Piecemeal trade liberalization on agriculture

- Theoretical and AGE based simulation Analysis -

Saito Katsuhiro[1] and Konomi Saito [2]

1. Introduction

A competitive equilibrium leads to an efficient allocation of resources under the condition that all economic agents behave to maximize their objective functions given prices. This is the well-known first theory of welfare economics. Even including foreign markets, a small country can attain a Pareto efficient allocation of resources under free trade when externality, price rigidity, wage differentials, and monopoly do not exist allowing the country to maximize its welfare. That is, an immediate implementation of free trade will be the first best policy for a small country if not considering the effects on domestic income distribution.

However, when we take into consideration the adjustment process in industrial structure caused by the implementation of free trade, the immediate and complete implementation of a free trade policy is not always the best policy or very realistic. Rather, the phased elimination of existing trade restrictive policies in order not to reduce a country's welfare level must be most realistic. Generally, analysis based on a two-goods model used in international trade textbooks provides the justification for gradual reduction of tariff rates when considering the speed of industrial structure adjustment because a slight reduction in the tariff rate will raise the countries' economic welfare under an import tariff enforced. This happens when the source of the distortion is the only *one* tariff [3].

However, in the case where the number of imported goods as well as the number of tariffs imposed on goods is more than one, a reduction of tariff rates will not necessarily increase the welfare level.

---

[1] The University of Tokyo
[2] Soka University
[3] Fukushima (1979) constructs a model which includes non-tradable goods and Falvey (1988) and Fukushima (1993) analyze a model including import quota.

The way in which reductions of tariff rates can be implemented is the main theme of this paper. Using a small country model similar to Hatta(1979a) and Hatta(1979b), the favorable procedure for altering policies such as reduction of tariff rates and relaxation of import quotas will be carefully discussed.

The paper is organized as follows. In Section 2, we explain that trade restrictive policies such as tariffs and quotas cause distortions and the policy to eliminate the distortion is not simply obtained when multiple (more than two) goods exist. In the next section, introducing a small country and multi-goods model, we will show a basic relation between reduction of tariff rates and welfare level and then analyze the piecemeal policy to reduce tariff rates. In both Sections 4 and 5, we will extend the model in Section 3 to the case including non-traded goods, analyze the relation between quotas and welfare, and develop the same argument as in Section 3.

2 Distortion and Economic Welfare

Firstly, we explain that in a two-good model free trade is most favorable to a small country. A two-good model is described as follows.

$$Q_1 = f(Q_2), E_1 = j(M_2), C_1 = Q_1 - E_1, C_2 = Q_2 + M_2 \quad (1)$$

where Q_i, C_i, E_i, M_i are respectively output, consumption, export, and import of the i-th good (i=1,2). The first equation shows the production possibility frontier and the second equation describes the foreign countries' offer curve. We assume that both curves satisfy the normal conditions. Since equation (1) is the condition which determines the consumption possibility sets of the counties after trade, the social utility function is defined as:

$$U = U(C_1, C_2) \quad (2)$$

Then, maximizing equation (2) given (1), the condition of resource allocation will be derived. That is,

$$DRS = DRT = FRT (= FP), \quad (3)$$

denoting the domestic rate of substitution, the domestic rate of transformation (in production), the foreign rate of transformation (in trade), and the foreign price by DRS, DRT, FRT, and FP. Needless to say, the condition that the marginal rates of technical substitution among factors are equalized is satisfied.

$$MRTS^{1}_{KL} = MRTS^{2}_{KL} \tag{3.1}$$

For a small country, free trade is the optimal policy when the Pareto optimal condition (3) is satisfied and efficient resource allocation will be obtained. However, when conditions (3) and (3.1) are not satisfied, free trade is no longer the first-best policy in terms of optimal allocation of resources. Depending on which equation fails to hold, that is to say, in which market distortion exists, four cases are possible as follows (Bhagwati).

(a) $DRS_{ij} = DRT_{ij} \neq FRT_{ij}$, $MRTS^{i}_{kl} = MRTS^{j}_{kl}$

(b) $DRS_{ij} = FRT_{ij} \neq DRT_{ij}$, $MRTS^{i}_{kl} = MRTS^{j}_{kl}$

(c) $DRT_{ij} = FRT_{ij} \neq DRS_{ij}$, $MRTS^{i}_{kl} = MRTS^{j}_{kl}$

(d) $DRS_{ij} = DRT_{ij} = FRT_{ij}$, $MRTS^{i}_{kl} \neq MRTS^{j}_{kl}$

Firstly, (a) is the case where a distortion exists in the non-domestic markets and $DP \neq FP$. Moreover, the existence of monopolistic power in trade is also present in this case so that $FP \neq FRT$ compared with $FP = DP$ under free trade. Equations, (b), (c) and (d) show the cases where distortions exist in domestic markets. External economy (diseconomy) in production or the presence of monopolistic factors in production lead to distortions such as (b). Equation (c) presents the case of distortion in consumption caused by external effects in consumption or when sellers put a premium on goods, whether imports or domestic products. The distortion in factor markets which (d) shows is the case where marginal rates of technical substitution differ across sectors with the result that allocation does not occur on the contract curve and the production point is inside the production possibility frontier. Wage differentiation among sectors, rigidities in factor prices, and immobility of factors across sectors lead to this type of distortion.

The first best policy for (b) (c) and (d) is to directly amend the causes through taxes or subsidies.

A trade policy such as tariff to amend a distortion in the domestic market is not the best policy since it will give birth to another distortion. That is; *if a policy which aims to eliminate a distortion causes another distortion, the policy does not necessarily increase the welfare level. Furthermore, when more than one distortion exists, if a rule to measure the degree of the whole distortion can be defined, then that will become a guideline for policy intervention.*

As discussed above, there are several types of distortion. In this paper we will focus our analysis on the (a) type of distortion and limit our discussion to the case where a distortion is caused by an artificial factor such as import restrictive trade policy.

