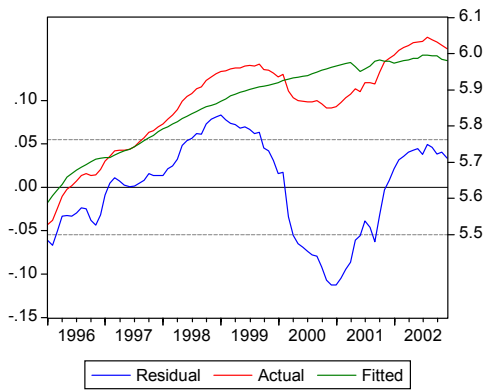
Milk



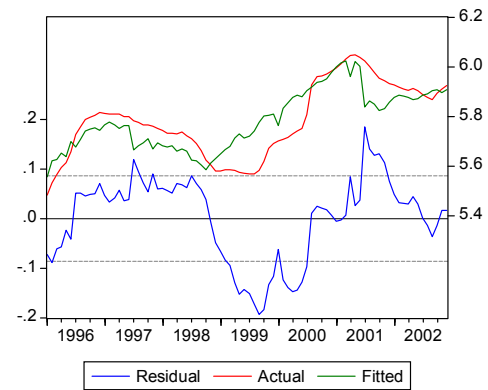
Products made of Milk



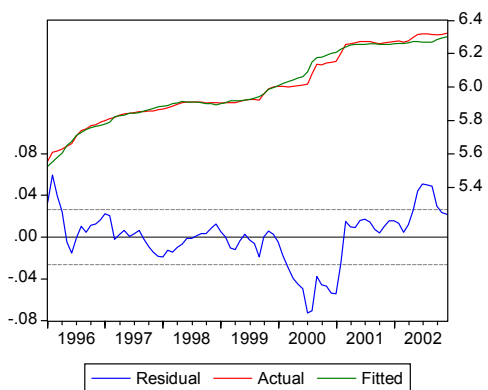
Vegetable Fat



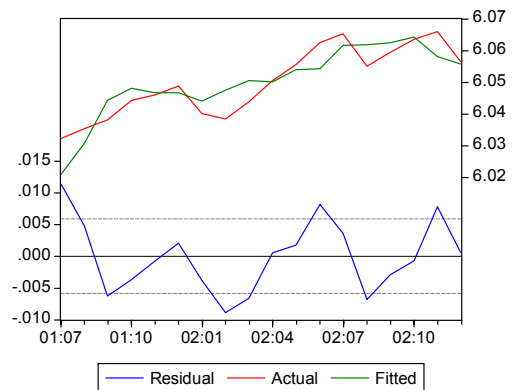
Flour



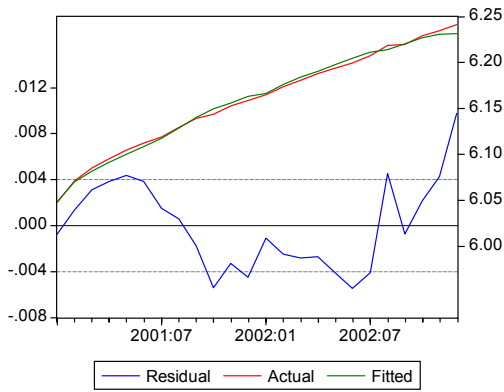
Breads And Rolls



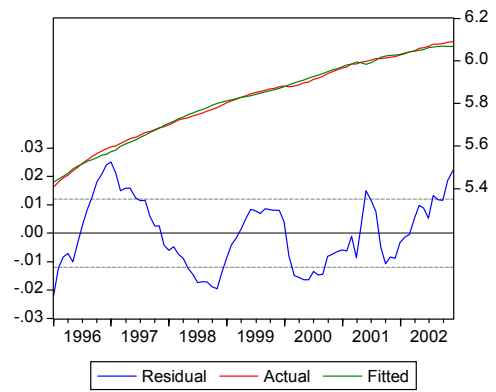
Sugar



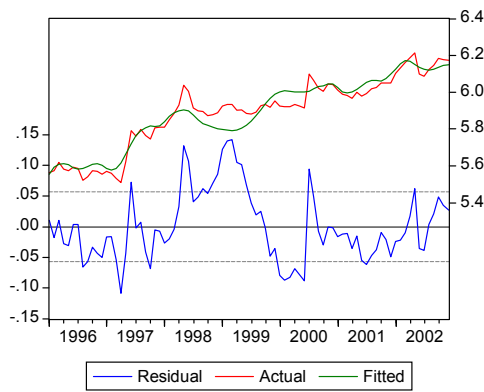
Sweets



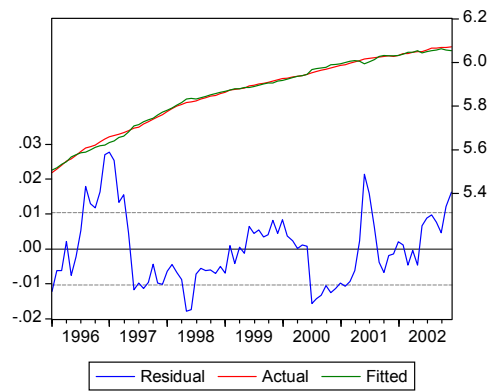
Other Cereals



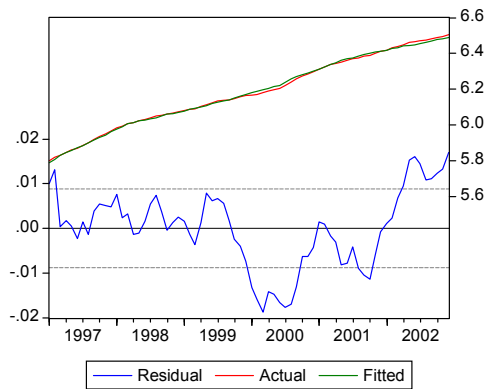
Fresh Vegetable and Fruit



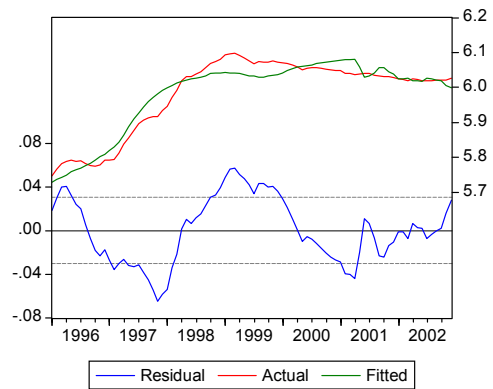
Preserved Food



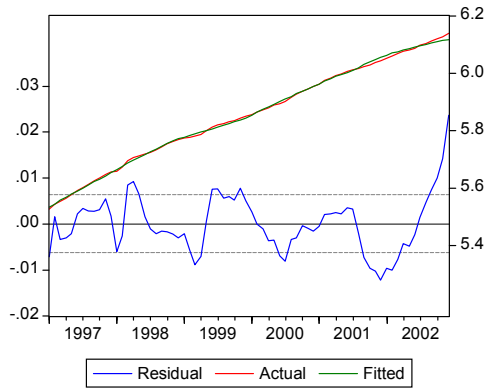
Meals



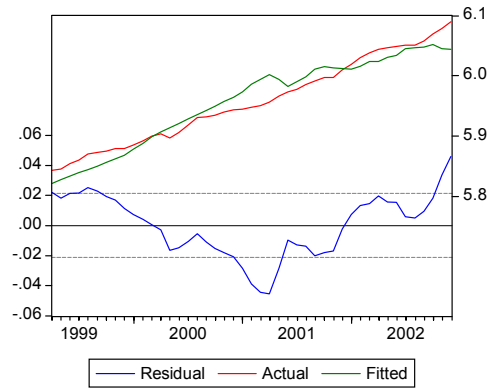
