On the Efficacy of Constraints on the Linear Combination Forecast Model

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Combination forecasting has been demonstrated to be a successful technique for enhanced forecast accuracy of economic and financial variables. An established method to generate the component forecast-weights is the ordinary-least-squares (OLS) regression technique. Actual values of a variable are regressed on within-sample values of forecasts generated by alternative forecast sources. The estimated regression coefficients then serve as weights for out-of-sample combination forecasts. The present study addresses the controversy regarding the efficacy of placing restrictions on the combining model when generating weights for out-of-sample forecasts. Combinations are formed of component earnings-growth forecasts generated separately by financial analysts and a statistical model. Both restricted and then unrestricted OLS are used in turn to generate the component-forecast weights. The findings suggest that combinations formed with weights generated by OLS with the constant suppressed and the sum-of-the-coefficients constrained to equal one lead to enhanced forecast accuracy and generally perform best. This study differs from a previous related study appearing in Applied Financial Economics[i] in at least three main ways:

1) Combination-forecasts are formed using actual regression-coefficients as forecast weights; 2) Forecast weights are generated using unrestricted OLS, as well as restricted OLS; 3) All combination-forecasts are strictly ex-ante simulated.

I) INTRODUCTION

Combining individual forecasts (generated separately by alternative forecast sources) into a composite forecast has proven to be an effective tool for increased forecast accuracy of a given forecast variable. [ii] Algebraically, a combination forecast may be expressed as the summation i to n of W_iF_i. The W_is are the forecast weights and the F_is are the component forecasts generated by n alternative sources. An established, widely used method to generate the component forecast-weights is the ordinary-least-squares (OLS) regression technique.[iii] Actual (realised) values of a particular variable are regressed on within-sample values of forecasts of that variable, generated by alternative forecast sources. The estimated regression coefficients then serve as weights for out-of-sample combination forecasts. [iv]

The present study is concerned with the controversy regarding the efficacy of placing restrictions on the regressions of the combination model when generating weights for out-of-sample forecasts: As a starting point, Granger and Ramanthan (1984) demonstrate that the method of unconstrained-least-squares results in unbiased estimators and minimum sum-of-squared errors for the data employed to fit the regression. Clemen (1986), however, argues that the objective is not to minimize the squared errors within the in-sample fitting data but to enhance the accuracy of the out-of-sample forecasts. Using restricted OLS Clemen demonstrates that constraining the linear combination leads to more efficient estimates of the regression coefficients, resulting in greater accuracy of the out-of-sample forecasts. Clemen's message is that the appropriate technique may be to constrain the sum-of-the coefficients to equal one, suppress the constant, or both, depending on the characteristics of the underlying forecast.

Further, it may be argued that if the process of constraining the linear combination leads to somewhat biased estimators, it may be worthwhile to trade off some incurred bias for more efficient estimators to enhance the accuracy of the out-of-sample forecasts: An estimator with lower dispersion about the mean (more efficient) and some bias will more closely approximate the true parameter than will an unbiased estimator with a larger dispersion about the mean.

Against this backdrop of reasoning, two subsequent, independent studies attempt to ascertain whether placing restrictions on the combining model leads to greater forecast accuracy. Guerard (1987) and Lobo (1991) each experiment with four alternative methods of combining annual earnings forecasts generated by security analysts and time-series models. Each of these two studies alternately employs restricted OLS (first with and then without a constant) and unrestricted OLS (with and without a constant) to generate component-forecast weights. Contrary to Clemen's argument, both studies find that the method of unrestricted OLS outperforms the restricted model in leading to superior forecasts, with one difference: Guerard finds that the method of OLS with a constant term and with no coefficient restrictions performs best. Lobo reports the unrestricted model with the constant suppressed does best in leading to superior forecasts.

To further investigate the efficacy of constraining the regressions of the linear combination model when estimating forecast weights, the present study forms combinations of average annual earnings-growth forecasts generated separately by financial analysts and a statistical model. [v]

Employing a similar methodology as in a previous study[vi], the present study differs from most other combination earnings-forecast studies in that an expected-return model is used, instead of a time-series model, to generate a statistical forecast-component. Specifically, an implicit earnings-growth forecast is extracted from the Capital Asset Pricing Model (CAPM), a risk-adjusted expected-return model.[vii]

A major reason for this approach is that risk-adjusted expected-return models have been shown to generate more accurate forecasts of earnings variables than a well-known representative modeller of the of the time-series behaviour of reported annual earnings, the submartingale.[viii] The implication, as explained in a previous study[ix], is that a risk-adjusted expected-return model may embody more independent information regarding the movements of an earnings forecast variable (and thus more useful for combination forecasting) than a time-series model.

The present study combines CAPM-generated forecasts of average annual earnings-growth with financial analysts' consensus forecasts of average annual earnings-growth, provided by International Brokers Estimate System Inc (IBES). All combination forecasts are strictly exante-simulated in that only information available prior to a forecast horizon is used in the construction of all forecasts. Congruous with the previous studies of Guerard (1987) and Lobo (1991), the present study calculates the combination-forecast weights using OLS and applies in turn each of four variations regarding the regression restrictions:

1) *OLS with a constant term and the coefficients unrestricted; 2) OLS with the constant term suppressed and the coefficients unrestricted; 3) OLS with a constant term and the sum-of-the-coefficients constrained to equal one; 4) OLS with the constant term suppressed and the sum-of-the-coefficients constrained to equal one.*

Each set of weights is then alternately used to form simulated ex-ante out-of-sample combination-forecasts of average annual earnings-growth for each firm in a given sample.[x]

Contrary to both Guerard (1987) and Lobo (1991), the present study's findings suggest that combinations formed with weights generated by OLS with the constant suppressed and the sum-of-the-coefficients constrained to equal one generally perform best, in leading to enhanced forecast accuracy over the financial analysts' consensus forecasts.