3. Small Economy under Free Trade

3.1 Outline

<u>Domestic demand</u>

The following general model is constructed to evaluate a distortion caused by tariff. The number of goods, price vector, the utility function defined in the non-negative quadrant of R^n and the compensated demand function of each good are respectively represented by n, p, $u(c)$ and $c = f(p,u)$. Representing the expenditure function by $E(p,u)$, according to Shepard's lemma the partial differential of this expenditure function with respect to price p_i equals the compensated demand function for that good, $f_i = \partial E/\partial p_i$. Defining the matrix composed of the coefficients of the partial differentials, $f_{ij} = \partial f_i/\partial p_j \ (i, j = 1,2, \cdots, n)$, by $F = \left(\P f_i/\P p_j\right)$ and defining $f_u = \left(\P f_i/\P u\right)$, the following properties are satisfied:

(D1) homogeneity $F p = 0$

(D2) symmetry $F = {}^tF$

(D3) semi-negative definite ${}^t y F y \le 0 \ \ for \ \forall y$.

Domestic production

Expressing the domestic supply of goods by $x = h(p)$ and $H = \left(\partial h_i / \partial p_j\right)$, the following properties are satisfied:

(S1) homogeneity $Hp = 0$

(S2) symmetry $H = {}^t H$

(S3) semi-positive definite ${}^t y H y \geq 0 \quad for \forall\, y$.

<u>Tariff and revenue</u>

As the country is a small open economy, the world price q is given. The tariff is specific with the rate t and the revenue is wholly returned to consumers. Then, the domestic price vector p is expressed as

$$p = \begin{pmatrix} q_1 & & & 0 \\ & q_2 & & \\ & & \ddots & \\ 0 & & & q_n \end{pmatrix} \begin{pmatrix} 1+t_1 \\ 1+t_2 \\ \vdots \\ 1+t_n \end{pmatrix} = Q(e+t)$$

$$\text{where } Q = \begin{pmatrix} q_1 & & & 0 \\ & q_2 & & \\ & & \ddots & \\ 0 & & & q_n \end{pmatrix}, \; e = \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix}, \; t = \begin{pmatrix} t_1 \\ \vdots \\ t_n \end{pmatrix}.$$ [4]

Then, the tariff revenue is

$$T = \sum_{i=1}^{n} t_i q_i (c_i - x_i) = {}^t t\, Q(c - x)$$

Also, the budget constraint of consumers is ${}^t pc = {}^t px + T$ and the trade balance equilibrium condition is satisfied as follows: ${}^t q(c - x) = 0$

[4] When $t < 0,\, p < q$ are satisfied, if i th good is exported, then t is a tariff, while if it is imported then t is a subsidy.

Import function and trade balance equilibrium condition

Defining the import function by $s(p,u) \equiv f(p,u) - h(p)$, $s_{ij} = f_{ij} - h_{ij}$ and $s_{iu} = f_{iu}$ where the import function of good i is respectively partially differentiated with respect to p_j and u. Hence, when $S = \left(s_{ij}\right)$, $S = F - H$ and the following properties are derived, after taking into consideration the properties of the demand function and supply function.

(import demand 1) homogeneity $\quad S\,p = 0$

(import demand 2) symmetry $\quad S = {}^tS$

(import demand 3) semi-negative definite $\quad {}^t y S y \leq 0 \quad for\ \forall\, y.$

Also, the trade balance equilibrium condition is that ${}^t q s(p,u) = 0$.

The substitute and complement relationships between goods i and j are defined as follows: if $f_{ij} > 0$ then goods i and j are substitutes, if $h_{ij} < 0$ then goods i and j are substitutes in production, and if $s_{ij} > 0$ then goods i and j are pure substitutes.

3.2 Comparative Statics

The equilibrium condition of this model is as follows.

(equilibrium condition) $\begin{cases} {}^t q s(p,u) = 0 \\ p = Q(e+t) \end{cases}$

u satisfying the above two equations determines the welfare level. Totally differentiating the equilibrium condition, we obtain

$$\begin{cases} {}^t q f_u du + {}^t q S dp = 0 \\ \qquad dp = Q dt \end{cases}$$

Assuming ${}^t q f_u \neq 0$,

$$du = -\frac{1}{{}^t q f_u} {}^t q S Q dt \tag{4}$$

This is the basic equation to used to evaluate welfare level when tariff rates are changed. When inferior goods do not exist, ${}^{t}q\,f_u > 0$ and a change in welfare level depends on the import function (s(p,u)). Using that $(1+r)\,{}^{t}qS={}^{t}(r-t)QS$ for all real numbers r [5] ($r = r\cdot e$), equation (4) becomes

$$du = -\frac{1}{{}^{t}q\,f_u}\cdot\frac{1}{1+r}{}^{t}(r-t)QSQdt \tag{5}$$

Since the matrix S is semi-negative definite by the properties of the import demand function, the welfare level can increase with the appropriately selected reduction in tariff rates.

3.3 The Effect of the Tariff Rate Reduction on Economic Welfare

Considering equation (5), let us suppose an alternative trade policy so as to enact $dt = k(r-t)$ where $0 < k \leq 1$ and $r={}^{t}(r,r,\cdots,r)$. This represents the policy to bring all tariff rates proportionately close to a given level r . Then, the welfare level becomes

$$\begin{aligned} du &= -\frac{1}{{}^{t}q\,f_u}\cdot\frac{k}{1+r}{}^{t}(r-t)QSQ(r-t) \\ &= -\frac{k}{1+r}\cdot\frac{1}{{}^{t}q\,f_u}{}^{t}\big(Q(r-t)\big)S\big(Q(r-t)\big) \geq 0 \quad (\because \text{semi-negative definite } S). \end{aligned}$$

This shows that the welfare level is improving. As is apparent from the proof, the semi-negative definite property of the substitution matrix S determines the sign and the assumption that all goods are pure substitutes is not necessary. That is, the following lemma is obtained. *If no inferior goods exist, then, the policy to bring all tariff rates proportionately close to a certain level r improves the country's economic welfare*. This alteration to bring the level of tariff rates to a certain proportional

[5] The poof is as follows. From the homogeneity of degree zero of the substitute matrix S

$$\begin{aligned} 0 = S\,p = SQ(e+t) = S\cdot\begin{pmatrix}(1+t_1)q_1\\ \vdots\\ (1+t_n)q_n\end{pmatrix} = S\cdot\begin{pmatrix}\big((1+r)-(r-t_1)\big)q_1\\ \vdots\\ \big((1+r)-(r-t_n)\big)q_n\end{pmatrix} \\ = (1+r)Sq - SQ(r-t) \end{aligned}$$

Using the symmetry of S and Q, $(1+r)\,{}^{t}qS={}^{t}(r-t)QS$.

level is the appropriate to eliminate the entire distortion without inducing or enlarging other distortions. Next, following the same procedure, let us analyze the policy to reduce only the highest tariff rate to the second highest level. When the highest rate falls, total distortion should also decrease, but what is the resultant change in the welfare level?

