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Administrative costs and optimal diversions from free trade in a small open economy

by

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As is broadly recognized, as a straightforward application of the Diamond-Mirrlees (1971) production efficiency theorem, that even when lump sum taxation is not available, it is optimal in a small open economy for the government to rely on taxes on the net demand of households rather than border taxes. However, for production efficiency, and by implication for free trade, to be desirable it most be possible to tax all market transaction at no costs. It is not likely for this condition to be satisfied, especially in developing countries and in new market economies where tax administration is associated with considerable costs. There is thus a need to provide guidance to the design and reform of tax-tariff systems taking into account the costs of administration. The paper address this challenge by characterising optimal tax-tariff systems taking and by identifying desirable tax-tariff reforms to achieve fiscal and distributional objectives in response to changes in the administrative costs and in response to international trade agreements restricting the use of tariffs

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1. Introduction

Even when lump sum taxation is not available, it is, as a corollary to the Diamond-Mirrlees (1971) production efficiency theorem, optimal in a small open economy for the government to rely on taxes on the net demand of households rather than border taxes (see for example Dixit 1985).

Stiglitz and Dasgupta (1971) undertook for an economy with one representative household an assessment of the robustness of the Diamond-Mirrlees' efficiency theorem. They established that production efficiency will not necessarily be desirable if certain tax instruments cannot be used suggesting that the conditions for the theorem to be valid are often not satisfied because in the real world all tax instruments cannot be used. One striking implication of their analysis is that under a general income tax, taxing all types of income at the same rate, productive efficiency may not be desirable. Considering the pervasiveness of general income taxation it is remarkable that this implication has been given so little attention in the literature. A possible explanation is that the original contributions[1] did not take distributional considerations into account and did not provide any explanation why certain tax instruments could not be used. Adopting a framework with heterogeneous households and an inequality averse government several authors have, however, recently given attention to the distributional aspect, Munk (1998) assuming a linear income tax, and Naito (1999) and Graube (2000) assuming a general income tax and shown that to achieve distributional objectives it is quite likely that a government will create production inefficiency.

The conditions for production efficiency and free trade to be desirable is far from satisfied in developing countries and in new market economies where tax administration is associated with considerable costs. There is thus a need to provide guidance for the design and reform of domestic tax-tariff systems taking into account the administrative costs of tax collection.

The condition for a tax-tariff system to be optimal is under the assumption that all market transaction and profit can be taxed at no costs fairly well understood (Dasgupta and Stiglitz 1974, Dixit and Norman 1980, Dixit 1985). The optimal tariff structure when tariffs is the only source of government has also been very well exposed by Hatta and Ogawa (2003)

There is also a considerable literature on desirable tax reform based on the Diamond-Mirrlees framework. Hatta (1977) made a seminal contribution to the analysis of the welfare effects of tariff reform when changes in government tax revenue can be made up by lump sum transfers. Although important in opening up the area for theoretical investigation the assumption that the government's revenue requirement can be adjusted by lump sum transfers clearly limit the practical usefulness of results. Subsequent contributions have taken into account that the revenue forgone by tariff reductions has to be replaced by tax revenue generated by other distortionary taxes in general within a framework where free trade is the ultimate aim of such reform, but provide little insight into desirable directions of coordinated tax-tariff reforms (see Keen and Ligthart 1999 on this point and for references).

Keen and Ligthart (1999) have explored the robustness of the Dixit and Norman results on coordinated tariff-tax reform focusing on considering piecemeal reform, but within a framework where distributional considerations and administrative costs are not taken into consideration, thus as

[1] The original analysis by Stiglitz and Dasgupta was recast using duality theory in Munk (1980).

the experience from the analysis of the optimal tax structure in a closed economy has shown severely limiting the scope for deriving relevant policy recommendations.

Anderson (1999) has developed a promising framework based on compensated marginal costs of funds for taking into account distributional considerations in identifying desirable directions of tax-tariff reform, but more as a framework to be used in connection with simulations with CGE models than for deriving general guidelines for reform.

Recent CGE simulation studies by Munk (1998) for a closed economy and by Anderson (1997, 1999) and Erbil (2002) for an open economy suggest that optimal diversion from production efficiency under realistic assumptions about the instruments available to the government may be quantitatively significant.

On this background it seems reasonable to conclude that there is a need for analysis of what characterise the optimal tax system in small open economies taking into account both administrative costs and distributional considerations. Although in practice guidelines for piecemeal tax reform in the end undoubtedly will turn out to be more relevant (if not based on theoretical considerations as advocated Keen and Ligthart 1999, but based on CGE simulations as advocated by Anderson 1999), experience shows that insight into desirable direction of reform (especially based on CGE simulations) critically depends on insight into what constitutes the optimal solution.[2] This paper will therefore focus on characterising the optimal tax-tariff system under realistic assumptions, i.e. by taking into account administrative costs.

The paper is structured as follows. Section 2 sets out the model framework which takes into account that different tax structures are associated with different levels of administrative costs. Section 3 characterises the optimal tax system under alternative assumptions about which tax-tariff instruments are feasible generalising Hatta and Ogawa (2003) analysis. Section 5 briefly considers the application of the framework to explain agricultural policies, and Section 6 concludes. Details on the derivation of equilibrium conditions and first order conditions for a social optimum are provided in a technical appendix.

2. The government's maximisation problem with tax-tariff constraints

We consider a small open economy comprising of H heterogeneous households, N perfectly competitive production sectors operating under non-increasing returns to scale each producing only one sector specific output, and a government. There is one primary factor and N produced commodities, all tradable.

Each sector only uses the primary factor as input and are confronted with producer prices $\mathbf{p}^{\mathrm{j}} \equiv \left(p_0^{\mathrm{j}}, p_{\mathrm{j}}^{\mathrm{j}} \right)$ $j \in \mathrm{A} \equiv (1,.,\mathrm{N})$. Profit maximisation thus yields profit functions $\Pi^j(p_0^{\mathrm{j}}, p_{\mathrm{j}}^{\mathrm{j}})$ $\mathrm{j} \in \mathrm{A}$.

[2] Consider for example the insight in optimal taxation provided by Corlett and Hague (1953) help clarify the often confused discussion on the double dividend (see for example Munk 2000). See also Hatta (1994) and Hatta and Ogawa (2003).

By *Hotelling Lemma* net supply functions are $\Pi_i^j\left(p_0^{\mathrm{j}}, p_{\mathrm{j}}^{\mathrm{j}}\right) \equiv \frac{\partial \Pi_i^j}{\partial p_i}\left(p_0^{\mathrm{j}}, p_{\mathrm{j}}^{\mathrm{j}}\right)$, i$\in$ *(0,j)* . The production sectors' net supply vectors are $\left(-v_0^j, Y^j\right)$, j$\in$ A .

The preferences of the households are characterised by expenditure functions, $E^{\mathrm{h}}\left(\mathbf{q}, u^{\mathrm{h}}\right)$, $\mathrm{h} \in H \equiv (1,.,\mathrm{H})$, defined over household prices $\mathbf{q} \equiv \left(q_0, \ldots, q_{\mathrm{N}}\right)$ and utility u^{h}. By Shephard's lemma, net demand is given by $E_j^h\left(\mathbf{q}, u^{\mathrm{h}}\right) \equiv \frac{\partial E^h}{\partial q_j}\left(\mathbf{q}, u^{\mathrm{h}}\right)$, h$\in H$, j$\in C$. The h^th^ household's share of the profit in the k^th^ production sector is a_k^h. The households' net demand vectors are $\mathbf{x}^{\mathrm{h}} \equiv \left(x_0^h, x_1^h, .., x_{\mathrm{N}}^h\right)$, h$\in$*H.*

The government's resource requirements, $\mathbf{x}^{\mathrm{G}} \equiv \left(x_0^G, x_1^G, .., x_{\mathrm{N}}^G\right)$, is financed by revenue derived from different sources: household taxes, $\left(t_0, t_1, \ldots, t_{\mathrm{N}}\right)$, on the supply of the primary factor and on the consumption of produced commodities and a uniformlump sum tax, L; sector specific producer taxes $(t_0^k, t_{\mathrm{k}}^k)$, $k \in A$ and profit taxes $\left(t^1, \ldots, t^{\mathrm{N}}\right)$; and taxes on international trade flows, $\mathbf{t}^{\mathrm{W}} \equiv \left(t_0^W, t_1^W, \ldots, t_N^W\right)$. World market prices are $\mathbf{p}^W \equiv \left(p_1^W, \ldots, p_{\mathrm{N}}^W\right)$. Domestic market prices are defined relative to world market prices as $\mathbf{p} \equiv \left(p_0, p_1, \ldots, p_{\mathrm{N}}\right) = \left(p_0^W + t_0^W, p_1^W + t_1^W, .., p_{\mathrm{N}}^W + t_{\mathrm{N}}^W\right)$. Domestic prices are defined relative to market prices: Household prices as $\mathbf{q} \equiv \left(p_0 + t_0, p_1 + t_1, .., p_{\mathrm{N}} + t_{\mathrm{N}}\right)$ and producer prices as $\mathbf{p}^{\mathrm{k}} \equiv \left(p_0 + t_0^{\mathrm{k}}, p_{\mathrm{k}} + t_{\mathrm{k}}^{\mathrm{k}}\right)$ k$\in A$[3]. As a matter of normalisation we assume that $p_0 = 1$.