II) OBJECTIVES:

As reported in a previous study[xi], financial analysts' consensus forecasts (IBES) of average annual growth of earnings-per-share (EPS) are generally found to be significantly more accurate than forecasts generated by a risk-adjusted expected-return model (CAPM). A comparison of forecast errors leads to the rejection of the null hypothesis, that the financial anaysts' consensus forecasts are no more accurate than the CAPM generated forecasts. (See table 1A in the present study.)

With this finding in hand, the present study moves forward with three main objectives: First, to determine if forecast accuracy can be enhanced by forming combinations of financial analysts' consensus forecasts and CAPM-generated forecasts of average annual growth of EPS, using as weights actual regression coefficients from unrestricted ordinary-least-squares (OLS). Thus the first null hypothesis to be tested, H1: Combination forecasts using as weights estimated regression coefficients from unrestricted OLS, are no more accurate than the financial analysts' consensus forecasts.

The second objective is to determine if forecast accuracy can be enhanced by forming combination forecasts, using as weights actual regression coefficients from constrained OLS. Thus the second null hypothesis to be tested, H2: Combinations formed with estimated regression coefficients from restricted OLS as weights, are no more accurate than the financial analysts' consensus forecasts.

The third objective is to determine if combination forecasts, using as weights actual regression coefficients from constrained OLS, lead to greater forecast accuracy than combinations using as weights actual regression coefficients from unrestricted OLS. Thus the third null hypothesis to be tested, H3: Combinations formed with estimated regression coefficients from restricted OLS as weights, are no more accurate than combinations formed with estimated regression coefficients from unrestricted OLS as weights.

III) METHODOLOGY

An implicit forecast of the five-year average annual growth rate of earnings-per-share (EPS) for each firm in a sample is obtained from the CAPM, using a technique introduced by Rozeff (1983) and modified in a later study[xii]. As a starting point, a firm's one-period expected return is taken as the sum of the expected end-of-period dividend and change of price, divided by the beginning-of-period price, as formulated in equations 1 and 2:

$$E(R_i) = \frac{P_{i1} + D_{i1} - P_{i0}}{P_{i0}}$$

$$E(R_i) = \frac{D_{i1}}{P_{i0}} + \frac{P_{i1} - P_{i0}}{P_{i0}} \qquad \text{EQ 1}$$

where

$E(R_i)$	=	expected one-period return of stock i;
P_{i1}	=	expected end-of-period price per share;
D_{i1}	=	expected dividend per share during the period;
P_{i0}	=	current price per share;
D_{i0}	=	current dividend per share

Hence,

$$\frac{D_{i1}}{P_{i0}} + \frac{P_{i1} - P_{i0}}{P_{i0}} = \frac{D_{i0}(1+g_{id})}{P_{i0}} + g_{ip} \qquad \text{EQ 2}$$

Where,

g_{id}	=	growth rate of dividends;
g_{ip}	=	growth rate of price.

Assuming $g_{id} = g_{ip} = g_{ie}$, where g_{ie} = growth rate of earnings:

Then,

$$E(R_i) = \frac{D_{i0}(1+g_{ie})}{P_{i0}} + g_{ie} \qquad \text{EQ 3}$$

Each firm's expected return $E(R_i)$ is then separately estimated by the CAPM.[xiii] The CAPM determined value of $E(R_i)$ is then inserted into equation 3, which can then be solved to obtain the CAPM implicit forecast of five-year average annual earnings growth, g_{ie}[xiv] :

$$g_{ie} = \frac{E(R_i) - \dfrac{D_{i0}}{P_{i0}}}{1 + \dfrac{D_{i0}}{P_{i0}}} \qquad \text{EQ 4}$$

Next, in equation 4, one modification is made in the model. A measure representing the firm's dividend-paying-ability is substituted for actual dividends. This measure is equal to the product of the historical average pay-out ratio of 0.45 (over the years of the data set) and the annual average of historical firm earnings for the five years immediately passed, NE_{i0}, (to smooth out any cyclical fluctuations).[xv]
The so respecified CAPM forecasting model takes the form of:

$$g_{ie} = \frac{E(R_i) - \dfrac{NE_{i0}(.45)}{P_{i0}}}{1 + \dfrac{NE_{i0}(.45)}{P_{i0}}} \qquad \text{EQ 5}$$

Thus, as indicated in equation 5, by estimating expected return, $E(R_i)$, from the CAPM, calculating normalised earnings, NE_{i0}, from historical data, setting the dividend pay-out ratio at .45, and observing current price, P_{i0}, a forecast of the firm i five-year average annual growth of earnings-per-share is extracted from the modified CAPM forecasting model. [xvi]

III) COMBINATION FORECASTS

Combinations of financial analysts' consensus forecasts (IBES) and implicit CAPM forecasts of five-year average annual earnings growth (for each firm in a given sample) can be expressed as:

$F_c = W_1(\text{IBES}) + W_2(\text{CAPM})$.