As the following equations are satisfied, we put the number of goods in order of tariff rates.

「$t_1 > t_2 \geq t_3 \geq \cdots \geq t_n (> -1)$」

Here, the policy to reduce the highest tariff rate (t_1) to the second highest rate (t_2) appears as follows.

「$dt = a\,|t_2 - t_1| \cdot {}^t(-1, 0, \cdots, 0)$ where $0 < a \leq 1$.」[6].

When $r = t_1$ in equation (5), the change in the welfare level is

$$du = -\frac{1}{{}^t q \quad f_u} \cdot \frac{1}{1+t_1} {}^t\begin{pmatrix} 0 \\ t_1 - t_2 \\ \vdots \\ t_1 - t_n \end{pmatrix} QSQdt = \frac{1}{1+t_1} \cdot \frac{|t_2 - t_1|}{{}^t q \quad f_u} {}^t\begin{pmatrix} 0 \\ (t_1 - t_2) q_2 \\ \vdots \\ (t_1 - t_n) q_n \end{pmatrix} \begin{pmatrix} s_{11} & s_{12} & \cdots & s_{1n} \\ s_{21} & s_{22} & \cdots & s_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ s_{n1} & s_{n2} & \cdots & s_{nn} \end{pmatrix} \begin{pmatrix} a \quad q_1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$

$$= \frac{|t_2 - t_1|}{1+t_1} \cdot \frac{a\, q_1}{{}^t q\ f_u} \sum_{i \geq 2} (t_1 - t_i) q_i s_{i1}$$

Since $(1+t_1)$, $a\,q_1$ and $(t_1 - t_i)$ are all positive, if ${}^t q f_u$ and $\sum_{i \geq 2} (t_1 - t_i) q_i s_{i1}$ have the same signs, a reduction of the highest tariff rate will raise the country's economic welfare.

If inferior goods do not exist, and if the goods whose tariff rates are reduced and all of the other goods are pure substitutes, then ${}^t q f_u > 0$ and $\sum_{i \geq 2} (t_1 - t_n) q_i s_{i1} > 0$ are satisfied. Hence, we conclude the following. ***If no inferior good exists and the goods whose tariff rates are reduced and all the other***

---

[6] The piecemeal policy to bring t_1 close to t_2 little by little corresponds to the parameter a moving from 0 to 1. The policy that t_1 is decreased to t_2 corresponds to $a = 1$. In this case, all the tariff rates are not reduced to zero at the same time, and this is the case of *piecemeal trade policy.*

goods are pure substitutes, then the alteration of the trade policy to reduce the highest tariff rate to the level of the second highest tariff rate improves the country's welfare level.[7]

4. The Small Country -Open Economy-Model with Non-Traded Goods

In this section as well as the previous section, a small country, n-good open economy model was considered. Now the model will be generalized to include non-traded goods. The first m goods are traded goods while the rest of the goods, $(n-m)$, are non-traded and each vector is defined in block form by the following set of equations:

$$c = \begin{pmatrix} c_T \\ c_N \end{pmatrix},\ x = \begin{pmatrix} x_T \\ x_N \end{pmatrix},\ s = \begin{pmatrix} s_1 \\ \vdots \\ s_m \\ s_{m+1} \\ \vdots \\ s_n \end{pmatrix} = \begin{pmatrix} s_T \\ s_N \end{pmatrix},$$

$$S = \begin{pmatrix} s_{11} & \cdots & s_{1,m} & s_{1,m+1} & \cdots & s_{1,n} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ s_{m,1} & \cdots & s_{m,m} & s_{m,m+1} & \cdots & s_{m,n} \\ s_{m+1,1} & \cdots & s_{m+1,m} & s_{m+1,m+1} & \cdots & s_{m+1,n} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ s_{n,1} & \cdots & s_{n,m} & s_{n,m+1} & \cdots & s_{n,n} \end{pmatrix} = \begin{pmatrix} S_{TT} & S_{TN} \\ S_{NT} & S_{NN} \end{pmatrix}、$$

[7]We do not deny the possibility that when the highest tariff rate t_1 decreases to a level higher than the secondly highest rate t_2, the welfare level might rise. For under the condition of ${}^t q\, f_u > 0$, if $\sum_{i\geq 2}(t_1 - t_i) q_i s_{i1} > 0$ is satisfied, it happens. The conditions obtained to increase the welfare level are all sufficient conditions.

$$p_T = \begin{pmatrix} p_1 \\ \vdots \\ p_m \end{pmatrix} = \begin{pmatrix} q_1 & & 0 \\ & \ddots & \\ 0 & & q_m \end{pmatrix} \begin{pmatrix} 1+t_1 \\ \vdots \\ 1+t_m \end{pmatrix} = Q_T(e+t_T)$$

By the small country assumption, $q_T = {}^t(q_1, \cdots, q_m)$ is a constant vector and the tariff revenue is returned to consumers as lump-sum subsidies.