The level of commodity taxation for domestic households may be expressed by ratios between household prices and market prices,

$$T_i \equiv q_i / p_i = (t_i + p_i) / p_i \qquad i \in C$$

for the different production sectors by ratios between producer prices and market prices,

$$T_j^k \equiv p_j^k / p_j = (t_j^k + p_j) / p_j \qquad k \in A \;\; j \in (0,k)$$

and for the foreign sector by ratios between world market prices and market prices,

$$T_j^W \equiv p_j / p_j^w = (t_j^W + p_j^W) / p_j^W \qquad j \in C$$

[3] The sign convention is thus: the tax rate on the net demand of the primary factor is negative when the supply is taxed, the tax rate on net demand of commodity k by sector k is positive when the output is taxed and negative when the input is taxed; and trade taxes are in the case of export subsidies and import taxes (tariffs) positive and in the case of export taxes and import subsidies negative.

A *tax-tariff system* $x \equiv \left(\mathbf{T}, (\mathbf{T}^k, \mathrm{k} \in \mathrm{C}), \mathbf{t}, L, \mathbf{T}^W\right)$, where $\mathbf{T} \equiv \left(T_0, ..., T_\mathrm{N}\right)$, $\mathbf{T}^k \equiv (T_1^k, ..., T_\mathrm{N}^k)$, $\mathrm{k} \in A$, $\mathbf{t} \equiv \left(t^1, ..., t^\mathrm{N}\right)$, may be characterised by the type of *tax-tariff structure* to which it belongs. A tax-tariff structure i, $\mathbf{?}^i$, is a set of tax systems, $x_i^j \in \mathbf{?}^i$, $j \in \mathrm{S}^i$, where the same restrictions are imposed on the set of tax instruments. The number of tax-tariff structures $i \in \mathrm{F}$, and the number of tax-tariff systems, $j \in \mathrm{S}^i, i \in \mathrm{F}$, are assumed to be finite. The administrative costs associated with tax-tariff systems belonging to the same tax-tariff structure are assumed to be associated with the same administrative costs. The government’s resource requirements other than for tax administration are considered exogenous. The government's total resource requirement, including the administrative costs associated with a tax-tariff system *i*, $\mathbf{x}^G\left(\mathbf{?}^i\right)$ is therefore independent of market prices $(\mathbf{p}, \mathbf{w})$. Furthermore we assume that the cost of tax-tariff administration, $B\left(\mathbf{?}^i\right)$, are greater the more differentiated the tax structure, i.e. if a set of tax rates are all zero or proportionally all the same, then the administrative costs of taxation are smaller than if they are differentiated.

The conditions for equilibrium may be expressed as (see the Technical appendix (TA) for details on the derivation or these conditions from the underlying model)

$$E^h\left(\mathbf{p}^W + \mathbf{t}^w + \mathbf{t},\ u^h\right) = \sum_{j \in A} (1 - t^j)\, a_j^h \Pi^j\left(p_0^W + t_j^W + t_0^j, p_j^W + t_j^W + t_j^j\right) - L \qquad \mathrm{h} \in H \tag{1}$$

which for each household requires the levels of individual utilities to be consistent with the level of unearned income, $I^h = \sum_{j \in A} (1 - t^j) a_j^h \Pi^j\left(p_0^W + t_j^W + t_0^j, p_j^W + t_j^W + t_j^j\right) - L$,; and

$$\begin{aligned}
\sum_{\mathrm{j} \in \mathrm{C}} p_j x_j^G\left(\mathbf{?}^i\right) = & \sum_{\mathrm{i} \in \mathrm{C}} t_i \sum_{h \in \mathrm{H}} E_\mathrm{i}^\mathrm{h}\left(\mathbf{p}^W + \mathbf{t}^w + \mathbf{t}, u^h\right) + \mathrm{H}L \\
& + \sum_{\mathrm{j} \in \mathrm{A}} \sum_{i \in (0,k)} t_\mathrm{i}^\mathrm{j} \Pi_i^j\left(p_0^W + t_j^W + t_0^j, p_j^W + t_j^W + t_j^j\right) + \sum_{j \in A} t^j \Pi^j\left(p_0^W + t_j^W + t_0^j, p_j^W + t_j^W + t_j^j\right) \\
& + \sum_{\mathrm{i} \in A} t_\mathrm{i}^\mathrm{W}\left(\Pi_i^i\left(p_0^W + t_j^W + t_0^j, p_j^W + t_j^W + t_j^j\right) - \sum_{h \in \mathrm{H}} E_i^h\left(\mathbf{p}^W + \mathbf{t}^w + \mathbf{t}, u^h\right) - x_i^G\left(\mathbf{?}^i\right)\right)
\end{aligned} \tag{2}$$

which represents the government’s budget constraint.

In finding an optimal solution reflecting administrative costs, we consider tax structures defined by the following restrictions on the government’s choice of commodity tax rates and tariff rates[4]

$$\{\, q_i / p_i = T_\mathrm{i}\ ,\ \mathrm{i} \in C \,\} \tag{3}$$

[4] A number of contributions assume as a matter of normalisaton one primary facor as untaxed, eg. Stiglitz and Dasgupta (1971) Lack of clarity with respect to when without loss of generality normalisation one commodity can be assumed untaxed often leads to misunderstandings with respect to the interpretation of the optimal tax structure (see Munk 1982). It is therefore important to emphasise that when the household receive profit income it is not possible as a matter of normalisation to assume one commodity untaxed (see Munk 1978).

which for produced commodities constrain the differences between household prices, q_i , and producer prices

$$\{ p_j^k / p_j = T_j^k , k \in A , j \in (0,k) \} \tag{4}$$

which constrain the differences between sector specific producer prices, p_j^k , and market prices; p_j ;

$$\{ p_j / p_j^w = T_j^W , j \in C \} \tag{5}$$

which for produced commodities constrain the differences between market prices p_j and world market prices, p_j^w .

The government maximises in a two-step procedure a Pareto social welfare function, $W\left(u^1, u^2, .., u^H\right)$, with respect to u^h, $(h = 1, .., H)$, L; t_i, $\left(i = 0, .., N\right)$; t_j^k, $k \in \left(1, ., N\right)$, $j \in (0, k)$; t^k, $\left(k = 0, .., N\right)$ and t_i^W, $i = 1, .., N$ subject to the constraints representing the structure of the economy, including the costs associated with tax administration[5]: First, it calculates the optimal tax system, x_i^*, for each tax structure, $i \in F$; then, in the second step, it chooses that tax structures, **?** **, which allows the highest level of social welfare to be attained, and thus by implication the overall optimal tax system, x_{**}^* .

3. The optimal tax- tariff system disregarding administrative costs (? ^{0})

That a tax-tariff structure where all market transactions and profit can be taxed should be optimal seems based on casual observations very unlikely. If for example it is desirable to tax the return of different primary factor and profits at the same rate, then this would at variance with this assumption which in general would require the rates on which different sources of factor income were taxed to be differentiated taxation. However, the understanding of what determines the optimal tax structure under the assumption that all market transaction and profit can be taxed at no costs provides a very helpful benchmark for the analysis under more realistic assumptions.

As the equilibrium conditions in this case are homogenous of degree zero in market prices, producer prices and household prices one commodity, say the supply of the primary factor, when all commodities can be taxed at no costs one commodity can be assumed untaxed as a matter of normalisation.