Combination weights (W_1,W_2) are generated using cross-sectional regressions, thus incorporating information from all firms in a given sample. Actual values are regressed on predicted values of the five-year average annual growth rate of earnings-per-share (EPS), for all firm in a given sample, in the following manner:

$$\mathbf{a_{it}} = \alpha + \beta\,({}_{t\text{-}5}\mathbf{g}_{i1t}) + \gamma\,({}_{t\text{-}5}\mathbf{g}_{i2t}) + \mu_t \qquad \text{EQ 6}$$

where,

a_{it} = actual five-year average annual growth rate of EPS of firm i over the 60 months preceding time t ;

${}_{t\text{-}5}g_{i1t}$ = consensus forecast of the five-year average annual EPS growth-rate of firm i, made by financial analysts (Model A) in period t-5, taken from the IBES data-source;

${}_{t\text{-}5}g_{i2t}$ = forecast of the five-year average annual EPS growth- rate of firm i, generated from the CAPM-based forecasting method (Model B), using only information available at time t-5 and using

the model's estimation procedure and forecasting method each period;

μ_t = error term;

α = constant term.

As detailed above, the regression model is estimated four ways:
With unconstrained OLS, first with and then without a constant; and with constrained OLS, with and without a constant.
Each of the four sets of estimated regression coefficients is then alternately used as weights for out-of-sample combination forecasts of five-year average annual EPS growth for each firm in a cross-sectional sample for a given time period.

IV) DIAGNOSTIC ANALYSIS AND CORRECTIVE PROCEDURES

Nonnormality is not an issue in the OLS regressions of the present study, due to the large, random samples and the Central Limit Theorem.

Serial correlation is not a concern, as the regressions are cross-sectional.

However, evidence of heteroskedasticity is found, using White's (1980) test. As noted in a previous study[xvii], it may be that firms with higher growth rates of earnings may have different variances of forecast error than firms with smaller growth rates of earnings. Therefore, errors in predicting growth rates may be associated with one of the right-hand variables. The White (1980) procedure corrects for heteroskedasticity caused by variance related to right-hand variables. White's (1980) procedure generates a heteroskedasticity-consistent estimate of the least-squares covariance matrix, regardless of the form of heteroskedasticity. Thus, the White (1980) procedure is employed in the present study to generate a heteroskedasticity-consistent covariance matrix to construct the required significance tests.

V) SAMPLES AND TEST PROCEDURES

A) Samples:

The first in-sample coefficient-estimation period is the five-year period from January 1982 to January 1987. Using only information available prior to January 1982, an estimation is made of the parameters of the CAPM-based forecasting model. For each firm in the sample, employing the CAPM-based forecasting model, a simulated ex-ante forecast of the average annual earnings-per-share (EPS) growth rate over the January 1982 - January 1987 period is then made. The actual average annual EPS growth rates over this period are then regressed against financial analysts' (IBES) consensus forecasts and CAPM-generated forecasts, to generate four sets of weights for the out-of-sample combination forecasts for each firm in a sample (as detailed in a previous section of this study).

The first out-of-sample forecast horizon is the adjacent five-year period from January 1983 to January 1988. For each firm in the sample, employing the CAPM-based forecasting model, a simulated ex-ante forecast of the average annual EPS growth rate over the January 1983 - January 1988 period is then made. For each firm in the sample, combinations of CAPM-generated forecasts and financial analysts' (IBES) consensus forecasts for this period are then formed, using the four different sets of weights for the combination forecasts. The four sets of estimated regression coefficients generated from the January 1982 - January 1987 in-sample coefficient-estimation period are also used to manufacture out-of-sample combination forecasts for the five-year period from January 1984 to January 1989; and also for the five-year period from January 1985 to January 1990.

The experiment is replicated twice more: The second coefficient-estimation period is from January 1983 to January 1988, generating four sets of weights for out-of-sample combination forecasts for the adjacent five-year period from January 1984 to January 1989; and also for the five-year period from January 1985 to January 1990.

The third coefficient-estimation period is from January 1984 to January 1989, leading to out-of-sample combination forecasts for the adjacent five-year period from January 1985 to January 1990 (the last year of the available data set).

The combination forecasts in this study may be considered out-of-sample in the sense that some portion of a combination forecast horizon is outside of the in-sample estimation period.

In terms of practical use: for a given firm an analyst/forecaster could compare, for example, the actual five-year average annual earnings growth for the January 1982-January 1987 period with a simulated exante combination-forecast of the average annual earnings growth for the January 1985-January 1990 period (made at the beginning of the January 1987-January 1990 period, being the present time for the analyst/forecaster). From any difference between the two values, the analyst/forecaster could conceivably infer a corresponding change in the prospects of the firm for the upcoming January 1987-January 1990 period, and adjust the firm's present valuation accordingly.