Coffee and Tea



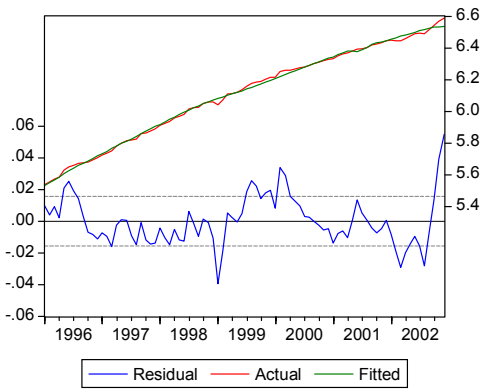
Alcoholic Beverage



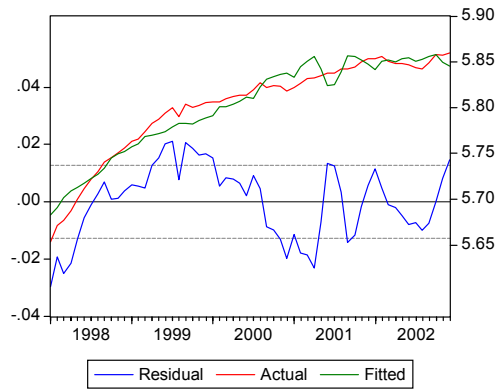
Non Alcoholic Beverage



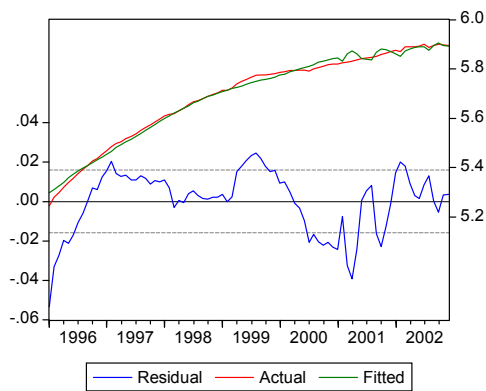
Tobacco



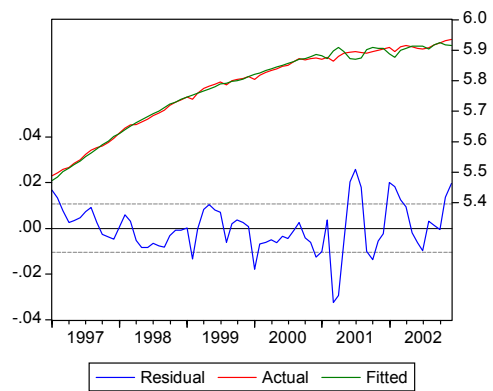
Clothing Materials



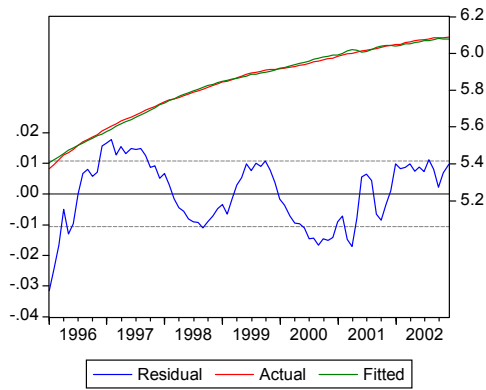
Shoes



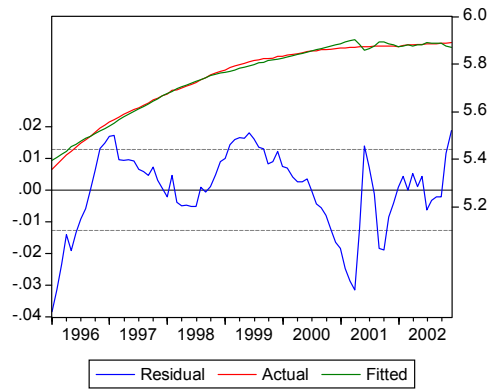
Clothing



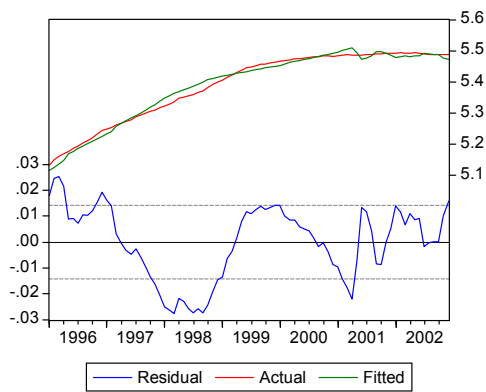
Underwear



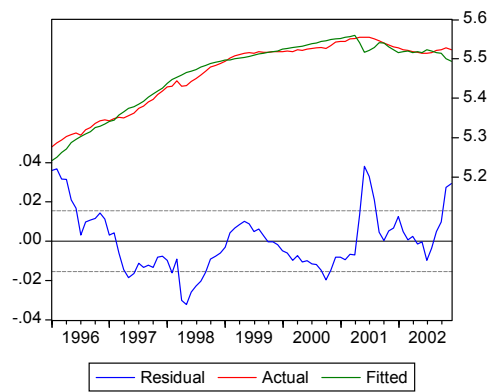
Furniture



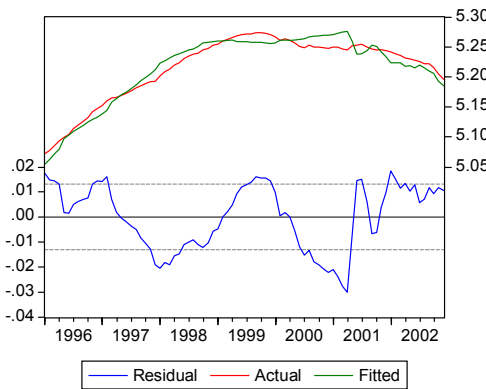
Durable Household Goods



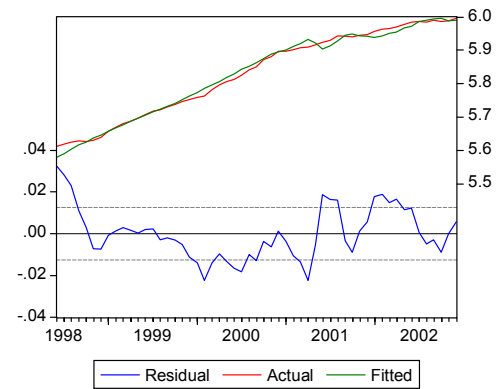
Vehicles



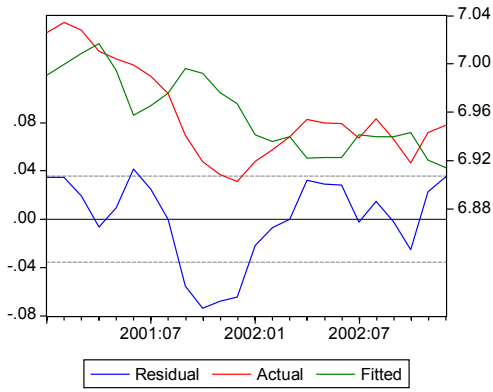