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[5] The maximisation problem is formulated such that $u^1, u^2, .., u^H$ and e technically are choice variables (Mirrlees (1976), Dixit and Munk 1977, Munk (1978), Munk (1980), Hatta (1993). This naturally should not be interpreted that these variables are instruments to be fixed by the government

3.1 The general case

When the profit tax rates are chosen optimally (see TA 2.19)

$$\sum_{i\in(0,j)} (t_{\mathrm{i}}^{\mathrm{j}} + t_{\mathrm{i}}^{\mathrm{W}})\Pi_{ij}^{j} \qquad =0 \qquad j=0,1,.,N \qquad (6)$$

These conditions are satisfied for $t_{\mathrm{i}}^{\mathrm{j}}$ =0 j=1, ., N, i=0,j $t_{\mathrm{i}}^{\mathrm{W}} = 0$, i=0,1, ., N. Therefore production efficiency is desirable, as could be deduced directly from the Diamond-Mirrlees (1971) production efficiency theorem. The optimal values of the commodity tax rates, t_{i} , must thus satisfy (see TA 2.15)

$$\sum_{i\in A} t_i \sum_{h\in \mathrm{H}} E_{ik}^{h} = -\frac{(\lambda - \mathrm{R_k})}{\lambda}\sum_{\mathrm{h=1}}^{\mathrm{H}} E_k^h \qquad j=1,.,N \qquad (7)$$

Interpretation

The optimal tax structure will be determined by three considerations: 1) not to distort the pattern of consumption of produced commodities, 2) the desire to discourage the untaxed consumption of the endowment and 3) to tax commodities with high $\mathrm{R_k}$ less than commodities with low $\mathrm{R_k}$, i.e. to tax commodities primarily consumed by the rich and by household with a high propensity to consume low taxed good higher than commodities primarily consumed by the poor and by household with a high propensity to consume high taxed goods (Myles 1985 and Munk 2002).

3.2 The case of only one representative household

In the case of only one representative household if the government's requirement is larger than the total value of the profits in all sectors, then it is optimal to tax profit at 100% and use distortionary taxes to raise the remaining revenue.

In this case (7) becomes

$$\sum_{i\in A} t_i E_{ik} = -\theta x_k \qquad \mathrm{k} \in \mathrm{C} \quad (8)$$

where $\theta = -\frac{(\lambda - m)}{\lambda}$.

or in matrix notation

$$\mathbf{Et}_{-0} = \mathbf{b}$$

where

$$\mathbf{E}^0 \equiv \begin{bmatrix} E_{11} & . & E_{1\mathrm{N}} \\ . & . & . \\ E_{\mathrm{N}1} & . & E_{\mathrm{NN}} \end{bmatrix},\ \mathbf{t}_{-0} \equiv \begin{bmatrix} t_1 \\ . \\ t_{\mathrm{N}} \end{bmatrix} \text{ and } \mathbf{b} \equiv \begin{bmatrix} -\theta x_1 \\ . \\ -\theta x_N \end{bmatrix}$$

By $E(\mathbf{q},u)$ being concave in **q,** $\{E_{ij}, i, j \in \mathrm{FC}\}$ is negative semi-definite. Assuming that the utility function is strictly quasi-concave $\mathbf{E}^0$ is negative definite and therefore $|\mathbf{E}^0|$>0.Therefore we can solve for the optimal commodity tax rates

$$\mathbf{t}_{-0} = \mathbf{E}^{0^{-1}}\mathbf{b} \tag{9}$$

using Cramer's Rule, i.e.

$$t_k = \frac{|\mathbf{E}_k^0|}{|\mathbf{E}^0|} \qquad k \in \mathrm{C} \tag{10}$$

where $\mathbf{E}_k^0$ is obtained from $\mathbf{E}^0$ by replacing the k$^{\text{th}}$ column by $\mathbf{b}$

In the case of many commodities it is difficult to identify the trade-off between the two objectives: 1) not to distort the pattern of consumption of produced commodities and 2) the desire to discourage the untaxed consumption of the endowment. However, in the case of an economy with only two produced commodities, as originally considered by Corlett and Hague (1953), the trade-off is easier to establish.

In the case of only two produced commodities

$$|\mathbf{E}^0| = \begin{vmatrix} E_{11} & E_{21} \\ E_{12} & E_{22} \end{vmatrix} = E_{11}E_{22} - E_{21}E_{12} \tag{11}$$

and

$$|\mathbf{E}_1^0| = \begin{vmatrix} -q X_1 & E_{12} \\ -q X_2 & E_{22} \end{vmatrix} = q\,(-E_{22}X_1 + E_{12}X_2) \tag{12}$$

$$|\mathbf{E}_2^0| = \begin{vmatrix} E_{11} & -q X_1 \\ E_{21} & -q X_2 \end{vmatrix} = q\,(-E_{11}X_2 + E_{21}X_1) \tag{13}$$

The optimal tax rates for the produced commodities are

$$t_1 = q\,\frac{\left(-E_{22}X_1 + E_{12}X_2\right)}{D} \tag{14}$$

$$t_2 = q\,\frac{\left(-E_{11}X_2 + E_{21}X_2\right)}{D} \tag{15}$$

where $D = E_{11}E_{22} - E_{21}E_{12} = \left|\mathbf{E}^0\right| > 0$, or

$$t_1 = q\,\frac{\left(e_{12} - e_{22}\right)}{e_{11}e_{22} - e_{21}e_{12}} > 0 \tag{16}$$

$$t_2 = q\,\frac{\left(e_{21} - e_{11}\right)}{e_{11}e_{22} - e_{21}e_{12}} > 0 \tag{17}$$

where $\varepsilon_{ij} \equiv E_{ij} \Big/ \frac{x_i}{q_j}$.

By the homogeneity of degree zero of compensated demand, $E_j(\mathbf{q},u)$, we have that $\sum_{j\in FC} e_{ij} = 0$, i$\in FC$, and therefore that $\varepsilon_{12} = -\varepsilon_{11} - \varepsilon_{10}$ and $\varepsilon_{21} = -\varepsilon_{22} - \varepsilon_{20}$. The optimal tax structure may therefore also be expressed as[6]

$$\frac{t_1 / q_1}{t_2 / q_2} = \frac{-e_{11} - e_{22} - e_{10}}{-e_{11} - e_{22} - e_{20}} \tag{18}$$

Interpretation

Which commodity will be taxed at the highest rate depends entirely on the sign of $e_{10} - e_{20}$ (representing the desire to discourage the untaxed consumption of the endowment), for given value of $-e_{11} - e_{22}$, the difference is the greater the greater the numerical value of $e_{10} - e_{20}$, and for given values e_{10} and e_{20} the difference is smaller the greater is $-e_{11} - e_{22}$ (representing the objective not to distort the pattern of consumption of produced commodities).[7]

[6] A formula similar to (18) was first obtained by Harberger (1964) and later published in Harberger (1974)
[7] See Munk (2002) for more details on this interpretation.

4. The optimal tax-tariff system taking administrative costs into account

4.1 *Optimal tax-tariff system when only border taxes are feasible* ?[1]

We now consider the case where government's revenue requirement can only be financed by border taxes. Border taxes are relatively easy to levy compared to domestic taxes as the number of transactions to be monitored typically are far smaller. The assumption that a tax structure where only border taxes are used is the optimal tax structure may therefore be pertinent to the design of tax-tariff systems for less developed countries with very weak administrative infrastructure.

4.1.1 The equilibrium conditions and normalisation

The conditions for equilibrium under this assumption may be expressed as (see TA)

$$E^h\left(\mathbf{p}^W+\mathbf{t}^w,\ u^h\right)=\sum_{j\in A} a_j^h \Pi^j\left(p_0^W+t_0^W,\ p_j^W+t_j^W\right) \qquad \text{h=1,..H} \tag{19}$$

and

$$\sum_{i\in C} t_i \sum_{h\in H} E_i^h\left(\mathbf{p}^W+\mathbf{t}^w,\ u^h\right) + \sum_{i\in A} t_i^W \left(\Pi_i^i\left(p_0^W+t_0^W,\ p_j^W+t_j^W\right) - \sum_{h\in H} E_i^h\left(\mathbf{p}^W+\mathbf{t}^w, u^h\right) - x_i^G\left(\mathbf{?}^i\right)\right) - \sum_{j\in C} p_j x_j^G\left(\mathbf{?}^i\right)=0 \tag{20}$$

The equilibrium conditions are unaffected by multiplying relative prices p_j / p_j^w by T. The allocation is therefore the same for { p_j / p_j^w =1, j$\in C$ }, i.e. without taxation as for a proportional tariff structure { $p_j / p_j^w = T \neq 1$, j$\in C$ }. A proportional tariff structure therefore cannot finance the government's resource requirement. We may thus without loss of generality assume that t_0^W =0. This may also be seen by considering the household's budget constraint (see Munk 1983 and Hatta and Ogawa 2003)

4.1.2 The general case

From TA 2.2 we have that

$$m^h = b^h + l\left(\sum_{i\in A} t_i^W \sum_{h\in H} \frac{\P x_i^h(\mathbf{q}, I^h)}{\P I^h}\right)$$

is the *net marginal social welfare of income for household h*.