To be included in a given sample a firm must have the necessary data available to:

i) generate a CAPM-based forecast, both in-sample and out-of-sample;

ii) construct in-sample regression-coefficient estimation;

iii) generate out-of-sample combination forecasts;

iv) allow the construction of, and the comparison of forecast errors. [xviii]

B. Deriving $E(R_i)$ from the CAPM:

The Capital Asset Pricing Model states that, in equilibrium, an individual security's expected return is a linear function of it covariance of return with the market portfolio. This relationship is depicted in ex-ante form by the equation:

$$E(R_i) = R_f + B_i [E(R_m) - R_f] \qquad \text{EQ 7}$$

A firm's expected return, $E(R_i)$, is calculated via the CAPM by the conventional two-stage technique. First, regression analysis is used to estimate a firm's beta, B_i . Actual, monthly security returns, $R_{i,t}$, (thirty-day geometric mean) are regressed against actual, monthly market returns, $R_{m,t}$, (thirty-day geometric mean) over the 60-month period prior to an earnings growth rate forecast horizon. This regression in equation form is:

$$R_{i,t} = B_i (R_{m,t}) \qquad \text{EQ 8}$$

The monthly market return, $R_{m,t}$, is a value-weighted measure of the returns of all stocks on the CRSP tape. All returns (firm and market) include both dividends and price changes. Once a firm's beta (B_i) is estimated, this value is inserted into equation 8 to solve for the firm's expected rate of return, $E(R_i)$. In equation 8 the risk-free rate, R_f, is taken as the yield-to-maturity on a five-year U.S. government security, prevailing at the beginning of a forecast horizon. The data source is Moody's Municipal and Government Manual. The mean market return, $E(R_m)$, is estimated as the average of the monthly market returns over the 60-month period prior to a forecast horizon. This measure is a value-weighted index of all stocks on the CRSP tape.

C. Test Procedures:

Let

a_i = actual five-year average annual growth rate of earnings-per-share (EPS) for firm i ;

and

g_{ij} = forecasted five-year average annual growth rate of EPS for firm i by method j.

In each test period a vector of forecast errors,

$$|a_i - g_{ij}| = e_{ij} \qquad \text{EQ 9}$$

is calculated for each method j. e_{ij} is the absolute value of the difference between the forecasted and realised growth rates. The mean absolute forecast error (MABE), defined as the sample average of $|a_i - g_{ij}|$, is then computed. This measure best reflects the overall forecasting performance since it takes into account the average error size. For hypothesis tests of different forecasting methods, the procedure of match-pairs case for each firm is utilised. The members of each pair are the absolute forecast errors (e_{ij}) from two forecasting methods. Each pair can be reduced to a single observation by taking the difference in the absolute forecast errors. The Wilcoxon sign rank test is used as a non-parametric test of the mean difference between the absolute forecast errors of two forecasting methods.

VI) EMPIRICAL RESULTS

As first reported in a previous study[xix], in all three test periods the financial analysts' consensus forecasts of five-year average EPS growth rates are found to be superior to the forecasts generated by the CAPM forecasting model. (See Tables 1A and 2A.) This result perhaps may be anticipated since the analysts' forecast generating mechanism certainly takes into account more subjective information about a company's prospects than the statistical forecasting model employed in the present study.

However, the present study does find that combining the analysts' consensus forecasts with the CAPM-generated forecasts generally leads to enhanced forecast accuracy (over the financial analysts' consensus forecasts) when using as weights the OLS coefficients that are constrained to sum-to-one: The mean absolute forecast error (MABE) of each of the constrained combination models is lower than the corresponding MABE of the the financial analysts' (IBES) forecasting mechanism, in eleven out of the twelve cases in the six out-of-sample test periods. (See tables 1A, 1B, and 1C.) And the mean difference in absolute forecast error between the financial analysts' forecasting mechanism (IBES) and each corresponding constrained combination model is significantly positive in these eleven out of twelve cases. [xx] (See tables 2A, 2B, and 2C.) Note that within this subset of combining models (OLS coefficients constrained to sum-to-one), the combining model with the constant suppressed generates the superior forecasts in five out of the six out-of-sample test periods. (See tables 1A, 1B, and 1C.)

On the basis of these findings, then, one may reasonably reject the null hypothesis (H2) that the combination forecasts of this study formed with restricted OLS regression coefficients as weights, are no more accurate than financial analysts' consensus forecasts.

The results of the present study also indicate that when employing forecast-weights generated from the the first coefficient-estimation period (1982-87), the unrestricted combining model with the constant supressed is generally successful in enhancing out-of-sample forecast accuracy (over the financial analysts' consensus forecasts) in all three possible cases: In comparison with the financial analysts' consensus forecasts, this combining model has lower corresponding MABEs; and significantly positive mean differences in absolute forecast error. (See tables 1A and 2A.) However, the unrestricted combining model from the the first coefficient-estimation period, with the constant included, is generally not successful in enhancing out-of-sample forecast accuracy, in first two out of the three possible cases. (See tables 1A and 2A.) Further, when employing forecast-weights generated from the second and third coefficient-estimation periods, the unrestricted combining model (both with and without a constant) is also not generally successful in enhancing forecast accuracy, in all possible cases. (See tables 1B, 1C; and tables 2B, 2C .)

Thus, on the basis of these findings, one perhaps cannot reasonably reject the null hypothesis (H1) that the combination forecasts of this study formed with unrestricted OLS regression coefficients as weights, are generally no more accurate than the financial analysts' consensus forecasts.