Durable Recreation Goods



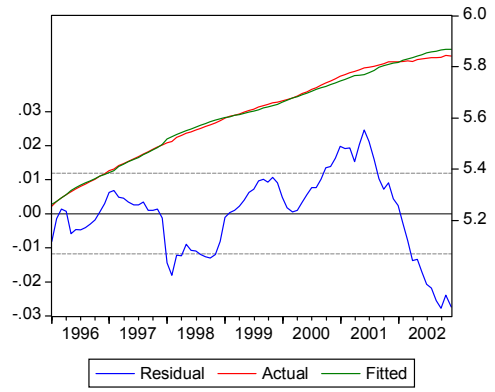
Coal and Wood



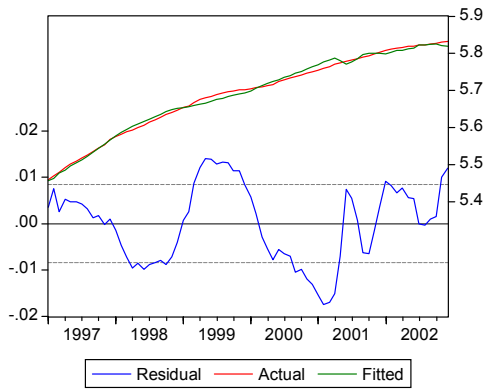
Propane Butane Gas



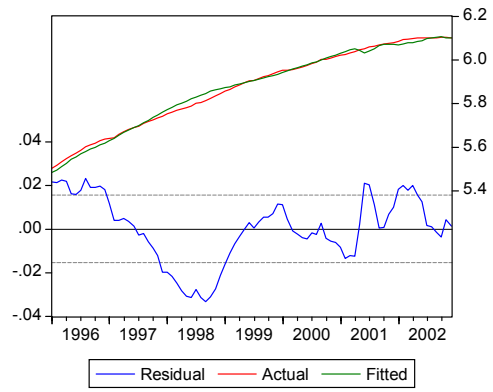
Household Rep. & Maint Goods



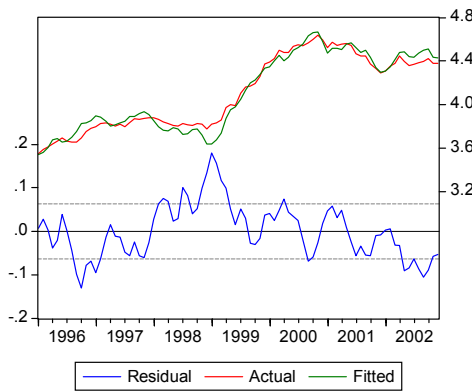
Household Goods



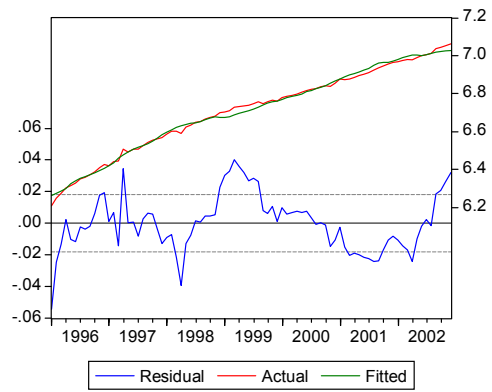
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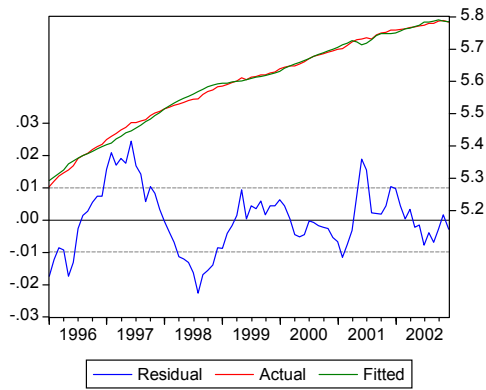
Net Price of Fuel



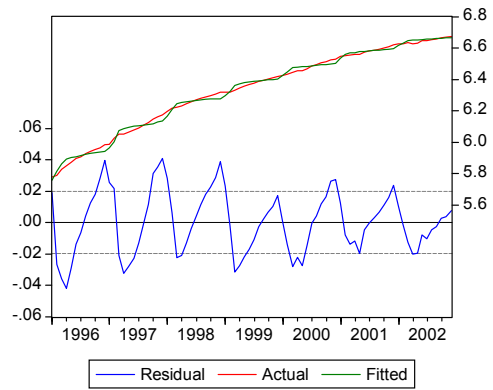
Newspapers and Books



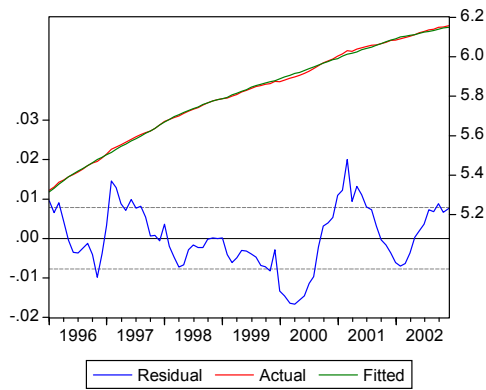
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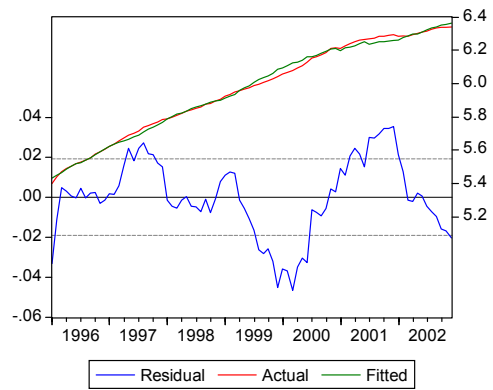
Maintenance Cost