From the first order conditions with respect to t_j^w, (see TA 2.8) we have

$$-\left(\sum_{h\in H}\mu^h a_j^h \Pi_j^j - \sum_{h\in H}\mu^h \sum_{h\in H} E_j^h\right) + l\left(\sum_{i\in A} t_i^W \left(\sum_{h\in H} E_{ij}^h - \Pi_{ij}^j\right) + \sum_{h\in H} E_j^h - \Pi_j^j\right) = 0 \qquad j=1,.,N \quad (21)$$

We define

$$R_k = \frac{\sum_{h\in H} m^h x_k^h}{\sum_{h\in H} x_k^h}, \text{ and}$$

$$P^j = \sum_{h\in H} m^h a_j^h$$

R_k *is the net distributional characteristics of the consumption of commodity k,* P^j *is the distributional characteristics of the profit in sector j.* We notice that the distributional characteristics are a weighted average of the household's net marginal social welfare where the weights are the households share of the total demand and of the share of profit in the sector in question, respectively.

Interpretation

Using these definitions we rewrite the first order conditions with respect to t_j^w, (21), (see TA 2.14) to obtain

$$-\left(P^j \Pi_j^j - R_j \sum_{h\in H} E_j^h\right) + l\left(\sum_{i\in A} t_i^W \sum_{h\in H} E_{ij}^h - t_j^W \Pi_j + \sum_{h\in H} E_i^h - \Pi_j\right) = 0 \quad j=1,.,N \qquad (22)$$

Interpretation

The optimal tax-tariff system is in this case determined by two considerations: the objective of increasing the real value of the profit in the sectors where $P^j = \sum_{h\in H} m^h a_j^h > l$ and of decreasing the value of the profit in the sectors where $P^j = \sum_{h\in H} m^h a_j^h < l$ (see Munk 1989); and the objective of indirectly taxing the untaxed consumption of the endowment, in this case the consumption of leisure. The optimal tariff structure thus depends on characteristics of both household net demand and net supply of the production sectors (see Munk 1978).

4.1.3 One representative household

The case of one representative household has been analysed by several authors, most comprehensively by Hatta and Ogawa (2003).

Equation (22) now becomes

$$\mu Z_j + l\left(\sum_{i\in C} t_i^W Z_{ij} + Z_j\right)=0 \qquad \text{j=1,.,N} \qquad (23)$$

which may be rewritten as

$$\sum_{i\in C} t_i^W Z_{ik} = -\frac{(\lambda - m)}{\lambda} Z_k \qquad \text{j=1, ., N} \qquad (24)$$

Using the same method as in the derivation of (18) we obtain (see also Hatta and Ogawa 2003)

$$\frac{t_i^W / q_1}{t_i^W / q_2} = \frac{-\tilde{e}_{11} - \tilde{e}_{22} - \tilde{e}_{10}}{-\tilde{e}_{11} - \tilde{e}_{22} - \tilde{e}_{20}} \qquad (25)$$

where $\tilde{\varepsilon}_{jj} \equiv Z_{ij} \Big/ \frac{Z_j}{q_j} = \frac{E_j}{Z_j}\varepsilon_{jj} - \frac{\Pi_j}{Z_j} z_{jj}$ and $\tilde{\varepsilon}_{j0} \equiv Z_{j0} \Big/ \frac{Z_j}{q_0} = \frac{E_0}{Z_j}\varepsilon_{j0} - \frac{\Pi_0}{Z_j} z_{j0}$ for j=1,2, and where $\tilde{\varepsilon}_{ij} \equiv E_{ij} \Big/ \frac{Z_i}{q_j} = \frac{E_i}{Z_j}\varepsilon_{ij}$ for $i \neq j$ (see Munk 1985) and $\tilde{\varepsilon}_{ij} \equiv E_{ij} \Big/ \frac{Z_i}{q_j} = \frac{E_i}{Z_j}\varepsilon_{ij}$.

Interpretation

The optimal tax tax-tariff system is in this case determined by two considerations: the objective to decrease the real value of the profit to the household and the objective of indirectly taxing the untaxed consumption of the endowment, in this case the consumption of leisure (see Munk 1978 and Munk 2002)

Which commodity will be taxed at the highest rate depends entirely on the sign of $\tilde{e}_{10} - \tilde{e}_{20}$ and, for given value of $-\tilde{e}_{11} - \tilde{e}_{22}$, the difference is the greater the greater the numerical value of $\tilde{e}_{10} - \tilde{e}_{20}$, and for given values $\tilde{e}_{10}$ and $\tilde{e}_{20}$ the difference is smaller the greater is $-\tilde{e}_{11} - \tilde{e}_{22}$. Notice that the optimal solution will involve export to be subsidies and import to be taxed. Production efficiency will not be desirable. If the case of 1) homothetic preferences in the produced commodities and separability between the their consumption and the untaxed consumption of the endowment and 2) a proportional tax Π_{j0}.

The cross elasticity $\tilde{\varepsilon}_{j0} \equiv Z_{j0} \Big/ \frac{Z_j}{q_0} = \frac{E_0}{Z_j}\varepsilon_{j0} - \frac{\Pi_0}{Z_j} z_{j0}$ will have different values for an imported and for an exported commodity. When the objective of indirectly taxing the untaxed consumption of the endowment dominates $t_i^W > 0$ for imports, and $t_i^W < 0$ for exports. This corresponds to the practice observed in many developing countries (see Munk 1998).

In the case of constant returns to scale producer prices become constant and the consumption of a produced commodity will either be entirely imported or domestically produced depending of whether the corresponding world market price is smaller or greater than the corresponding domestic prices. In this case there is no real profit which can be influenced by taxes and only the last consideration apply, i.e. the solution thus becomes identical to the Corlett and Hague (1953) result considered above.

4.2 Optimal tax-tariff system when only a tax on the primary factor and border taxes are feasible $?^2$

We now consider the case where government's revenue requirement can only be financed by a tax on the supply of the primary factor and by a border taxes. For less developed countries to establish a tax on primary factor income may be the first step in establishing a domestic tax system less distorting than border taxes.

4.2.1 The equilibrium conditions and normalisation

The conditions for an equilibrium under this assumption may be expressed as (see (1))

$$E^h\left(\mathbf{p}^W + \mathbf{t}^w + \tilde{\mathbf{t}},\ u^h\right) = \sum_{j\in A} a_j^h \Pi^j\left(p_0^W + t_0^W,\ p_j^W + t_j^W\right) \qquad \text{h=1,..H} \tag{26}$$

where $\tilde{\mathbf{t}} = (t_0, 0,...,0)$; and

$$\sum_{i\in C} t_i \sum_{h\in H} E_i^h\left(\mathbf{p}^W + \mathbf{t}^w + \tilde{\mathbf{t}},\ u^h\right) + \sum_{i\in A} t_i^W \left(\Pi_i^i\left(p_0^W + t_0^W,\ p_j^W + t_j^W\right) - \sum_{h\in H} E_i^h\left(\mathbf{p}^W + \mathbf{t}^w + \tilde{\mathbf{t}},\ u^h\right) - x_i^G\left(?^i\right)\right) - \sum_{j\in C} p_j x_j^G\left(?^i\right) = 0 \tag{27}$$

We may thus as for the tax structure $?^1$ without loss of generality assume that t_0^W =0.