Finally, the results of the present study also indicate that the unrestricted combining model is generally inferior to the restricted model for all six out-of-sample test periods. Here, the relevant comparisons are Model 1 (the combination model with weights generated by unconstrained OLS with a constant included) versus Model 3 (the combination model with weights generated by constrained OLS with a constant included); and Model 2 (the combination model with weights generated by unconstrained OLS with the constant suppressed) versus Model 4 (the combination model with weights generated by constrained OLS with the constant suppressed). Model 3 generally outperforms Model 1: Model 3 has lower corresponding MABEs than Model 1 in all six cases. And the mean difference in absolute forecast error between Model 1 and Model 3 is significantly positive in all six cases. (See tables 1A, 1B, 1C; and tables 2A, 2B, 2C.) Model 4 also generally outperforms Model 2: Model 4 has lower corresponding MABEs than Model 2 in all cases. However, the mean difference in absolute forecast error between Model 2 and Model 4 is significantly positive in only the latter three out of six cases. (See tables 1A, 1B, 1C; and tables 2A, 2B, 2C.) Thus overall, in comparison with their unconstrained combining-model counterparts, the constrained combining-models have the lower MABE in twelve out of twelve cases; and a significantly positive mean difference in absolute forecast error in nine out of twelve cases.

Thus it follows that one may reasonably reject the null hypothesis (H3) that the combination forecasts of this study formed with restricted OLS regression coefficients as weights, are generally no more accurate than the combination forecasts formed with unrestricted OLS regression coefficients.

The results of the OLS regressions may help to explain the success of the constrained combination model over the unconstrained model. (See table 3A, table 3B and table 3C.) A comparison of the estimated coefficients of each of the unconstrained OLS and constrained OLS regressions for each estimation period, leads to the following observations:

First, for the initial coefficient-estimation period (1982-1987) the estimated coefficients for the analysts' consensus forecasts (IBES) are all significantly positive in each of the four regressions, ranging in magnitude from 0.6679 to 0.8522. The values of the CAPM coefficients are also positive in all four regressions, ranging in magnitude from 0.1424 to 0.1713. However, only in the constrained regression with the constant suppressed is the CAPM coefficient significantly positive. It is also observed that none of the constants are significant in this test period. Of perhaps greatest relevance and importance are the results concerning the standard errors. When comparing the standard errors for the unconstrained and constrained regressions with a constant, it is observed that the constrained regression results in lower standard errors for both the IBES and CAPM coefficients. When comparing the standard errors for the unconstrained and constrained regressions with the constant suppressed, it is observed that the standard errors for the IBES coefficients are about the same. However, the standard error for the CAPM coefficient is much lower in the constrained regression. The regression with the lowest overall pair of IBES and CAPM coefficient standard errors is the constrained regression with the constant suppressed. Thus the constrained coefficients are generally more efficient, which may account for the greater forecast accuracy of the constrained combination models over their unconstrained-counterparts for this time period. (See table 3A.)

For the second coefficient-estimation period (1983-1988) it is observed that the constants are all significantly positive and close in magnitude. The coefficients for the analysts' consensus forecasts (IBES) in the two unconstrained models are significantly positive (ranging in magnitude from 0.6413 to 0.8676), have about the same standard errors and are generally close in value, compared to their counterparts in the two constrained models, which are also significantly positive. The matter is different however regarding the estimated coefficients of the CAPM forecasts. Here it is observed that in the two unconstrained models the CAPM coefficients have a negative sign and are not statistically significant. The CAPM coefficients for the counterparts in the two constrained models are significantly positive, ranging in magnitude from 0.1368 to 0.3283. Again, of perhaps greatest importance and relevance is the fact that the CAPM coefficients in the two constrained models have much lower standard errors than their counterparts in the two unconstrained cases. Thus these constrained CAPM coefficients are more efficient, which may again account for the greater forecast accuracy of the constrained combination models over their unconstrained-counterparts in this second time period. The regression with the lowest overall pair of IBES and CAPM coefficient standard errors again is the constrained regression with the constant suppressed. (See table 3B.)

For the third coefficient-estimation period (1984-1989) it is observed that the constants are all close in magnitude, but not significant.. The coefficients for the analysts' forecasts (IBES) in the two unconstrained models are generally close in value (ranging in magnitude from 0.8298 to 0.9494), significantly positive, and have about the same standard errors, compared to their counterparts in the two constrained models, which are again also significantly positive. The matter again is different however regarding the estimated coefficients of the CAPM forecasts. Here it is observed that in the two unconstrained models the coefficients are significantly negative. For their counterparts in the two constrained models the coefficients are positive (ranging in magnitude from 0.0531 to 0.1686) but not significant.. But again, of perhaps greatest relevance is the fact that the CAPM coefficients in the two constrained models have much lower standard errors than their counterparts in the two unconstrained cases. Thus these constrained CAPM coefficients are more efficient, which may again account for the greater forecast accuracy of the constrained combination models over their unconstrained-counterparts in the third time period. Again, the regression with the lowest overall pair of IBES and CAPM standard errors is the constrained regression with the constant suppressed. (See table 3C.)

VI) SUMMARY AND CONCLUSIONS

The findings of the present study seem to lend support to the premise that restricted OLS leads to more efficient estimates of the in-sample regression coefficients of the combining model, resulting in greater accuracy of the out-of-sample forecasts.

Using only information that would be available to a forecaster prior to an out-of-sample forecast horizon, simulated ex-ante combinations are formed of financial analysts' consensus forecasts and CAPM-generated forecasts of five-year average annual earnings growth. Forecast weights generated by unrestricted OLS (with and without a constant) and restricted OLS (with and without a constant) are alternately applied, in turn. Thus for each firm in a given sample, four combination forecasts are formed.

In the present study the technique of OLS with the coefficients constrained to sum-to-one and with the constant suppressed generated the most efficient in-sample regression coefficients, which in turn generally led to superior out-of-sample combination forecasts.