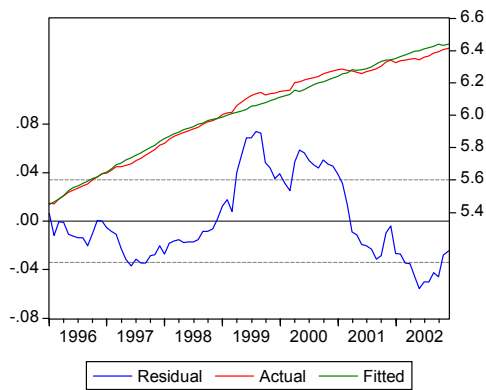
Repairs and maintenance of dwellings



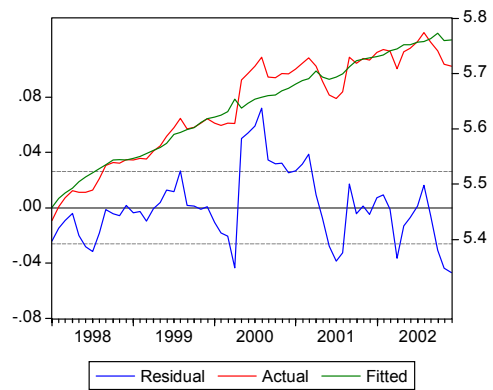
Transportation



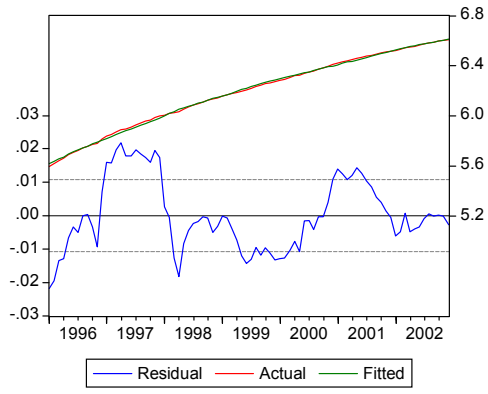
Recreation in the Country



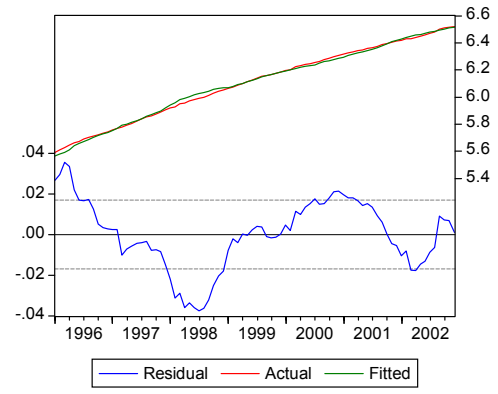
Recreation Abroad



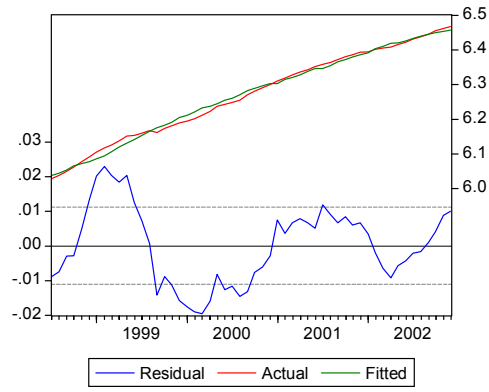
Repairs



Cultural Services



Other Services



B Appendix: The Model of Non-Parametric Distributed Lag

In this appendix, we present the method of estimating the short-run dynamics of cost pass-through, paraphrasing the quadratic programming problem (5) and its notations.

Before doing this, it seemed useful to compare our estimation method to the 'smoothness prior' based Bayesian estimation approach developed by *Shiller* [1973]. The non-parametric treatment of *a priori* knowledge of 'smooth shaped' distributed lag and the modification on ordinary least square technique in order to make the model estimable are common. But, while Shiller in his paper deals with the distributed lag of only one explanatory variable, our model can have different distributed lag structures of several explanatory variables with parameter restrictions at the same time. A further difference is the sign-restrictions of our parameters that yield a real quadratic programming problem, while Shiller's approach only requires a standard econometric software which implements the ordinary least squares regression.

Our starting equation for estimating short-run dynamics – without any parameter restriction for the present – is the (3) formula presented in the main part of our paper:

$$\begin{aligned} \Delta p_{i,t} = & \lambda_i + \gamma_1 B_1(L) \Delta c_{1,t} + \gamma_2 B_2(L) \Delta c_{2,t} + \dots + \gamma_{n-1} B_{n-1}(L) \Delta c_{n-1,t} + \\ & + (1 - \gamma_1 - \gamma_2 \dots - \gamma_{n-1}) B_n(L) \Delta c_{n,t} - \phi_i \varepsilon_{i,t-1} + \xi_{i,t}, \end{aligned} \quad (6)$$

In order to clarify the similarities and differences between the ordinary least square method and our estimation approach of non-parametric distributed lag, firstly, we manipulate the (6) formula into a more concise form. Let:

$$\mathbf{y} = \begin{bmatrix} \Delta p_{i,t} \\ \Delta p_{i,t-1} \\ \vdots \\ \Delta p_{i,t-m} \end{bmatrix}, \dots \boldsymbol{\xi} = \begin{bmatrix} \xi_{i,t} \\ \xi_{i,t-1} \\ \vdots \\ \xi_{i,t-m} \end{bmatrix} \quad (7)$$