4.2.2 The general case

From TA 2.2 we obtain

$$\mu^h = \beta^h + \lambda \left(t_0 \sum_{h \in H} \frac{\partial x_0^h(\mathbf{q}, I^h)}{\partial I^h} + \sum_{i \in A} t_i^W \sum_{h \in H} \frac{\partial x_i^h(\mathbf{q}, I^h)}{\partial I^h} \right)$$

as the *net marginal social welfare of income for household h.*

From the first order conditions with respect to t_0, (see TA 2.4),

$$\sum_{h \in H} \mu^h E_0^h - \lambda \left(t_0 \sum_{h \in H} E_{00}^h + \sum_{i \in A} t_i^W \sum_{h \in H} E_{i0}^h + \sum_{h \in H} E_0^h \right) = 0 \qquad k=1, .,N \ (28)$$

From the first order conditions with respect to t_j^w, j=1,.,N (see TA 2.8)

$$-\left(\sum_{h \in H} \mu^h \alpha_j^h \Pi_j^j - \sum_{h \in H} \mu^h \sum_{h \in H} E_j^h \right) + \lambda \left(t_0 \sum_{h \in H} E_{0j}^h + \sum_{i \in A} t_i^W \left(\sum_{h \in H} E_{ij}^h - \Pi_{ij}^j \right) + \sum_{h \in H} E_j^h - \Pi_j^j \right) = 0 \qquad j=1,.,N \ (29)$$

Interpretation

The optimal tax tax-tariff system is in this case determined by two considerations: the objective of increasing the real value of the profit in the sectors where $\Pi^j = \sum_{h \in H} \mu^h \alpha_j^h > \lambda$ and of decreasing the value of the profit in the sectors where $\Pi^j = \sum_{h \in H} \mu^h \alpha_j^h < \lambda$ and the objective of indirectly taxing the untaxed consumption of the endowment, in this case the consumption of leisure (see Munk 1978 and Munk 2002). This latter consideration will in this case be less important compared with the previous case, as the endowment used for domestic production and export will be taxed by the domestic factor tax. In this case the untaxed endowment is now only the consumption of (non-market use of time) leisure.

4.2.3 The case of one representative household

This corresponds to the framework analysed by several authors, most comprehensively by Hatta and Ogawa (2003).

Equation (28) now becomes

$$\mu E_0 - \lambda \left(t_0 E_{00} + \sum_{i \in A} t_i^W E_{i0} + E_0 \right) = 0 \qquad k=1, .,N \ (30)$$

which may be written as

$$t_0 E_{00} + \sum_{i \in C} t_i^W Z_{i0} \;\; = -\frac{(\lambda - m)}{\lambda} E_0$$

and (29) becomes

$$\mu Z_j + l\left(t_0 E_{0j} + \sum_{i \in A} t_i^W Z_{ij} + Z_j \;\right) = 0 \qquad \text{j=1,.,N} \qquad (31)$$

which may be rewritten as

$$t_0 E_{0k} + \sum_{i \in C} t_i^W Z_{ik} \;\; = -\frac{(\lambda - m)}{\lambda} Z_k \qquad \text{j=1, ., N} \qquad (32)$$

Interpretation

The generation of tax revenue will mainly rely on the primary income tax. The border taxes will still be used to decrease the real value of the profit to the household and indirectly to tax the untaxed consumption of leisure.

4.3 Optimal tax-tariff system when border taxes are not feasible $?^3$

We finally consider the optimal tax system when a country by its international trading partners or by international financial institutions have been forced to adopt free trade and where government's revenue requirement must now be financed by a tax on the supply of the primary factor and by differential taxes/subsidies on sectoral income. For less developed countries to tax on sectoral income may be a natural extension of the tax on primary factor income. Differential taxation of sectoral income may serve as an instrument to achieve the distributional objectives which under the tax structures $?^1$ and $?^2$ is achieved by border taxes.

The conditions for an equilibrium under this assumption may be expressed as

$$E^h\left(\mathbf{p}^W + \tilde{\mathbf{t}},\; u^h\right) = \sum_{j \in A} a_j^h \Pi^j\left(p_0^W + t_0, p_j^W\right) \qquad \text{h=1,..H} \qquad (33)$$

where $\tilde{\mathbf{t}} = (t_0, 0, \ldots, 0)$

$$\sum_{i \in C} t_i \sum_{h \in H} E_i^h\left(\mathbf{p}^W + \tilde{\mathbf{t}}, u^h\right)$$
$$+ \sum_{i \in A} t_i^W \left(\Pi_i^i\left(p_0^W + t_0, p_j^W + t_j^W\right) \; - \sum_{h \in H} E_i^h\left(\mathbf{p}^W + \tilde{\mathbf{t}}, u^h\right) - \; x_i^G\left(?^i\right)\right) - \sum_{j \in C} p_j x_j^G\left(?^i\right) = 0 \qquad (34)$$

4.3.1 The general case

From TA 2.2 we have

$$m^h = b^h + l\left(t_0 \sum_{h \in \mathrm{H}} \frac{¶\, x_0^h(\mathbf{q}, I^h)}{¶\, I^h} \right)$$

as the *net marginal social welfare of income for household h.*

From the first order conditions with respect to t_0, (see TA 2.4),

$$\sum_{h\in \mathrm{H}} \mu^{\mathrm{h}} E_0^h - l\left(t_0 \sum_{h\in \mathrm{H}} E_{i0}^h \right) \qquad =0 \qquad \text{k=1, .,N} \quad (35)$$

From the first order conditions with respect to t^k, (TA 2.7), we have

$$-\sum_{h\in \mathrm{H}} \mu^{\mathrm{h}} a_j^h \Pi^j + l\,\Pi^j \qquad =0 \qquad (36)$$

Interpretation

The optimal tax tax-tariff system is in this case determined by two considerations: achieving distributional objectives mainly trough the differentiation of the taxation (or subsidisation) of sectoral income which is associated with not distortionary costs and limiting the distortionary costs associated with the taxation of the primary factor.

Using the definitions of R_0, and P^j we rewrite the first order conditions with respect to t_0, (see (35))

$$t_0 \sum_{h\in \mathrm{H}} E_{0k}^h = -\frac{(\lambda - \mathrm{R}_0)}{\lambda} \sum_{\mathrm{h=1}}^{\mathrm{H}} E_k^h \qquad \text{k=0,1, N} \quad (37)$$

from the first order conditions with respect to t^j (see (36))

$$\mathrm{P}^j = l \qquad \text{j=1, ., N,k=0,j} \quad (38)$$

Interpretation

The optimal tax tax system is in this case determined by two considerations: achieving distributional objectives (mainly trough the differentiation of the taxation (or subsidisation) of sectoral income which is associated with not distortionary costs) and by limiting the distortionary costs associated with the taxation of the primary factor by increasing the average level of taxation of sectoral income.

4.3.2 The case of one representative household

It will it the governments requirement is larger than the total profit be optimal to tax profit at 100%, and the tax on the primary factor supply will provide the remaining tax revenue.

4.4 The optimal tax system

Two principles

- the more instruments the higher welfare disregarding administrative costs
- the more instruments the higher administrative costs

Based on these principles the social welfare of the optimal tax-tariff system under $\mathbf{?}^0$ will be greater than under $\mathbf{?}^1, \mathbf{?}^2$ and $\mathbf{?}^3$, whereas the administrative costs will be larger under $\mathbf{?}^0$. Which tax system will be the over all optimal tax system, x_*^*, the optimal tax system for the tax structure without restrictions, $x_*^1 \in \mathbf{?}^0$, the optimal tax system for the tax structure when only border taxes and an income tax is feasible, $x_*^1 \in \mathbf{?}^1$, the optimal tax system for the tax structure when a tax on primary factor income is feasible, $x_*^2 \in \mathbf{?}^2$, or the optimal tax system for the tax structure when a tax on primary factor income is supplemented with sectoral taxes, but where border taxes are not feasible, $x_*^3 \in \mathbf{?}^3$, thus depends on a trade-off between allocation benefits and administrative costs. If $\mathbf{?}^1$ and $\mathbf{?}^2$ dominates $\mathbf{?}^0$ the optimal tax structure will be characterised by production inefficiency and by diversions from free trade being desirable. Under $\mathbf{?}^3$ free trade is not desirable, but enforced. Furthermore, in the case of $\mathbf{?}^1$ and $\mathbf{?}^2$ diversions from production efficiency within the domestic production sector through the differentiation of output and input taxes (subsidies) between sectors will be desirable if the administrative costs involved do not dominate the benefits.

5. Application: the evolution of agricultural policy

Worldwide market price support, i.e. tariff protection/ export subsidies have been used to increase agricultural income. To analyse what determines the optimal level of market price support we consider an economy where agricultural sector profit is not taxed. In such an economy a tax structure $\mathbf{?}^1$ may be optimal if the costs of less distortionary support instruments than market price support are high and difficult to target to low income households in the agricultural sector, as will be the case in countries with a weak administrative infrastructure.