The equilibrium condition for this economy is

$$\begin{cases} {}^t q_T s_T(p,u) = 0 \\ s_N(p,u) = 0 \\ p_T = Q_T(e+t_T) \end{cases}$$

where the third equation shows that the domestic prices of traded goods are presented as world prices multiplied by tariff rates. Substituting the third equation, we obtain

$$\begin{cases} {}^t q_T s_T(Q_T(e+t_T), p_N, u) = 0 \\ s_N(Q_T(e+t_T), p_N, u) = 0 \end{cases}$$

The first equation presents the trade balance equilibrium condition and the second shows the demand-supply equilibrium in the non-traded goods market. Totally differentiating the above equilibrium equations we obtain,

$$\begin{cases} {}^t q_T s_{Tu} du + {}^t q_T s_{TN} dp_N + {}^t q_T s_{TT} Q_T dt_T = 0 \\ s_{Nu} du + s_{NN} dp_N + s_{NT} Q_T dt_T = 0 \end{cases}$$

Then, expressing the equations in matrix form,

$$\begin{pmatrix} {}^t q_T s_{Tu} & {}^t q_T s_{TN} \\ s_{Nu} & s_{NN} \end{pmatrix} \begin{pmatrix} du \\ dp_N \end{pmatrix} = -\begin{pmatrix} {}^t q_T s_{TT} Q_T dt_T \\ s_{NT} Q_T dt_T \end{pmatrix}$$

Next we need to derive the inverse of the coefficient matrix. [8]

---

[8] Dividing a matrix, so that $A = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}$,

$$\begin{pmatrix} {}^t q_T s_{Tu} & {}^t q_T s_{TN} \\ s_{Nu} & s_{NN} \end{pmatrix}^{-1} = \begin{pmatrix} \tilde{A} & -\tilde{A}\,{}^t q_T S_{TN} S_{NN}^{-1} \\ -S_{NN}^{-1} S_{Nu} \tilde{A} & S_{NN}^{-1} + S_{NN}^{-1} S_{Nu} \tilde{A}\,{}^t q_T S_{TN} S_{NN}^{-1} \end{pmatrix}$$

where $\tilde{A} = \left({}^t q_T S_{Tu} - {}^t q_T S_{TN} S_{NN}^{-1} S_{Nu}\right)^{-1}$

Hence,

$$du = -\left({}^t q_T \left(S_{Tu} - S_{TN} S_{NN}^{-1} S_{Nu}\right)\right)^{-1} \cdot \left({}^t q_T \left(S_{TT} - S_{TN} S_{NN}^{-1} S_{NT}\right)\right) Q_T \cdot dt_T \tag{6}$$

$$= -\frac{1}{\left({}^t q_T \left(S_{Tu} - S_{TN} S_{NN}^{-1} S_{Nu}\right)\right)} \cdot \left({}^t q_T \left(S_{TT} - S_{TN} S_{NN}^{-1} S_{NT}\right)\right) Q_T \cdot dt_T$$

$$= -\frac{{}^t q_T D Q_T \cdot dt_T}{{}^t q_T Z \cdot S_u}$$

using $D = S_{TT} - S_{TN} S_{NN}^{-1} S_{NT}$ and $Z = \left(\mathrm{I} \ \vdots \ -\mathrm{S}_{\mathrm{TN}} \mathrm{S}_{\mathrm{NN}}^{-1}\right)$.

Then, since S is homogeneous of degree zero ($S p = 0$),

$$\begin{cases} S_{TT} p_T + S_{TN} p_N = 0_T \\ S_{NT} p_T + S_{NN} p_N = 0_N \end{cases}$$

Also by the existence of S_{NN}^{-1}, we can solve the second equation for p_N. Substituting this into the first equation,

$$\left(S_{TT} - S_{TN} S_{NN}^{-1} S_{NT}\right) p_T = D\, p_T = 0_T .$$

Since the matrix S is symmetric , as is the matrix D, then ${}^t p_T D = 0$.

As shown in the last section, since for any i satisfying $i \in T = \{1, \cdots, m\}$ it follows that

$$p_i = (1 + t_i) q_i = \left(1 + r - (r - t_i)\right) q_i ,$$

and we obtain

$${}^t p_T = (1 + r)\, {}^t q_T - {}^t (r - t_T) Q_T .$$

$$A^{-1} = \begin{pmatrix} \tilde{A} & -\tilde{A} A_{12} A_{22}^{-1} \\ -A_{22}^{-1} A_{21} \tilde{A} & A_{22}^{-1} + A_{22}^{-1} A_{21} \tilde{A} A_{12} A_{22}^{-1} \end{pmatrix} \text{ where } \tilde{A} = \left(A_{11} - A_{12} A_{22}^{-1} A_{21}\right)^{-1} .$$

The proof is to divide a matrix B following matrix A, and to determine the B so as to satisfy

$AB = I$ (*identity matrix*). Finally confirm $BA = I$.

Substituting this into ${}^{t}p_T D = 0$,

$$(1+r)\,{}^{t}q_T D = {}^{t}(r - t_T)Q_T D \tag{7}$$

using that $r = {}^{t}(r, r, \cdots, r) \in R^m$ and r is any real number.

Substituting (7) into(6),

$$du = -\frac{1}{\left({}^{t}q_T (S_{Tu} - S_{TN} S_{NN}^{-1} S_{Nu})\right)} \cdot \frac{1}{1+r} \cdot {}^{t}(r - t_T)Q_T (S_{TT} - S_{TN} S_{NN}^{-1} S_{NT})Q_T \cdot dt_T \tag{8}$$

by $r \neq -1$.

Then, suppose that $dt_T = a(r - t_T)$, $0 < a \leq 1$. The numerator of equation (8) is

$$a \cdot {}^{t}(Q_T(r - t_T))(S_{TT} - S_{TN} S_{NN}^{-1} S_{NT})(Q_T(r - t_T)),$$

where the sign depends on the matrix $D = (S_{TT} - S_{TN} S_{NN}^{-1} S_{NT})$, while the sign of the denominator depends on

$$(S_{Tu} - S_{TN} S_{NN}^{-1} S_{Nu}) = \begin{pmatrix} \mathrm{I} & \vdots & -S_{TN} S_{NN}^{-1} \end{pmatrix} \cdot \begin{pmatrix} S_{Tu} \\ \cdots \\ S_{Nu} \end{pmatrix} = \begin{pmatrix} \mathrm{I} & \vdots & -S_{TN} S_{NN}^{-1} \end{pmatrix} \cdot s_u \,.$$

Next, assuming that no inferior goods exists, ($s_u \geq 0$),

$$D = (S_{TT} - S_{TN} S_{NN}^{-1} S_{NT}) = \begin{pmatrix} \mathrm{I} & \vdots & -S_{TN} S_{NN}^{-1} \end{pmatrix} \cdot \begin{pmatrix} S_{TT} \\ \cdots \\ S_{NT} \end{pmatrix}$$

$$\begin{pmatrix} \mathrm{I} & \vdots & -S_{TN} S_{NN}^{-1} \end{pmatrix} \cdot \begin{pmatrix} S_{TN} \\ \cdots \\ S_{NN} \end{pmatrix} = S_{TN} - S_{TN} S_{NN}^{-1} S_{NN} = \mathrm{O} \cdot$$