$$\mathbf{X}' = \begin{bmatrix} \gamma_1 \Delta c_{1,t} & \gamma_1 \Delta c_{1,t-1} & \dots & \gamma_1 \Delta c_{1,t-m} \\ \gamma_1 \Delta c_{1,t-1} & \gamma_1 \Delta c_{1,t-2} & \dots & \gamma_1 \Delta c_{1,t-m-1} \\ \vdots & \vdots & & \vdots \\ \gamma_1 \Delta c_{1,t-q_1} & \gamma_1 \Delta c_{1,t-q_1-1} & \dots & \gamma_1 \Delta c_{1,t-m-q_1} \\ \gamma_2 \Delta c_{2,t} & \gamma_2 \Delta c_{2,t-1} & \dots & \gamma_2 \Delta c_{2,t-m} \\ \gamma_2 \Delta c_{2,t-1} & \gamma_2 \Delta c_{2,t-2} & \dots & \gamma_2 \Delta c_{2,t-m-1} \\ \vdots & \vdots & & \vdots \\ \gamma_2 \Delta c_{2,t-q_2} & \gamma_2 \Delta c_{2,t-q_2} & \dots & \gamma_2 \Delta c_{2,t-m-q_2} \\ \vdots & \vdots & \dots & \vdots \\ \gamma_n \Delta c_{n,t} & \gamma_n \Delta c_{n,t-1} & \dots & \gamma_n \Delta c_{n,t-m} \\ \gamma_n \Delta c_{n,t-1} & \gamma_n \Delta c_{n,t-2} & \dots & \gamma_n \Delta c_{n,t-m-1} \\ \vdots & \vdots & & \vdots \\ \gamma_n \Delta c_{n,t-q_n} & \gamma_n \Delta c_{n,t-q_n} & \dots & \gamma_n \Delta c_{n,t-q_n-1} \\ 1 & 1 & \dots & 1 \\ \varepsilon_{i,t-1} & \varepsilon_{i,t-2} & \dots & \varepsilon_{i,t-1-m} \end{bmatrix}, \quad \boldsymbol{\beta} = \begin{bmatrix} b_{1,1} \\ b_{1,2} \\ \vdots \\ b_{1,q_1} \\ b_{2,1} \\ b_{2,2} \\ \vdots \\ b_{2,q_2} \\ \vdots \\ b_{n,1} \\ b_{n,2} \\ \vdots \\ b_{n,q_n} \\ \lambda_i \\ \phi_i \end{bmatrix}, \quad (8)$$

where we have data for $m + 1$ periods, γ_n denotes $(1 - \gamma_1 - \gamma_2 \dots - \gamma_{n-1})$ and we defined the transposed matrix of \mathbf{X} to make the print more readable. The vectors $\boldsymbol{\beta}$ and \mathbf{y} , $\boldsymbol{\xi}$ have $2 + \sum_{i=1}^n q_i$ and $m + 1$ rows respectively and the matrix \mathbf{X} has a dimension of $2 + \sum_{i=1}^n q_i \times m + 1$. Using (8)-(7) the equation (6) can be summarized into form $\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\xi}$, where the least square estimation of $\boldsymbol{\beta}$ vector-parameter is the solution of the quadratic programming problem below:

$$\min_{\boldsymbol{\beta}} \boldsymbol{\xi}' \boldsymbol{\xi} = \min_{\boldsymbol{\beta}} (\mathbf{y} - \mathbf{X}\boldsymbol{\beta})' (\mathbf{y} - \mathbf{X}\boldsymbol{\beta}). \quad (9)$$

Obviously, it is quadratic, as $(\mathbf{y} - \mathbf{X}\boldsymbol{\beta})' (\mathbf{y} - \mathbf{X}\boldsymbol{\beta}) = \mathbf{y}'\mathbf{y} - 2\boldsymbol{\beta}'\mathbf{X}'\mathbf{y} + \boldsymbol{\beta}'\mathbf{X}'\mathbf{X}\boldsymbol{\beta}$.