The a level of market price support is likely to be high when

a) when it increases the income of factor owners with relatively low income,

b) when it decreases the real income of consumers and taxpayers with relatively high income,

c) when governments have relatively strong redistributional preferences

d) when "the distortionary costs" of market price are relatively low. This will be the case if supply and demand elasticities are small, if for example the support is only provided for a short period.

e) when direct administrative costs are low which will be the case if the country is an importer and already has an administrative system for collecting border taxes,

f) when the budget costs and hence the "indirect costs" are low or even negative, i.e. when the country is an importer,

g) when for an importer (exporter) the administrative and distortionary costs of raising government revenue using general tax instruments are high (low),

6. Concluding remarks

The costs of using specific instruments are seldom explicitly considered in theoretical public economic analysis. However, as the present analysis demonstrates taking these cost into account may explain the why government policies often create productive inefficiencies. The framework may be used both to explain why market price support in all OECD countries has been provided to agricultural at times where these agricultural income were particular under pressure and where the transaction costs of less distortionary support instruments were relatively high, and why these policies are being changed in these countries now when the pressure on the income is less and the costs of direct income support much smaller (see Munk 1985). The same combinations of high administrative costs of non-distortionary support policies, low mobility and strong redistributional concerns characterise many other sectors under structural adjustment pressure. This suggests that the framework may also contribute to the explanation of policies toward other sectors.

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Technical appendix:

1. The Model: Notation and equilibrium conditions

We consider an economy with N production sectors, H households, a foreign sector and a government. There is one primary factor and N produced commodities. Each sector produces under conditions of non-increasing returns to scale one sector specific commodity using only the primary factor as input. Each household has an endowment of the primary factor and a share of the profit in each industry. Each household consumes all produced commodities and has an endogenous supply of the primary factor. All produced commodities are traded with the rest of the world at fixed world market prices. The government has a fixed requirement in terms of the primary factor that is financed by taxes on domestic and international transactions.

Notation

<u>Index sets</u>

Commodities:	$C = (0,1, .\ \ , N)$
Sectors	$A = (1, . , N)$
Households:	$H = (1, .\ \ , H)$

<u>Variables</u>

Quantities

Production levels, outputs:	Y_i	$i \in A$
Labour:	v_0^j	$j \in A$
Household demand of produced commodities:	x_i^h	$i \in A, h \in H$
Household net demand(-supply) of labour:	x_0^h	$h \in H$
Rest of the world net demand of produced commodities:	x_i^W	$i \in A, h \in H$
Rest of the world demand of labour:	x_0^W	
Labour endowments:	w_0^h	$h \in H$
Consumption of produced commodities:	$c_i^h \equiv x_i^h$	$i \in A, h \in H$

Leisure: $l^{\mathrm{h}} \equiv w_0^h + \mathrm{x}_0^{\mathrm{h}}$ $\quad \mathrm{h} \in \mathrm{H}$

Household net demand vector: $\mathbf{x}^{\mathbf{h}} \equiv \left(\mathrm{x}_0^{\mathrm{h}}, \mathrm{x}_1^{\mathrm{h}}, ..., \mathrm{x}_{\mathrm{N}}^{\mathrm{h}}\right)$ $\quad \mathrm{h} \in \mathrm{H}$

Aggregate household net demand vector: $\mathbf{X} \equiv \left(\mathrm{X}_0, \mathrm{X}_1, ..., \mathrm{X}_{\mathrm{N}}\right)$

Taxes

Rate of tax on household demand of produced commodities: t_{i} $\quad \mathrm{i} \in \mathrm{A}$

Rate of tax on net demand of labour : t_0

Household commodity tax vector $\mathbf{t} \equiv \left(\mathrm{t}_0, \mathrm{t}_1, ..., \mathrm{t}_{\mathrm{N}}\right)$

Rate of tax on Rest of the World net demand of commodities:

$\mathrm{t}_{\mathrm{i}}^{\mathrm{W}}$ $\quad \mathrm{i} \in \mathrm{C}$

Rest of the World commodity tax vector $\mathbf{t}^{\mathrm{W}} \equiv \left(\mathrm{t}_0^{\mathrm{W}}, \mathrm{t}_1^{\mathrm{W}}, ..., \mathrm{t}_{\mathrm{N}}^{\mathrm{W}}\right)$

Profit tax rate: t^{j}

Income

Profit in sector i: Π^{i} $\quad \mathrm{i} \in \mathrm{A}$

Share of profit in sector i for household h: a_i^h $\quad \mathrm{i} \in \mathrm{A}$ $\mathrm{h} \in \mathrm{H}$

where $\sum_{h \in H} a_i^h = 1$, $\mathrm{i} \in \mathrm{A}$

Non labour income: I^{h} $\quad \mathrm{h} \in \mathrm{H}$

Prices

Producer prices for produced commodities: p_{i} $\quad \mathrm{i} \in \mathrm{C}$

Producer price for labour: p_0

Household prices for produced commodities: $q_i \equiv p_i + t_i$ $\quad i \in C$

Household prices for labour: $q_0 \equiv p_0 + t_0$

Rest of the World prices for produced commodities: $p_i^W \equiv p_i + t_i^W$ $\quad i \in C$

Producer price vector $\mathbf{p} \equiv (p_0, p_1, ..., p_N)$

Household price vector: $\mathbf{q} \equiv (q_0, q_1, ..., q_N)$

Rest of the world price vector $\mathbf{p}^W \equiv (p_0^W, p_1^W, ..., p_N^W)$

<u>Functions</u>

Household net demand functions:

$$x_i^h * = x_i^h(\mathbf{q}, I^h), \qquad i \in A,\ h \in H$$

$$x_0^h * = x_0^h(\mathbf{q}, I^h), \qquad h \in H$$

solves

$$V(\mathbf{q}, I^h) = \underset{\{\mathbf{x^h}\}}{Max}\ u^h(\mathbf{x^h}) \text{ s.t. } \mathbf{qx^h} \equiv I^h \qquad h \in H$$

Output contingent input demands

$$v_0^{j*} = C_0^j(p_0^j, Y_j) \qquad j \in A$$

solves

$$C^j(p_0, Y_j) \equiv \min_{v_0^j} p_0^j v_0^j \text{ s.t. } Y_j = f^j(v_0^j) \qquad j \in A$$

Profit function

$$\Pi^j(p_0^j, p_j^j) = \underset{Y_j, v_0^j}{Max}\ p_j Y_0^j - p_0^j v_j^j \quad \text{s.t.} \quad Y_j = f(v_0^j) \qquad j \in A$$

Equilibrium conditions

Profit maximisation:

$$v_0^j = \Pi_0^j(p_0^{\mathrm{j}}, p_{\mathrm{j}}^{\mathrm{j}}) \qquad \mathrm{j} \in \mathrm{A} \qquad (1.1)$$

$$Y_j = \Pi_j^j(p_0^{\mathrm{j}}, p_j^j) \qquad \mathrm{j} \in \mathrm{A} \qquad (1.2)$$

Determination of non-labour income

$$I^h = \sum_{j \in A} (1 - t^{\mathrm{j}}) a_j^h \Pi^j (p_0^{\mathrm{j}}, p_{\mathrm{j}}^{\mathrm{j}}) \qquad (1.3)$$

Utility maximisation

$$x_{\mathrm{i}}^{\mathrm{h}} = x_{\mathrm{i}}^{\mathrm{h}} \left(\mathbf{q}, I^{\mathrm{h}}\right) \qquad \mathrm{i} \in \mathrm{A}\ \mathrm{h} \in \mathrm{H} \qquad (1.4)$$

$$x_0^{\mathrm{h}} = x_0^{\mathrm{h}} \left(\mathbf{q}, I^{\mathrm{h}}\right) \qquad \mathrm{h} \in \mathrm{H} \qquad (1.5)$$

Balance in foreign trade

$$\sum_{\mathrm{i} \in \mathrm{C}} p_i^W x_i^W = 0 \qquad (1.6)$$

Material balance

$$Y_i = \sum_{\mathrm{h} \in \mathrm{H}} x_i^h + x_i^W \qquad \mathrm{i} \in \mathrm{A} \qquad (1.7)$$

$$0 = \sum_{\mathrm{j} \in \mathrm{A}} v_0^j + \sum_{\mathrm{h} \in H} x_0^h + x_0^W + x_0^G \qquad (1.8)$$

Government budget constraint

$$\sum_{\mathrm{i} \in C} t_i \sum_{\mathrm{h} \in \mathrm{H}} x_i^h + \sum_{\mathrm{i} \in \mathrm{C}} t_i^W x_i^W + t_0^j \Pi_0^{\mathrm{j}}(p_0^{\mathrm{j}}, p_{\mathrm{j}}^{\mathrm{j}}) + t_j^j \Pi_{\mathrm{j}}^{\mathrm{j}}(p_0^{\mathrm{j}}, p_{\mathrm{j}}^{\mathrm{j}}) + \sum_{\mathrm{j} \in \mathrm{A}} t^j \sum_{\mathrm{h} \in H} a_j^h \Pi^j (p_0^{\mathrm{j}}, p_{\mathrm{j}}^{\mathrm{j}}) - p_0 x_0^G = 0 \qquad (1.9)$$

Tax-price equations

Household prices for produced commodities

$$\mathrm{q_i} = \mathrm{p_i} + \mathrm{t_i} \qquad \mathrm{i} \in \mathrm{A} \qquad (1.10)$$

Household prices for primary factors

$$q_0 = p_0 + t_0 \tag{1.11}$$

Border prices

$$p_i^W = p_i - t_i^W \qquad i \in C \tag{1.12}$$

We may, as a matter of normalisation, assume that $p_0 = \overline{p}_0$.