Using

$$\begin{pmatrix} \mathrm{I} & \vdots & -S_{TN} S_{NN}^{-1} \end{pmatrix} \cdot \begin{pmatrix} S_{TT} & S_{TN} \\ S_{NT} & S_{NN} \end{pmatrix} = \begin{pmatrix} D & \vdots & \mathrm{O} \end{pmatrix}$$

$$\begin{pmatrix} D & \vdots & \mathrm{O} \end{pmatrix} \cdot \begin{pmatrix} \mathrm{I} \\ \cdots \\ * \end{pmatrix} = D \ ,$$

$$D = \left(S_{TT} - S_{TN} S_{NN}^{-1} S_{NT}\right) = \left(\mathrm{I} \;\; \vdots \;\; -S_{TN} S_{NN}^{-1} \right) \cdot \begin{pmatrix} S_{TT} & S_{TN} \\ S_{NT} & S_{NN} \end{pmatrix} \begin{pmatrix} \mathrm{I} \\ \cdots \\ -S_{NN}^{-1} S_{NT} \end{pmatrix} = ZS\,{}^{t}Z$$

From the symmetry of S and D, $D={}^{t}ZSZ$ and the sign of the numerator is determined by the negative definite condition.

Let us consider the sign of the denominator. As previously mentioned, under the condition that $s_u \geq 0$, the sign depends on the matrices S_{TN} and S_{NN}^{-1}. If non-traded goods are substitutes for all of the other goods, S_{TN} is a non-negative matrix ($S_{TN} > 0$). Moreover, the diagonal components of S_{NN} are negative and the non-diagonal components are non-negative. As is well known from the Hawkins-Simon (1949) condition, the matrix $(-S_{NN})$ has positive diagonal elements and non-positive non-diagonal elements. If the determinants of the principal minors are all positive, then

$$\left(-S_{NN}\right)^{-1} = S_{NN}^{-1} \cdot \left(-\mathrm{I}\right)^{-1} = -S_{NN}^{-1} \geq 0 .$$

This is a non-negative matrix. Actually, since S is semi-negative definite, using、

$$p={}^{t}(p_T, p_N)={}^{t}(0, p_N),$$

$${}^{t}p(-S)p={}^{t}p_N\left(-S_{NN}\right)p_N > 0 .$$

So, for any p_N, $\left(-S_{NN}\right)$ becomes positive definite and the determinants of the principal minors are all positive. That is, $-S_{NN}^{-1} \geq 0$ and

$$(S_{Tu} - S_{TN} S_{NN}^{-1} S_{Nu}) = \left(\mathrm{I} \;\; \vdots \;\; -S_{TN} S_{NN}^{-1} \right) \cdot s_u > 0 .$$

Summarizing the above,

Suppose that no inferior goods exists and all non-traded goods are substitutes for all other goods. Then, the policy to bring all tariff rates proportionally close to a certain level improves the country's economic welfare.

Although this theorem holds when $r > -1$[9], a policy to make r =0 is best in the small open economy

---

[9]Since when $r \leq -1$, domestic prices become negative or zero, it is a natural hypothesis that $r > -1$.

case. That is, ***suppose that no inferior goods exist and all non-traded goods are substitutes for all other goods. Then, a policy to bring all tariff and subsidy rates proportionately close to zero improves the country's economic welfare.***

Next, as in section 3, we arrange the number of goods in order of tariff rates such that $t_1 > t_2 \geq t_3 \geq \cdots \geq t_m$ and examine the policy to reduce the highest rate (t_1) to the second highest rate level (t_2). Since we obtain

$dt_T = a \cdot {}^t(t_2 - t_1, 0, 0, \cdots, 0) = a \cdot |t_2 - t_1| \cdot {}^t(-1, 0, 0, \cdots, 0) \in R^m,\ 0 < a \leq 1,$[10]

substituting this into ⑻ and using $r = t_1$,

$$du = -\frac{a}{\left({}^t q_T (S_{Tu} - S_{TN} S_{NN}^{-1} S_{Nu})\right)} \cdot \frac{|t_2 - t_1|}{1+t_1} \cdot {}^t\begin{pmatrix} 0 \\ t_1 - t_2 \\ \vdots \\ t_1 - t_m \end{pmatrix} Q_T \left(S_{TT} - S_{TN} S_{NN}^{-1} S_{NT}\right) Q_T \cdot \begin{pmatrix} -1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$

$$= -\frac{a}{\left({}^t q_T (S_{Tu} - S_{TN} S_{NN}^{-1} S_{Nu})\right)} \cdot \frac{|t_2 - t_1|}{1+t_1} \cdot {}^t\begin{pmatrix} 0 \\ t_1 - t_2 \\ \vdots \\ t_1 - t_m \end{pmatrix} Q_T \left(S_{TT} - S_{TN} S_{NN}^{-1} S_{NT}\right) \cdot \begin{pmatrix} -q_1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$

If non-traded goods are substitutes for all other goods, as we have seen above, all the elements of the matrix $\left(-S_{TN} S_{NN}^{-1} S_{NT}\right) = \left(b_{ij}\right)$ are non-negative. Adding $S_{TT} = \left(a_{ij}\right)$, we obtain

$$du = -\frac{a}{\left({}^t q_T (S_{Tu} - S_{TN} S_{NN}^{-1} S_{Nu})\right)} \cdot \frac{|t_2 - t_1|}{1+t_1} \cdot {}^t\begin{pmatrix} 0 \\ t_1 - t_2 \\ \vdots \\ t_1 - t_m \end{pmatrix} Q_T \begin{pmatrix} -(a_{11} + b_{11}) \cdot q_1 \\ -(a_{21} + b_{21}) \cdot q_1 \\ \\ -(a_{m1} + b_{m1}) \cdot q_1 \end{pmatrix}$$

---

[10] When there exist l goods having the highest tariff rate, $(t_1 = t_2 = \cdots = t_l > t_{l+1} \geq \cdots \geq t_m > -1)$,