For our non-parametric smooth distributed lag estimation process we defined a smoothness criterion that measures the smoothness of cost pass-through profiles (See formula (4) in the main part of this paper):

$$\sum_{i=1}^n w_i \sum_{j=1}^{q_i-1} ((b_{i,j} - b_{i,j-1}) - (b_{i,j} - b_{i,j+1}))^2. \quad (10)$$

The lower value of this expression is the more smooth pass-through profiles (the less variance in $b_{i,j}$). To write this expression in matrix arithmetic form, let \mathbf{W} a matrix with a dimension of $(2 + \sum_{i=1}^n q_i) \times (2 + \sum_{i=1}^n q_i)$:

$$\mathbf{W} = \begin{bmatrix} \overbrace{w_1 \quad 0 \quad \dots \quad 0}^{q_1} & \overbrace{0 \quad 0 \quad \dots \quad 0}^{q_2} & \dots & \overbrace{0 \quad 0 \quad \dots \quad 0}^{q_n} & 0 & 0 & 0 \\ 0 & w_1 & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \ddots & 0 & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \dots & 0 & w_1 & 0 & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \dots & \dots & 0 & w_2 & 0 & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \dots & \dots & \dots & 0 & w_2 & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & & & & \ddots & 0 & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \dots & \dots & \dots & \dots & 0 & w_2 & \vdots & \vdots & \vdots & \vdots \\ \vdots & & & & & \ddots & 0 & \vdots & \vdots & \vdots & \vdots \\ 0 & \dots & \dots & \dots & \dots & \dots & 0 & w_n & 0 & \vdots & \vdots \\ 0 & \dots & \dots & \dots & \dots & \dots & \dots & 0 & w_n & \vdots & \vdots \\ \vdots & & & & & & & & \ddots & 0 & \vdots \\ 0 & \dots & \dots & \dots & \dots & \dots & \dots & \dots & 0 & w_n & 0 & 0 \\ 0 & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & 0 & 0 & 0 \\ 0 & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & 0 & 0 & 0 \end{bmatrix}. \quad (11)$$

Let $\mathbf{Q}^{k \times k}$ a symmetric square matrix with a dimension of $k \times k$:

$$\mathbf{Q}^{k \times k} = \begin{bmatrix} 1 & -2 & 1 & 0 & \dots & \dots & \dots & 0 \\ -2 & 5 & -4 & 1 & 0 & \dots & \dots & 0 \\ 1 & -4 & 6 & -4 & 1 & & & \vdots \\ 0 & 1 & -4 & 6 & \ddots & \ddots & & \vdots \\ 0 & 0 & 1 & \ddots & \ddots & \ddots & 1 & 0 \\ \vdots & & & \ddots & \ddots & 6 & -4 & 1 \\ 0 & \dots & \dots & 0 & 1 & -4 & 5 & -2 \\ 0 & \dots & \dots & \dots & 0 & 1 & -2 & 1 \end{bmatrix}.$$

With the help of above defined different sized $\mathbf{Q}^{k \times k}$ matrices let \mathbf{Q} the following matrix with a dimension of $(2 + \sum_{i=1}^n q_i) \times (2 + \sum_{i=1}^n q_i)$:

$$\mathbf{Q} = \begin{bmatrix} \mathbf{Q}^{q_1 \times q_1} & \mathbf{0} & \dots & \mathbf{0} & 0 & 0 \\ \mathbf{0} & \mathbf{Q}^{q_2 \times q_2} & & \vdots & \vdots & \vdots \\ \vdots & & \ddots & \vdots & \vdots & \vdots \\ \mathbf{0} & \dots & \dots & \mathbf{Q}^{q_n \times q_n} & \vdots & \vdots \\ 0 & \dots & \dots & \dots & 0 & 0 \\ 0 & \dots & \dots & \dots & 0 & 0 \end{bmatrix}. \quad (12)$$

Then the smoothness criteria (10) can be transcribed into a $\beta' \mathbf{S} \beta$ form, where $\mathbf{S} = \mathbf{W} \mathbf{Q}$. Combining this expression with (9) yields a modified formula of least square estimation:

$$\min_{\beta} \xi' \xi + \beta' \mathbf{S} \beta, \quad (13)$$

where $\xi' \xi + \beta' \mathbf{S} \beta = (\mathbf{y} - \mathbf{X} \beta)' (\mathbf{y} - \mathbf{X} \beta) + \beta' \mathbf{S} \beta = \mathbf{y}' \mathbf{y} - 2 \beta' \mathbf{X}' \mathbf{y} + \beta' (\mathbf{X}' \mathbf{X} + \mathbf{S}) \beta$. If we neglect the parameter restrictions of the main part of our paper on β again for a moment, then the solution of the problem (13) can be derived from the first order

condition:²⁴

$$\frac{\partial (\boldsymbol{\xi}'\boldsymbol{\xi} + \boldsymbol{\beta}'\mathbf{S}\boldsymbol{\beta})}{\partial \boldsymbol{\beta}} = -2\mathbf{X}'\mathbf{y} + \mathbf{2}(\mathbf{X}'\mathbf{X} + \mathbf{S})\boldsymbol{\beta} = \mathbf{0}. \quad (14)$$

If the inverse of $(\mathbf{X}'\mathbf{X} + \mathbf{S})$ exists, then the solution for $\boldsymbol{\beta}$ can be expressed as follows:

$$\boldsymbol{\beta} = (\mathbf{X}'\mathbf{X} + \mathbf{S})^{-1} \mathbf{X}'\mathbf{y}, \quad (15)$$

that is analogous to the ordinary least square approach, the appearance of matrix \mathbf{S} forms a dissimilarity only.