Assuming that world market prices are exogenously determined we have

$$p_i = p_i^W + t_i^W \qquad i \in C \tag{1.13}$$

By substitution the equilibrium conditions may be reduced to the following equations.

$$\Pi_i^i(p_0 + t_0^j + t_0^W, p_j^W + t_j^j + t_j^W) = \sum_{h\in H} x_i^h\left(\mathbf{p}^W + \mathbf{t}^w + \mathbf{t}, I^h\right) + x_i^W \qquad i \in C$$

$$\sum_{j\in A}\Pi_0^j(p_0 + t_0^j + t_0^W, p_j^W + t_j^j + t_j^W) + \sum_{h\in H} x_0^h\left(\mathbf{p}^W + \mathbf{t}^w + \mathbf{t}, I^h\right) + x_0^G = 0 \tag{1.14}$$

$$\begin{aligned}&\sum_{i\in C} t_i \sum_{h\in H} x_i^h\left(\mathbf{p}^W + \mathbf{t}^w + \mathbf{t}, I^h\right) + t_0 \sum_{h\in H} x_0^h\left(\mathbf{p}^W + \mathbf{t}^w + \mathbf{t}, I^h\right) + \sum_{i\in C} t_i^W x_i^W \\ &+ \sum_{j\in A} t^j \sum_{h\in H} a_j^h \Pi^j\left(p_0 + t_0^j + t_0^W, p_j^W + t_j^j + t_j^W\right) - p_0 x_0^G = 0\end{aligned} \tag{1.15}$$

where

$$I^h = (1-t)^j \sum_{j\in A} a_j^h \Pi^j\left(p_0 + t_0^j + t_0^W, p_j^W + t_j^j + t_j^W\right)$$

By further substitution we obtain the following two equations

$$\sum_{j\in A}\Pi_0^j\left(p_0 + t_0^j + t_0^W, p_j^W + t_j^j + t_j^W\right) + \sum_{h\in H} x_0^h\left(\mathbf{p}^W + \mathbf{t}^w + \mathbf{t}, I^h\right) + x_0^G = 0 \tag{1.16}$$

$$\begin{aligned}&\sum_{i\in C} t_i \sum_{h\in H} x_i^h\left(\mathbf{p}^W + \mathbf{t}^w + \mathbf{t}, I^h\right) + t_0 \sum_{h\in H} x_0^h\left(\mathbf{p}^W + \mathbf{t}^w + \mathbf{t}, I^h\right) \\ &+ \sum_{i\in C}\left(t_i^W - p_i^W\right)\left(\Pi_i^i(p_0 + t_0^j + t_0^W, p_j^W + t_j^j + t_j^W) - \sum_{h\in H} x_i^h\left(\mathbf{p}^W + \mathbf{t}^w + \mathbf{t}, I^h\right)\right. \\ &+ \sum_{j\in A} t^j \sum_{h\in H} a_j^h \Pi^j\left(p_0 + t_0^j + t_0^W, p_j^W + t_j^j + t_j^W\right) - \sum_{i\in C} p_i x_i^G = 0\end{aligned} \tag{1.17}$$

where

$$I^h = \sum_{j\in A}(1-t^j) a_j^h \Pi^j\left(p_0 + t_0^j + t_0^W, p_j^W + t_j^j + t_j^W\right)$$

Using the expenditure function approach equilibrium conditions may be expressed by the following equations

$$(1-t)^j \sum_{j\in A} a_j^h \Pi^j\left(p_0+t_0^j+t_0^W,\, p_j^W+t_j^j+t_j^W\right)+\, \mathrm{E}^h(\mathbf{p}^W+\mathbf{t}^w+\mathbf{t},u^h)=0 \tag{1.18}$$

$$\sum_{j\in A}\Pi_0^j(p_0+t_0^j+t_0^W,\, p_j^W+t_j^j+t_j^W) \quad + \sum_{h\in H} E_0^h(\mathbf{p}^W+\mathbf{t}^w+\mathbf{t},u^h) \; + \; x_0^G \; = 0 \tag{1.19}$$

$$\begin{aligned}&\sum_{i\in C} t_i \sum_{h\in H}\mathrm{E}_i^h(\mathbf{p}^W+\mathbf{t}^w+\mathbf{t},u^h)\; + \; t_0 \sum_{h\in H} E_0^h(\mathbf{p}^W+\mathbf{t}^w+\mathbf{t},u^h)\\ &+\sum_{i\in C} t_i^W\left(\Pi_i^i\left(p_0+t_0^j+t_0^W,\, p_j^W+t_j^j+t_j^W\right)-\sum_{h\in H} E_i^h(\mathbf{p}^W+\mathbf{t}^w+\mathbf{t},u^h)-\, x_i^G\right)\\ &+\sum_{j\in A} t^j \sum_{h\in H} a_j^h \Pi^j\left(p_0+t_0^j+t_0^W,\, p_j^W+t_j^j+t_j^W\right)-\sum_{i\in C} p_i x_i^G \quad =0\end{aligned} \tag{1.20}$$

By Walras' law the equilibrium conditions may therefore be expressed by the last equation which may be interpreted as the government budget constraint.

To find a solution for a given government requirement, exogenous values must therefore be specified for all consumer prices and government purchases, except one.

2. Derivation of first order conditions

The Lagrangian expression corresponding to the government's maximisation problem assuming that the tax systems has to belong to a given tax structure, $\mathbf{?}^s$, may be formulated as

$$\begin{aligned}L =\, & W\left(u^1,u^2,..,u^H\right)+\sum_{h\in H} m^h\left(\sum_{j\in A}(1-t^j)a_j^h\Pi^j\left(p_0+t_0^j+t_0^W,\, p_j^W+t_j^j+t_j^W\right)-L-E^h\left(\mathbf{p}^W+\mathbf{t}^w+\mathbf{t},u^h\right)\right)\\ &+\, l\left(\sum_{i\in C} t_i \sum_{h\in H} E_i^h\left(\mathbf{p}^W+\mathbf{t}^w+\mathbf{t},u^h\right)+\mathrm{H}L\right.\\ &-\sum_{j\in A}\sum_{i\in(0,k)} t_i^j\Pi_i^j\left(p_0+t_0^j+t_0^W,\, p_j^W+t_j^j+t_j^W\right)+\sum_{j\in A} t^j\Pi^j\left(p_0+t_0^j+t_0^W,\, p_j^W+t_j^j+t_j^W\right)\\ &\left.+\sum_{i\in A} t_i^W\left(\Pi_i^i\left(p_0+t_0^j+t_0^W,\, p_j^W+t_j^j+t_j^W\right)\; - \sum_{h\in H} E_i^h\left(\mathbf{p}^W+\mathbf{t}^w+\mathbf{t},u^h\right)-\, x_i^G\left(\mathbf{?}^i\right)\right)-\sum_{j\in C} p_j x_j^G\left(\mathbf{?}^i\right)\right)\end{aligned} \tag{2.1}$$