$dt_T = a|t_{l+1} - t_1| \cdot {}^t(-1, -1, \cdots, -1, 0, \cdots, 0) \in R^m$ where $0 < a \leq 1$. Here, the values of the first element through the l-th element in the vector are all (-1).

$$= \frac{a}{\left({}^{t}q_T(S_{Tu} - S_{TN}S_{NN}^{-1}S_{Nu})\right)} \cdot \frac{|t_2 - t_1|}{1 + t_1} \cdot \left((t_1 - t_2)(a_{21} + b_{21}) \cdot q_1 q_2 + \cdots + (t_1 - t_m)(a_{m1} + b_{m1}) \cdot q_1 q_m\right)$$

If the first good, having the highest tariff rate, is a substitute for all other goods (i.e. from the second good through the m-th good), then $a_{i1} > 0$. The following lemma holds: ***If no inferior good exists, non-traded goods are substitutes for all other goods, and the highest tariff rate good and all other traded goods are substitutes, then the trade policy to reduce the highest tariff rate to the level of the second highest tariff rate improves the country's welfare level.***

5. A Small Country Model Including Import Quotas

In the preceding section we generalized the basic model so as to include non-traded goods. In this section a model which includes import quotas will be constructed. In particular, when import of a good is zero due to a quota, the model built in this chapter will be identical to the model with non-traded goods developed in the previous chapter. Therefore, the model constructed here will, in a sense, be a generalization of the previously constructed[11] model.

Let us assume a small open economy enacting an import quota, resulting in two groups of goods, goods with tariff (T) and goods with quota (R).

Let the domestic price and world price vectors be defined by $p = \begin{pmatrix} p_T \\ \cdots \\ p_R \end{pmatrix}$, $q = \begin{pmatrix} q_T \\ \cdots \\ q_R \end{pmatrix}$, and let import quotas, compensated excess demand functions and the substitution matrix be represented by z, $s = \begin{pmatrix} S_T(p_T, p_R, u) \\ S_R(p_T, p_R, u) \end{pmatrix}$ and $S = \begin{pmatrix} S_{TT} & S_{TR} \\ S_{RT} & S_{RR} \end{pmatrix}$, giving $p_T = Q_T(e + t_T)$.

Then, the equilibrium condition is

[11] Strictly it is not a generalization, for we do not classify traded goods by type of import restriction such as tariff or quota.

$$\begin{cases} {}^t q_T s_T + {}^t q_R s_R = 0 \\ s_R = z \end{cases}.$$

Totally differentiating these equations,

$$\begin{cases} {}^t q_T s_{TT} dp_T + {}^t q_T s_{TR} dp_R + {}^t q_T s_{Tu} du + {}^t q_R s_{RT} dp_T + {}^t q_R s_{RR} dp_R + {}^t q_R s_{Ru} du = 0 \\ s_{RT} dp_T + s_{RR} dp_R + s_{Ru} du = dz \end{cases} \tag{9}$$

and from the second equation,

$$dp_R = S_{RR}^{-1}(dz - S_{RT} dp_T - s_{Ru} du).$$

Substituting this into the first equation and rearranging it, we have

$$Edu = -{}^t q_T (S_{TT} - S_{TR} S_{RR}^{-1} S_{RT}) dp_T - ({}^t q_T S_{TR} S_{RR}^{-1} + {}^t q_R) dz \tag{10}$$

$$E \equiv {}^t q_T (s_{Tu} - S_{TR} S_{RR}^{-1} s_{Ru}) = {}^t q_T \begin{pmatrix} \mathrm{I} & \vdots & -\mathrm{S}_{\mathrm{TR}} S_{RR}^{-1} \end{pmatrix} \begin{pmatrix} S_{Tu} \\ \cdots \\ S_{Ru} \end{pmatrix}$$

First consider the case of tariff liberalization under quota implementation. When an import quantity is fixed at a given level ($dz = 0$), from equation (9)

$$\begin{cases} {}^t q_T s_{TT} dp_T + {}^t q_T s_{TR} dp_R + {}^t q_T s_{Tu} du = 0 \\ s_{RT} dp_T + s_{RR} dp_R + s_{Ru} du = 0 \end{cases}$$

As mentioned previously this is the same equation as in the model including non-traded goods. Hence, if we rewrite the condition such that no inferior good exists and the imported goods under quota and all of the other goods are substitutes, then the argument of the small country model including non-traded goods holds with no modification. Also, even though there exists a distortion such as a quota, the optimal tariff rate becomes zero and the quota does not influence any other markets. Accordingly, the distortion, i.e. quota, can not be a justification for restricting trade by implementing tariffs on the other goods.

Next, let us analyze a change in welfare level when the quota level changes with no change in tariff rates. Substituting $dp_T = 0$ into (10) and using both conditions of homogeneity of degree zero and symmetry of the substitution matrix[12],

$$Edu = \left({}^t(p_R - q_R) + {}^t(p_T - q_T)S_{TR}S_{RR}^{-1} \right) dz \tag{11}$$

This shows that the effects of a change in quota level to country's welfare level are divided into a direct effect, $({}^t(p_R - q_R)dz)$, plus an indirect effect, $({}^t(p_T - q_T)S_{TR}S_{RR}^{-1}dz)$.

Under the conditions that $dp_T = 0$ and $du = 0$, from $dz = S_{RR}dp_R$, if the quota is binding[13], $dp_R = S_{RR}^{-1}dz$. Since $dS_T = S_{TR}dp_R = S_{TR}S_{RR}^{-1}dz$, the indirect effect is (keeping the utility level constant) a change in tariff revenue caused by a change in demand for goods on which tariffs are enacted. This change in tariff revenue results from the change in domestic prices of goods on which quotas are enacted when the level of the quotas is altered.

When a quota level on good $j \in R$ is relaxed, if the demand for goods $i \in T$ on which tariffs are enacted decreases (increase), then we define this to be a substitutionary (complementary) relation $\left(\frac{\partial S_i}{\partial z_j} < 0 \ (> 0)\right)$ [14] and as shown, the effect of a change in quota on welfare level is indeterminate.