Shiller derives essentially the same expression in his paper. (See *Shiller* [1973] equation (8) on page 778.) However, for the issues reviewed in the main part of our paper, some elements of $\boldsymbol{\beta}$ must be non-negative, moreover we have to impose some restrictions on the sums of elements of $\boldsymbol{\beta}$. Therefore the formula (15) cannot be used, and we have to solve a true quadratic programming problem instead. In order to get the same closed form of the problem as in problem (5), we have to take some further steps. Denote:

$$\boldsymbol{\beta}_{low} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ -\infty \\ -1 \end{bmatrix}, \quad \boldsymbol{\beta}_{up} = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \\ \infty \\ 0 \end{bmatrix}, \quad (16)$$

²⁴Our assumptions on matrices guarantee the fulfilment of second order condition of optima. Namely, one can easily justify that $\mathbf{X}'\mathbf{X} + \mathbf{S}$ is a positive definit matrix that is a sufficient condition of optima.

$$\mathbf{A} = \left[\begin{array}{cccccccccccc}
\overbrace{1 \dots \dots}^{q_1} & \overbrace{1 \ 0 \dots \dots}^{q_2} & \overbrace{0 \dots \dots}^{q_n} & 0 & 0 & 0 & 0 \\
0 & \dots & \dots & 0 & 1 & \dots & \dots & 1 & \vdots & \dots & \vdots & 0 & 0 \\
\vdots & & & \vdots & 0 & \dots & \dots & 0 & \dots & \vdots & \vdots & \vdots & \vdots \\
\vdots & & & \vdots & \vdots & & & \vdots & 0 & \dots & \dots & 0 & \vdots & \vdots \\
0 & \dots & \dots & 0 & 0 & \dots & \dots & 0 & 1 & \dots & \dots & 1 & 0 & 0
\end{array} \right], \quad \mathbf{d} = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ \vdots \\ 1 \end{bmatrix}, \quad (17)$$

where vector β_{low} and β_{up} have $2 + \sum_{i=1}^n q_i$ rows, vector \mathbf{d} has n rows, and finally matrix \mathbf{A} has a dimension of $n \times (2 + \sum_{i=1}^n q_i)$.

Using expressions (16)-(17) and the problem of type (13), our programming problem of estimation of cost pass-through profiles can be written as:

$$\min_{\beta} \xi' \xi + \beta' \mathbf{S} \beta = (\mathbf{y} - \mathbf{X}\beta)'(\mathbf{y} - \mathbf{X}\beta) + \beta' \mathbf{S} \beta, \quad (18)$$

$$\mathbf{A}\beta = \mathbf{d}$$

$$\beta_{low} \leq \beta \leq \beta_{up}.$$

In programming problem (18), the restriction $\mathbf{A}\beta = \mathbf{d}$ ensures that the sums of each pass through profiles equal to 1. The non-negativity of pass-through parameters guaranteed by inequality $\beta_{low} \leq \beta$.

B.1 Some Basic Features of the Model of Non-Parametric Distributed Lag

We have already made some comment on the relationship between the number of estimated parameters in problem (18) (i.e. the degree of freedom of the problem) and the weights w_i . In this short section, we review this relation presenting some other statistical feature of this problem (18). We analyze whether our estimation converges to a better known estimation approach when the values of w_i becomes infinitely high or low.

Firstly, let us start with investigating the case when every weight parameter measuring the smoothness of pass through profiles equals zero. ($\forall i : w_i = 0$) Then using definition (11) the W matrix becomes $\mathbf{W} = \mathbf{0}$ yielding $\mathbf{S} = \mathbf{0}$, hence $\mathbf{S} = \mathbf{WQ}$. Thus, the estimation problem (18) reduces to the next form:

$$\min_{\boldsymbol{\beta}} \boldsymbol{\xi}' \boldsymbol{\xi} = \mathbf{y}'\mathbf{y} - 2\boldsymbol{\beta}'\mathbf{X}'\mathbf{y} + \boldsymbol{\beta}'\mathbf{X}'\mathbf{X}\boldsymbol{\beta}, \quad (19)$$

$$\mathbf{A}\boldsymbol{\beta} = \mathbf{d}$$

$$\boldsymbol{\beta}_{low} \leq \boldsymbol{\beta} \leq \boldsymbol{\beta}_{up},$$

which is *ex-post* an restricted least square estimation. We say *ex-post* because if some of the inequalities are not binding, then they can be regarded as being not present, and if some of the inequalities are really binding, then they can be regarded as common constraints. Nevertheless, this kind of estimation has different statistical properties compared to least square estimation, because *ex-ante* we don't know which inequalities are going to be binding.