The first order conditions with respect to $u^1,u^2,..,u^{\mathrm{H}}$; $t_i,\, i=0,..,N$; $t^k,\, k=0,..,N$; and $t_i^W,\, i=1,..,N$; may therefore be expressed as:

$$\frac{\partial L}{\partial u^h} = \frac{\partial W}{\partial u^h} - \mu^h E_u^h + \lambda \left(\sum_{i\in C} t_i E_{iu}^h - \sum_{i\in A} t_i^W E_{iu}^h \right) = 0 \qquad \text{h=1,., H (2.2)}$$

$$\frac{\partial L}{\partial L} = -\sum_{h\in H} \mu^h + \lambda H = 0 \qquad (2.3)$$

$$\frac{\partial L}{\partial t_k} = -\sum_{h\in H} \mu^h E_k^h + \lambda \left(\sum_{i\in C} t_i \sum_{h\in H} E_{ik}^h + \sum_{i\in A} t_i^W \sum_{h\in H} E_{ik}^h + \sum_{h\in H} E_k^h \right) = 0 \qquad \text{k=0,1, .,N(2.4)}$$

$$\frac{\partial L}{\partial t_0^j} = \left((1-t^j) \sum_{h\in H} \mu^h a_j^h \Pi_0^j \right) + \lambda \left(-t_0^j \Pi_{00}^j - \left(t_j^j + t_j^W \right) \Pi_{j0}^j - \Pi_0^j + t^j \Pi_0^j \right) = 0 \qquad (2.5)$$

$$\frac{\partial L}{\partial t_j^j} = \left((1-t^j) \sum_{h\in H} \mu^h a_j^h \Pi_j^j \right) + \lambda \left(-t_0^j \Pi_{0j}^j - \left(t_j^j + t_j^W \right) \Pi_{jj}^j - \Pi_j^j + t^j \Pi_j^j \right) = 0 \qquad \text{j=1,., N (2.6)}$$

$$\frac{\partial L}{\partial t^k} = -\sum_{h\in H} \mu^h a_j^h \Pi^j + \lambda \Pi^j = 0 \qquad \text{k=1,., N (2.7)}$$

$$\begin{aligned} \frac{\partial L}{\partial t_j^w} = & -\left((1-t^j) \sum_{h\in H} \mu^h a_j^h \Pi_j^j - \sum_{h\in H} \mu^h \sum_{h\in H} E_j^h \right) \\ & + \lambda \left(t_0 \sum_{h\in H} E_{0j}^h + t_0^j \Pi_{0j}^j + t_j^j \Pi_{jj}^j + t^j \Pi_j^j \right. \\ & \left. + \sum_{i\in C} t_i^W \sum_{h\in H} E_{ij}^h + t_0^W \Pi_{0j}^j + t_j^W \Pi_{jj}^j + \sum_{h\in H} E_j^h - \Pi_j^j \right) = 0 \end{aligned} \qquad \text{j=1,.,N (2.8)}$$

We define the marginal social welfare of income for household h as $\beta^h = \frac{\partial W}{\partial u^h} / E_u^h$. Using this definition we get (in analogy with the definition by Diamond (1975) for a closed economy) from the first order conditions with respect to u^h, (2.2)

$$\mu^h = \beta^h + \lambda \sum_{i\in C} \left(t_i + t_i^W \right) \frac{\partial x_i^h(\mathbf{q}, I^h)}{\partial I^h} \qquad (2.9)$$

as the *net marginal social welfare of income for household h*.

From from the first order conditions with respect to *L*, (2.3) we get

$$\frac{\sum_{h\in H} \mu^h}{H} = \lambda$$

From the first order conditions with respect to t_k, (2.4),

$$\sum_{h\in H} \mu^h E_k^h - \lambda \left(\sum_{i\in C} t_i \sum_{h\in H} E_{ik}^h + \sum_{i\in A} t_i^W \sum_{h\in H} E_{ik}^h + \sum_{h\in H} E_k^h \right) = 0 \qquad \text{k=1, .,N (2.10)}$$

From the first order conditions with respect to t_0^j, t_j^j, (2.5) and (2.6) we have

$$\left((1-t^{j})\sum_{h\in H}\mu^{h}a_{j}^{h}\Pi_{0}^{j}\right)+l\left(-\left(t_{0}^{j}+t_{0}^{W}\right)\Pi_{00}^{j}-\left(t_{j}^{j}+t_{j}^{W}\right)\Pi_{j0}^{j}-\Pi_{0}^{j}+t^{j}\Pi_{0}^{j}\right)=0 \qquad (2.11)$$

$$\left((1-t^{j})\sum_{h\in H}\mu^{h}a_{j}^{h}\Pi_{j}^{j}\right)+l\left(-\left(t_{0}^{j}+t_{0}^{W}\right)\Pi_{0j}^{j}-\left(t_{j}^{j}+t_{j}^{W}\right)\Pi_{jj}^{j}-\Pi_{j}^{j}+t^{j}\Pi_{j}^{j}\right)=0 \qquad j=1,., N \ (2.12)$$

From the first order conditions with respect to t^{k}, (2.7) we have

$$-\sum_{h\in H}\mu^{h}a_{j}^{h}\Pi^{j}+l\,\Pi^{j} \quad =0 \qquad (2.13)$$

From the first order conditions with respect to t_{j}^{w}, (2.8)

$$-\left((1-t^{j})\sum_{h\in H}\mu^{h}a_{j}^{h}\Pi_{j}^{j}-\sum_{h\in H}\sum_{h\in H}\mu^{h}E_{i}^{h}\right)+$$
$$l\left(t_{0}E_{0j}^{h}+\sum_{i\in A}t_{i}E_{ij}+t_{0}^{j}\Pi_{0j}^{j}+t_{j}^{j}\Pi_{jj}^{j}+t^{j}\Pi_{j}^{j}+\sum_{i\in A}t_{i}^{W}E_{ij}+\sum_{i\in A}t_{i}^{W}\Pi_{ij}+\sum_{h\in H}E_{i}^{h}-\Pi_{j}\right)=0 \qquad j=1,.,N \ (2.14)$$

Rewriting (28) we get from the first order conditions with respect to t_{k},

$$\sum_{i\in C}(t_{i}+t_{i}^{W})\sum_{h\in H}E_{ik}^{h}=-\frac{(\lambda-R_{k})}{\lambda}\sum_{h=1}^{H}E_{k}^{h} \qquad k=0,1, N \qquad (2.15)$$

from the first order conditions with respect to t_{0}^{j}, t_{j}^{j}, (2.11) and (2.12)

$$\sum_{i\in(0,j)}(t_{i}^{j}+t_{i}^{W})\Pi_{ij}^{j}=-\left(1-t^{j}-(1-t^{j})P_{j}^{j}/l\right) \qquad j=0,1, ., N \qquad (2.16)$$

from the first order conditions with respect to t^{k}, (36)

$$P^{j}=l \qquad j=1, ., N,k=0,j \qquad (2.17)$$

from the first order conditions with respect to t_{j}^{w}, (2.14)

$$-\left((1-t^{j})P^{j}\Pi_{j}^{j}-R_{j}\sum_{h\in H}E_{j}^{h}\right)+$$
$$l\left(t_{0}E_{0j}^{h}+\sum_{i\in A}t_{i}E_{ij}+t_{0}^{j}\Pi_{0j}^{j}+t_{j}^{j}\Pi_{jj}^{j}+t^{j}\Pi_{j}^{j}+\sum_{i\in A}t_{i}^{W}E_{ij}+\sum_{i\in A}t_{i}^{W}\Pi_{ij}+\sum_{h\in H}E_{i}^{h}-\Pi_{j}\right)=0 \qquad j=1,.,N \qquad (2.18)$$

where

$$\mathrm{R}_{\mathrm{k}} = \frac{\sum_{\mathrm{h}\in \mathrm{H}} m^{\mathrm{h}} x_k^h}{\sum_{\mathrm{h}\in \mathrm{H}} x_k^h}, \text{ and}$$

$$\mathrm{P}^j = \sum_{\mathrm{h}\in \mathrm{H}} m^{\mathrm{h}} a_j^h$$

R_{k} is the *net distributional characteristics of the consumption commodity k,* P_k^j *is the distributional characteristics of the change in profit by the changes of the producer price* p_k^j*, and* P^j *is the distributional characteristics of the change in profit by the changes of the producer price* t^j We notice that the distributional characteristics are a weighted average of the household's net marginal social welfare where the weights are the households share of the total demand and of the profit in question respectively.

When the tax rates t^k, k=1, ., N, on the profit in the various sectors are chosen optimally we have form (2.16) and (2.17)

$$\sum_{i\in(0,j)} (t_{\mathrm{i}}^{\mathrm{j}} + t_{\mathrm{i}}^{\mathrm{W}})\Pi_{ij}^j = 0 \qquad \text{j=1, ., N} \qquad (2.19)$$