Let us evaluate three cases of quota change. Firstly, the tariff rate is zero $(p_T = q_T)$. In this case, (11) becomes

$$Edu = {}^t(p_R - q_R)dz = \sum_{i \in R}(p_i - q_i)dz_i$$

[12] From the conditions of homogeneity of degree zero and symmetry of the substitution matrix, we have

$$\begin{cases} S_{TT}p_T + S_{TR}p_R = 0 \\ S_{RT}p_T + S_{RR}p_R = 0 \end{cases}$$

Multiplying both sides of the second equation by S_{RR}^{-1} and transposing the matrix, we have ${}^t p_R + {}^t p_T S_{TR}S_{RR}^{-1} = 0$, and then subtract this from (10).

[13] When we say that a quota implemented on good ($j \in R$) is binding, it means $p_j > q_j$. That is, it is the case that the domestic price of the good changes when the quota level changes.

[14] The *substitutionary relation* defined here is different from *substitute* which we have used as a pure substitute relation. Needless to say, $\frac{\partial S_i}{\partial z_j} = \left(S_{TR}S_{RR}^{-1}\right)_{ij}$.

and there exists no indirect effect. When $E > 0$, if there exists a good on which a quota is binding, then the relaxation of the quota will improve the country's welfare level. If there exists no good on which a quota is binding, then the relaxation of the quota will not improve welfare. On the other hand, when decreasing a quota level, the level of welfare will fall when the restriction becomes binding.

That is, ***when there exists a good whose tariff rate is zero and quota is binding, if there exist no inferior goods and goods on which quotas are enacted are substitutes for all other goods, then relaxation of quotas will improve the welfare level.***

The following case is that of non-zero tariff rates on all goods. In this case there exists an indirect effect. First, let us examine the simple case where no binding quota exists (i.e. $p_R = q_R$). Then, we have

$$Edu = {}^t(p_T - q_T) S_{TR} S_{RR}^{-1} dz = \sum_{j \in R} \sum_{i \in T} (p_i - q_i)\left(S_{TR} S_{RR}^{-1}\right)_{ij} dz_j \ .$$

The effect of the quota depends on the matrix $S_{TR} S_{RR}^{-1}$. If no inferior good exists and all of the goods on which quotas are enacted are substitutes for all other goods, then $E > 0$, $S_{TR} > 0$ and $S_{RR}^{-1} \le 0$. Therefore, we obtain the following lemma. ***In the case where all tariff rates are non-zero and there exists no binding quota, if there exists no inferior good and all goods on which quotas are imposed are substitutes for all of the other goods, then the welfare level will increase by introducing binding quotas on any goods on which quotas are imposed.***

Finally, let us investigate the case of binding quotas. When binding quotas are present, we need to evaluate both direct and indirect effects at the same time. The equation to evaluate the change in welfare level is

$$\begin{aligned} Edu &= \left({}^t(p_R - q_R) + {}^t(p_T - q_T) S_{TR} S_{RR}^{-1}\right) dz \\ &= \left({}^t(p_R - q_R) + \left(\sum_{i \in T} (p_i - q_i)\left(S_{TR} S_{RR}^{-1}\right)_{ij}\right)\right) dz \\ &= \sum_{j \in R} \left((p_j - q_j) + \sum_{i \in T} (p_i - q_i)\left(S_{TR} S_{RR}^{-1}\right)_{ij}\right) dz_j \end{aligned}$$

$$= \sum_{j \in R} \left((p_j - q_j) + \sum_{i \in T} (p_i - q_i) \frac{\partial S_{Ti}}{\partial z_j} \right) dz_j \, . \qquad (12)$$

Accordingly, under the condition that $E > 0$ for both all $i \in T$ and all $j \in R$, when $\partial S_i / \partial z_j > 0$, i.e. the goods on which quotas are imposed and the goods on which tariffs are imposed are subject to the substitutionary relation, the relaxation of quota level will improve welfare. However,we should consider whether or not the conditions $E > 0$ and $\partial S_i / \partial z_j > 0$ (for all $i \in T$ and all $j \in R$) are consistent. The condition we used, that there exists no inferior good and all of the goods on which quotas are imposed are substitutes for all other goods, is the sufficient condition for $E > 0$. This is also the condition used to define the goods on which tariffs are imposed and the goods on which quotas are imposed as subject to the substitutionary relation. That is, when all of the tariff rates are non-zero and there exist binding quotas, if there exists no inferior good, and all of the goods on which quotas are imposed are substitutes for all other goods, then the effects on welfare are indeterminate because the direct effect and the indirect effect work in opposite directions. Modifying the equation for evaluating welfare change, we can see which quota should be relaxed.

$$\begin{aligned}
Edu &= \left({}^t(p_R - q_R) + {}^t(p_T - q_T) S_{TR} S_{RR}^{-1} \right) dz \\
&= \sum_{j \in R} \left((p_j - q_j) + \sum_{i \in T} (p_i - q_i) \left(S_{TR} S_{RR}^{-1} \right)_{ij} \right) dz_j \\
&= \sum_{j \in R} p_j \left(\left(\frac{p_j - q_j}{p_j} \right) + \sum_{i \in T} \frac{p_i}{p_j} \left(\frac{p_i - q_i}{p_i} \right) \left(S_{TR} S_{RR}^{-1} \right)_{ij} \right) dz_j \\
&= \sum_{j \in R} p_j \left(\tilde{t}_j^R - \sum_{i \in T} (-1) \frac{p_i}{p_j} \left(S_{TR} S_{RR}^{-1} \right)_{ij} \cdot \tilde{t}_i^T \right) dz_j
\end{aligned}$$

where

$$\tilde{t}_j^R = \frac{p_j - q_j}{p_j} \quad (j \in R)$$ (tariff equivalent to domestic price) and

$$\tilde{t}_i^{\,T} = \frac{p_i - q_i}{p_i} \quad (i \in T) \quad \text{(tariff rate measured by domestic price).}$$

Moreover, when all goods subject to quota are substitutes for all other goods,

$$d_{ij} = -\frac{p_i}{p_j}\left(S_{TR}S_{RR}^{-1}\right)_{ij} \quad (i \in T,\ j \in R)$$

This is non-negative and the sum for all $i \in T$ is

$$\sum_{i \in T} d_{ij} = -\frac{1}{p_j}\sum_{i \in T} p_i \left(S_{TR}S_{RR}^{-1}\right)_{ij} = -\frac{1}{p_j}\cdot {}^t p_T \cdot j^{th} column of\ S_{TR}S_{RR}^{-1}.$$

Since from ${}^t p_R + {}^t p_T S_{TR}S_{RR}^{-1} = 0$, obtained from the homogeneity of the substitution matrix, this sum equals zero, where d_{ij} can be treated as a weight. Then, the equation to evaluate the welfare change becomes

$$Edu = \sum_{j \in R} p_j \left(\tilde{t}_j^{\,R} - \sum_{i \in T} d_{ij} \cdot \tilde{t}_i^{\,T} \right) dz_j .$$

This result shows that a relaxation of the quota on the good whose tariff-equivalent domestic price is higher than the highest tariff rate measured by domestic price will improve the welfare level.