Secondly, let us investigate the case when every w_i weight parameter goes to infinity. ($\forall i : w_i \rightarrow \infty$) Then in optimum problem (18) $\text{grad}(\mathbf{y}'\mathbf{y} - 2\boldsymbol{\beta}'\mathbf{X}'\mathbf{y} + \boldsymbol{\beta}'\mathbf{X}'\mathbf{X}\boldsymbol{\beta}) \ll \text{grad}(\boldsymbol{\beta}'\mathbf{S}\boldsymbol{\beta})$ for every $\boldsymbol{\beta} \geq \mathbf{0}$, i.e. the value of object function in the programming problem is very sensitive to $\boldsymbol{\beta}'\mathbf{S}\boldsymbol{\beta}$, thus the tag $\mathbf{y}'\mathbf{y} - 2\boldsymbol{\beta}'\mathbf{X}'\mathbf{y} + \boldsymbol{\beta}'\mathbf{X}'\mathbf{X}\boldsymbol{\beta}$ can be neglected. Therefore the optimal $\boldsymbol{\beta}$ will be close to $\boldsymbol{\beta}^*$ where the value $\boldsymbol{\beta}^{*\prime}\mathbf{S}\boldsymbol{\beta}^*$ is minimal. This latter is

minimal when the pass-through profiles are arithmetic progressions, in other words, the profiles follow a linear distributed lag structure ($b_{i,j} = d_i + b_{i,j-1}$). In this case $\beta' \mathbf{S} \beta = 0$ equals zero, as it can be easily seen from formula (10).²⁵ The minimum value of $\beta' \mathbf{S} \beta$ is in fact zero, as \mathbf{S} is positive semi definite thus $\beta' \mathbf{S} \beta \geq 0$ for $\forall \beta : \beta \in \mathcal{R}^n$. Hence, the pass-through profiles are going to be non-negative arithmetic progression where the restriction $\mathbf{A}\beta = \mathbf{d}$ holds. (i.e. $\sum_{j=0}^{q_i} b_{i,j} = 1$ for $\forall i$) Practically, only one parameter has to be estimated for each pass-through profile of each cost element, because profiles forming arithmetic progression and the rest of parameters can be derived from the restrictions posed on the sum of weights. To illustrate, this let us consider $b_{i,0}$. Then using lags of length q_i and marking the increment of arithmetic progression with d_i we can write:

$$\sum_{j=0}^{q_i} b_{i,j} = b_{i,0} + (b_{i,0} + d_i) + (b_{i,0} + 2d_i) + \dots + (b_{i,0} + q_i d_i) = 1,$$

that rearranging yields:

$$d_i = \frac{2}{q_i} \left(\frac{1}{1 + q_i} - b_{i,0} \right). \quad (20)$$

As we restricted our pass-through profiles to be non-negative, we can not choose an arbitrary $b_{i,0}$, as on the one part $b_{i,0}$ must $b_{i,0} \geq 0$, on the other part $b_{i,0}$ has an upper limit ensuring that the last element of profiles (in our case arithmetic progressions) is non-negative:

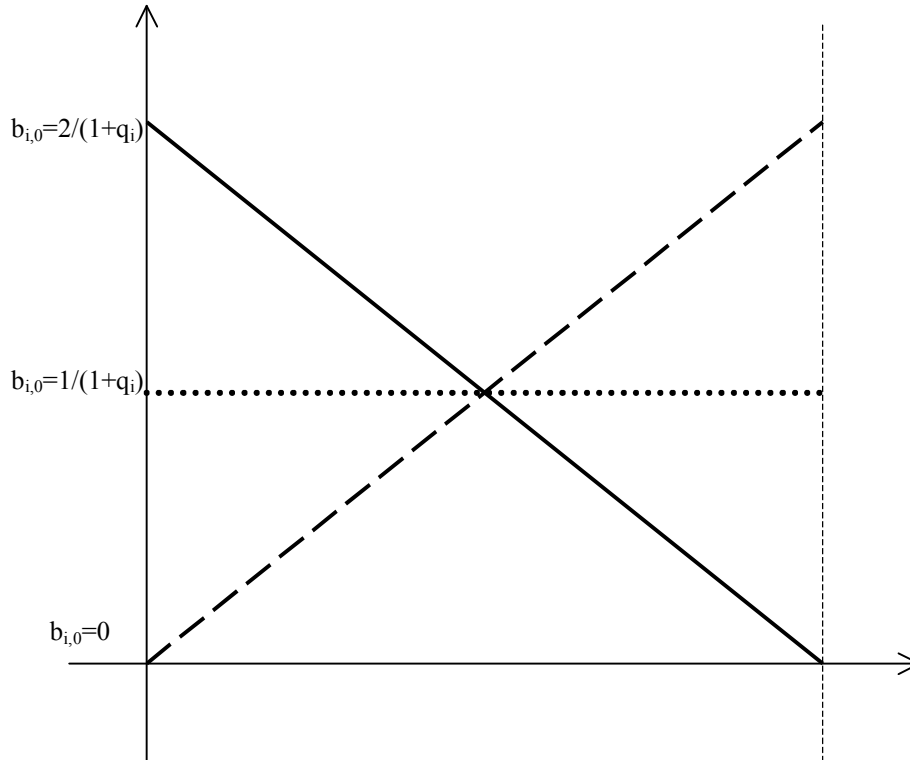
$$b_{i,0} + q_i d_i \geq 0. \quad (21)$$

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$$\begin{aligned} & w_i \sum_{j=1}^{q_i-1} ((b_{i,j} - b_{i,j-1}) - (b_{i,j} - b_{i,j+1}))^2 = \\ & = \sum_{i=1}^n w_i \sum_{j=1}^{q_i-1} ((d_i + b_{i,j-1} - b_{i,j-1}) - (b_{i,j} - (d_i + b_{i,j})))^2 = \\ & = \sum_{i=1}^n w_i \sum_{j=1}^{q_i-1} (d_i - d_i)^2 = \sum_{i=1}^n w_i \sum_{j=1}^{q_i-1} 0 = 0 \end{aligned}$$

Substituting (20) into (21) we get:

$$b_{i,0} \leq \frac{2}{1 + q_i}. \quad (22)$$



The Chart depicts three possible pass-through profiles strating from different $b_{i,0}$.

Summing up the case where the weight parameters go to infinity in problem (18), the estimation reduces to least square approach where the pass-through profiles are linear, and it requires to estimate only one parameter for each cost element with an upper and a lower limit.

But between these two extreme cases we do not know anything about the statistical properties of our estimation, nor does Shiller's paper give guidance for this, unfortunately.