It is perhaps worth noting that a set of in-sample, restricted regression coefficients, generated from one five-year estimation period (January 1982-January 1987) were successfully used to generate superior combination-forecasts for three successive, adjacent out-of-sample forecast horizons (January 1983-January 1988; January 1984-January 1989; January 1985-January 1990). And another set of in-sample, restricted regression coefficients, generated from another five-year estimation period (January 1983-January 1988) were also successfully used to generate superior combination-forecasts for two successive, adjacent out-of-sample forecast horizons (January 1984-January 1989); January 1985-January 1990). Thus, a temporal stability of the combination forecast weights is demonstrated; which in turn lends support to the justification of the use of the OLS technique to generate combination forecast weights in the present study.[xxi]

In any particular combination-forecasting experiment, the optimal combination-method perhaps depends on the particular design of the underlying model. For example, Guerard (1987) and Lobo (1991) each employ a times-series model to generate statistical alternative forecasts. The current study instead uses a risk-adjusted expected-return model (the CAPM) to generate statistical forecasts. Guerard (1987) uses cross-sectional data from the year previous to a forecast horizon to generate combination forecast weights. Lobo (1991) uses cross-sectional data from the previous two years. The current study uses cross-sectional data covering the five-year period prior to an out-of-sample forecast horizon.

Perhaps the ultimate indicator (in an ex-ante sense) of the optimal combination-method in any particular combination-forecasting attempt is whether or not, and to what extent constrained OLS leads to more efficient estimates of the in-sample regression coefficients of the combining model.[xxii]

Table 1A

Mean Absolute Forecast Error (MABE) Summary Table

(In Percentages)

(Note: all out-of-sample combination forecasts formed with forecast-weights generated from the 1982-1987 coefficient estimation period.)

Forecast horizon:	1983-88	1984-89	1985-90
Model A (IBES)	10.2015	10.9918	13.0300
Model B (CAPM)	13.4298	14.2684	17.4012
Model 1	10.3647	11.0038	12.9282
Model 2	9.9311	10.7829	12.8758
Model 3	9.9405	10.7954	12.8764
Model 4	9.9204	10.7622	12.8736

Notes:
Model A represents the financial analysts' forecasting mechanism (IBES).
Model B is the CAPM-based statistical forecasting model.
Model 1 is the combination model with weights generated by unconstrained OLS with a constant.
Model 2 is the combination model with weights generated by unconstrained OLS with the constant suppressed.
Model 3 is the combination model with weights generated by constrained OLS with a constant.
Model 4 is the combination model with weights generated by constrained OLS with the constant suppressed.

Table 1B

Mean Absolute Forecast Error (MABE) Summary Table

(In Percentages)

(Note: all out-of-sample combination forecasts formed with forecast-weights generated from the 1983-1988 coefficient estimation period.)

Forecast horizon:	1984-89	1985-90
Model A (IBES)	10.9918	13.0300
Model B (CAPM)	14.2684	17.4012
Model 1	11.5455	13.6478
Model 2	11.2859	13.4120
Model 3	11.0010	12.9212
Model 4	10.7541	12.8710

Notes:
Model A represents the financial analysts' forecasting mechanism (IBES).
Model B is the CAPM-based statistical forecasting model.
Model 1 is the combination model with weights generated by unconstrained OLS with a constant.
Model 2 is the combination model with weights generated by unconstrained OLS with the constant suppressed.
Model 3 is the combination model with weights generated by constrained OLS with a constant.
Model 4 is the combination model with weights generated by constrained OLS with the constant suppressed.

Table 1C
Mean Absolute Forecast Error (MABE) Summary Table
(In Percentages)
(Note: all out-of-sample combination forecasts formed with forecast-weights generated from the 1984-1989 coefficient estimation period.)

Forecast horizon:	1985-90
Model A (IBES)	13.0300
Model B (CAPM)	17.4012
Model 1	13.6652
Model 2	13.4267
Model 3	12.9205
Model 4	12.9248

Notes:
Model A represents the analysts' forecasting mechanism (IBES).
Model B is the CAPM-based statistical forecasting model.
Model 1 is the combination model with weights generated by unconstrained OLS with a constant.
Model 2 is the combination model with weights generated by unconstrained OLS with the constant suppressed.
Model 3 is the combination model with weights generated by constrained OLS with a constant.
Model 4 is the combination model with weights generated by constrained OLS with the constant suppressed.

Table 2A

Mean Difference in Absolute Forecast Error (ABE) Summary Table

(In Percentages)

(Note: all out-of-sample combination forecasts formed with forecast-weights generated from the 1982-1987 coefficient estimation period.)

Forecast horizon:	1983-88	1984-89	1985-90
E[ABE(Model A) - ABE(Model B)]	-4.3712	-3.2283	-3.2766
E[ABE(Model A) - ABE(Model 1)]	-0.1632*	-0.0121*	0.1018
E[ABE(Model A) - ABE(Model 2)]	0.2705	0.1300	0.1543
E[ABE(Model A) - ABE(Model 3)]	0.2610	0.1965	0.1536
E[ABE(Model A) - ABE(Model 4)]	0.2812	0.2297	0.1565
E[ABE(Model 1) - ABE(Model 3)]	0.4243	0.2084	0.0518
E[ABE(Model 2) - ABE(Model 4)]	0.0107*	0.0208*	0.0023*

Notes:
Model A represents the analysts' forecasting mechanism (IBES).
Model B is the CAPM-based statistical forecasting model.
Model 1 is the combination model with weights generated by unconstrained OLS with a constant.
Model 2 is the combination model with weights generated by unconstrained OLS with the constant suppressed.
Model 3 is the combination model with weights generated by constrained OLS with a constant.
Model 4 is the combination model with weights generated by constrained OLS with the constant suppressed.
All values significant at the 5% level or better (except where otherwise indicated).
* denotes not statistically significant.