That is, ***assuming that there exists no inferior good and all of the goods with quotas are substitutes for all other goods, a relaxation of the quota on the good whose tariff-equivalent domestic price is higher than the highest tariff rate measured by domestic price will improve the welfare level.***

6. Conclusion

In this paper, we focused on the case in which import restrictive trade policy results in distortions that cause differences between world prices and domestic prices and examined options to alter trade policy without lowering welfare level In particular, in the case of a more-than-three-good model, we showed that a relaxation of a restriction in order to reduce a given distortion can increase the effects of another distortion and does not necessarily improve the welfare level.

Let us summarize some of the sufficient conditions to increase welfare level derived in the paper.

First, if no inferior good exists, and if the good whose tariff rate is to be reduced and all the other goods are pure substitutes, then alteration of trade policy, either to reduce the highest tariff rate to the level of the second highest tariff rate or to bring all the tariff rates proportionately close to a given level, will improve the welfare level. Secondly, even when there exist non-traded goods, the policy to bring all the tariff and subsidy rates proportionately close to zero improves the country's welfare level. Lastly, in the case that quotas exist, if the quota level is fixed at a given level, the results are identical to the case of a small open economy with non-traded goods. When a quota level is altered without changes in tariff rates, the effects of the change in the quota level on the country's welfare level is divided into direct effects and indirect effects; the total effect on welfare level is indeterminant.

References

Bhagwati, J ; " Immiserizing Growth: A Geometrical Note." *Review of Economic Studies* 25, 1958.

Bhagwati, J; " Distortions and Immiserizing Growth: A Generalization, "*Review of Economic Studies* 35, 1968.

Bhagwati, J.;."The Generalized Theory of Distortions and Welfare." In Jagdish N. Bhagwati, 1971 *Ronald W. Jones, Robert A. Mundell, and Jaroslav Vanek* (eds.).*Trade, Balance of Payments, and Growth: Papers in International Economics in Honor of Charles P. Kindleberger.* Amsterdam: North-Holland.

Bhagwati, Jagdish and V. K. Ramaswami." Domestic Distortions, Tariff, and the Theory of Optimum Subsidy." *Journal of Political Economy* 71, pp. 44-50. 1963.

Bhagwati, Jagdish, V. K. Ramaswami and T. N. Srinivasan. "Domestic Distortions, Tariff, and the Theory of Optimum Subsidy: Some Further Results." *Journal of Political Economy* 77, pp.1005-10, 1962.

Corden, W. M. and Rodney E. Falvey; " Quotas and Second Best." *Economics Letters* 18, pp.67-70, 1985.

Davis, O. A. and A. B. Whinston ; " Piecemeal Policy in the Theory of Second Best." *Review of Economic Studies* 34, pp.323-331, 1965.

Diewert W. E., A. H. Turunen-Red and A. D. Woodland ;" Tariff Reform in a Small Open Multi-Household Economy with Domestic Distortions and Non Traded Goods." *International Economic Review* 32, pp.937-957, 1991.

Dixit, Avinash ; ." Tax Policy in Open Economies." In A. J. Auerbach and M. Feldstein (eds.) *Handbook of Public Economics* 1 , 1985, North Holland.

Falvey, Rodney E. " Tariffs, Quotas and Piecemeal Policy Reform." *Journal of International Economics* 25, 177-183., 1988.

Foster , Edward and Sonnenschein ;" Price Distortion and Economic Welfare." *Econometrica* 38, pp. 281-297, 1970.

Fukushima, Takashi;"Tariff Structure, Non Traded Goods and theory of Piecemeal Policy Recommendations." *International Economic Review* 20, 427-435, 1979.

Hatta, Tatsuo; " A Recommendation for A Better Tariff Structure." *Econometrica* 45, pp.1859-1869., 1977.

Hatta, Tatsuo ." A Theory of Piecemeal Policy Recommendations." *Review of Economic Studies* , pp. 1-21, 1977.

Hawkins, D. and H.A. Simon (1949)." Note: Some Conditions of Macroeconomic Stability." *Econometrica* 17, pp. 245-248. 1949.

Johnson, H.G. ."The Possibility of Income Losses from Increased Efficiency of Factor Accumulation in the Presence of Tariffs." *Economic Journal* 77, 1967.

Kawamata, K.; " Price Distortion and the Second Best Optimum" *Review of Economic Studies* 44, pp.23-29.,1977.

Kowalczyk, Carsten; " Trade Negotiations and World Welfare, "*American Economic Review* 79, pp.552-559., 1989.

Lipsey, R.G. and K. Lancaster. "The General Theory of Second Best." *Review of Economic Studies* 24, 1954.

Lloyd, P. J.; "A More General Theory of Price Distortions in Open Economies." *Journal of International Economics* 4, pp.365-386., 1974.

Samuelson, P.A.;"The Gains from International Trade." *Canadian Journal of Economics and Political Science* 5, 1939.

Thomas W. Hertel ed; *Global Trade Analysis*, Cambridge University Press, 1997.

Tyers, Rod and Rod Falvey; "Border Price Changes and Domestic Welfare in the Presence of Subsidised Exports." *Oxford Economic papers* 41, pp. 434-451. 1989.

Wong, Kar-yiu. ; *International Trade in Goods and Factor mobility.* MIT Press. 1995.

Woodland, Alan D. *International Trade and Resource Allocation.* Amsterdam: Northholland, 1982.