Table 2B

Mean Difference in Absolute Forecast Error (ABE) Summary Table

(In Percentages)

(Note: all out-of-sample combination forecasts formed with forecast-weights generated from the 1983-1988 coefficient estimation period.)

Forecast horizon:	1984-89	1985-90
E[ABE(Model A) - ABE(Model B)]	-3.2766	-4.3712
E[ABE(Model A) - ABE(Model 1)]	-0.5537	-0.6177
E[ABE(Model A) - ABE(Model 2)]	-0.2942	-0.3820
E[ABE(Model A) - ABE(Model 3)]	-0.0092*	0.1089
E[ABE(Model A) - ABE(Model 4)]	0.2377	0.1590
E[ABE(Model 1) - ABE(Model 3)]	0.5446	0.7266
E[ABE(Model 2) - ABE(Model 4)]	0.5318	0.5411

Notes:
Model A represents the analysts' forecasting mechanism (IBES).
Model B is the CAPM-based statistical forecasting model.
Model 1 is the combination model with weights generated by unconstrained OLS with a constant.
Model 2 is the combination model with weights generated by unconstrained OLS with the constant suppressed.
Model 3 is the combination model with weights generated by constrained OLS with a constant.
Model 4 is the combination model with weights generated by constrained OLS with the constant suppressed.
All values significant at the 5% level or better (except where otherwise indicated).
* denotes not statistically significant.

Table 2C

Mean Difference in Absolute Forecast Error (ABE) Summary Table

(In Percentages)

(Note: all out-of-sample combination forecasts formed with forecast-weights generated from the 1984-1989 coefficient estimation period.)

Forecast horizon:	1985-90
E[ABE(Model A) - ABE(Model B)]	-4.3712
E[ABE(Model A) - ABE(Model 1)]	-0.6351
E[ABE(Model A) - ABE(Model 2)]	-0.3966
E[ABE(Model A) - ABE(Model 3)]	0.1096
E[ABE(Model A) - ABE(Model 4)]	0.1053
E[ABE(Model 1) - ABE(Model 3)]	0.7448
E[ABE(Model 2) - ABE(Model 4)]	0.5019

Notes:
Model A represents the analysts' forecasting mechanism (IBES).
Model B is the CAPM-based statistical forecasting model.
Model 1 is the combination model with weights generated by unconstrained OLS with a constant.
Model 2 is the combination model with weights generated by unconstrained OLS with the constant suppressed.
Model 3 is the combination model with weights generated by constrained OLS with a constant.
Model 4 is the combination model with weights generated by constrained OLS with the constant suppressed.
All values significant at the 5% level or better.

Table 3A

Ordinary Least Squares (OLS) with White's Corrective Procedure

$$a_{it} = \alpha + \beta\,({}_{t-5}g_{1t}) + \gamma\,({}_{t-5}g_{2t}) + \mu_t$$

Horizon: January 1982-87 **Sample Size:** 461 firms

	α	β	γ
(Unconstrained OLS)			
estimated coefficients	2.8962	0.6679	0.1424
(t - statistic)	(1.3976)	(3.9418)	(1.0087)
standard error	2.0723	0.1694	0.1411
(Unconstrained OLS with the constant supressed)			
estimated coefficients	NC	0.8454	0.1387
(t - statistic)		(13.4611)	(0.9645)
standard error		0.0628	0.1438
(Constrained OLS)			
estimated coefficients	0.6615	0.8287	0.1713
(t - statistic)	(0.3696)	(6.6555)	(1.3753)
standard error	1.7897	0.1245	0.1245
(Constrained OLS with the constant supressed)			
estimated coefficients	NC	0.8522	0.1478
(t - statistic)		(11.9497)	(2..0718)
standard error		0.0713	0.0713

Table 3B

Ordinary Least Squares (OLS) with White's Corrective Procedure

$$a_{it} = \alpha + \beta\,({}_{t\text{-}5}g_{1t}) + \gamma\,({}_{t\text{-}5}g_{2t}) + \mu_t$$

Horizon: January 1983-88 **Sample Size:** 459 firms

	α	β	γ
(Unconstrained OLS)			
estimated coefficients	3.8284	0.6413	–3.4312
(t - statistic)	(2.9184)	(6.7665)	(-1.6103)
standard error	1.3118	0.0948	2.1308
(Unconstrained OLS with the constant suppressed)			
estimated coefficients	NC	0.8676	–2.9115
(t - statistic)		(13.2334)	(-1.4215)
standard error		0.0656	2.0481
(Constrained OLS)			
estimated coefficients	3.2242	0.6717	0.3283
(t - statistic)	(2.5742)	(7.2487)	(3.5426)
standard error	1.2525	0.0927	0.0927
(Constrained OLS with the constant suppressed)			
estimated coefficients	NC	0.8632	0.1368
(t - statistic)		(13.9049)	(2.2030)
standard error		0.0621	0.0621

Table 3C

Ordinary Least Squares (OLS) with White's Corrective Procedure

$$a_{it} = \alpha + \beta\,({}_{t\text{-}5}g_{1t}) + \gamma\,({}_{t\text{-}5}g_{2t}) + \mu_t$$

Horizon: January 1984-89 **Sample Size:** 464 firms

	α	β	γ
(Unconstrained OLS)			
estimated coefficients	1.8322	0.8298	–0.4913
(t - statistic)	(0.7095)	(4.7618)	(-0.2559)
standard error	2.5822	0.1743	1.9194
(Unconstrained OLS with the constant suppressed)			
estimated coefficients	NC	0.9494	–0.4689
(t - statistic)		(14.9484)	(-0.2442)
standard error		0.0635	1.9201
(Constrained OLS)			
estimated coefficients	1.7599	0.8314	0.1686
(t - statistic)	(0.6765)	(4.7614)	(0.9653)
standard error	2.6016	0.1746	0.1746
(Constrained OLS with the constant suppressed)			
estimated coefficients	NC	0.9469	0.0531
(t - statistic)		(14.8639)	(0.8328)
standard error		0.0637	0.0637

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Footnotes:

[i] See Terregrossa (1999).

[ii] Early work in this area focused on combination forecasts of macroeconomic variables. See, for example, Newbold and Granger (1974), Cooper and Nelson (1975), Granger and Newbold (1975), Makridakis and Winkler (1983), Bischoff (1989), and Fair and Shiller (1990). For examples of combination forecasts of financial variables (eg., firm earnings) see Malkiel and Cragg (1980), Fried and Givoly (1982), Ashton and Ashton (1985), Conroy and Harris (1987), Newbold, Zumwalt, and Kannan (1987), Guerard (1987) Lobo (1991), Lobo (1992), and Terregrossa (1999).)

[iii] See, for example, Nelson (1972), Cooper and Nelson (1975), Clemen (1986), Guerard (1987), Lobo (1991), and Terregrossa (1999).

[iv] The OLS technique is appropriate when there is believed to be temporal stability of the weights underlying the combination forecast. If this is not the case then other weighting schemes may be more appropriate, such as the Granger-Newbold (1975) method which gives more weight to forecasts which have performed better in the recent past and which allows for a non-stationary relationship between respective forecast performances. See Bischoff (1989) for a thorough discussion of this issue.

[v] In a related paper (Terregrossa (1999)) all employed forecast weights were rounded approximations, based on estimated coefficients from constrained OLS; actual regression coefficients were not utilised as forecast weights. In the present study actual regression coefficients generated from in-sample regressions are employed as forecast weights for out-of-sample combination forecasts. Forecast weights are generated from both unconstrained and constrained OLS, to allow more direct and meaningful comparison with previous studies in this area.

[vi] See Terregrossa (1999).

[vii] The Capital Asset Pricing Model (CAPM) was jointly developed by Markowitz (1959), Sharpe (1964) and Lintner (1965).

[viii] See Rozeff (1983).

[ix] See Terregrossa (1999).

[x] The justification of the appropriateness of the use of OLS to generate forecast weights presented in Terregrossa (1999) is also applicable in the present study.

[xi] See Terregrossa (1999).

[xii] See Terregrossa (1999).

[xiii] See section V, part B which explains the CAPM estimation technique.

[xiv] The growth rate extracted from the CAPM forecasting model is considered to be the five-year average growth rate of EPS, because a five-year risk-free rate (R_f), taken as the yield-to-maturity on a five-year U.S. government security; is employed in the CAPM when estimating $E(R_i)$, (following precedent set by Rozeff [1983], and later applied by Terregrossa [1999]).

[xv] This type of precise measure of dividend-paying-ability was successfully applied by Sorensen and Williamson (1985) in identifying under-and over-valued stocks; and also successfully applied by Terregrossa (1999) and Terregrossa (2001) in establishing an independent information content in CAPM generated forecasts.

[xvi] Specifying the model in this manner has various benefits, as pointed out in Terregrossa (1999). Among them, setting the pay-out ratio at a constant level (0.45 in the present study) mathematically ensures the equality of the growth rates of dividends-, price-, and earnings-per share. This allows the extraction of a single earnings growth rate from the model, without having to make any restrictive assumptions about a firm's actual pay-out policy. Modelling dividend-paying-ability in this manner also allows the inclusion of non-dividend-paying as well as dividend-paying firms in our samples. This results in a greater generalisation of the findings.

[xvii] See Terregrossa (2001), in which a diagnostic analysis was applied to the restricted OLS regressions, employed to demonstrate an independent information content in the CAPM-based EPS growth forecasts. In the present study the diagnostic analysis is applied to the unrestricted, as well as the restricted OLS regressions.

[xviii] For a detailed list and explanation of the criteria each firm must satisfy to be included in a given sample, see Terregrossa (1999). The same criteria is exactly applicable in the present study.

[xix] See Terregrossa (1999).

[xx] Although the improvements in the forecasting errors are slight, small differences in compound growth rates may translate into large changes in the absolute level of future expected earnings. Current stock-value is a function of the absolute size of future expected-earnings.

[xxi] This finding of a temporal stability of the in-sample, restricted OLS regression coefficients employed as out-of-sample combination forecast weights in the present study, supports and strenghtens a similar finding in a previous study (see Terregrossa [1999]).

[xxii] Unfortunately, no direct comparison of OLS regression results can be made with the Guerard (1987) and Lobo (1991) studies, since the regression results of those studies are not explicitly reported.