Shifting regimes in the relationship between interest rates and inflation: a threshold cointegration approach.

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Abstract

This paper examines the long-run relationship between short term nominal interest rates and inflation using American data. As the ex ante real interest rate is supposed to be either I(0) or I(1) without a real consensus in empirical studies, we test for a unit root in the framework of a complete cointegration analysis and ECM methods with changing regimes. As a first step, we conduct cointegration tests, while innovating by allowing a break in the cointegrating vector as well as a mean shift for the constant in the long-run equation following Gregory & Hansen (1996) methodology. This will help us to specify correctly any sudden and exogenous change in the process. As a second step, we undertake Threshold AutoRegressive (TAR) tests for the residuals of the cointegration relationship and for the real interest rates, as well as a test of non-linearity allowing a smooth transition from one regime to another. The null hypothesis is the unit root hypothesis while the alternative is the stationary Logistic Smooth Transition Autoregressive (LSTAR) model. An application to the US data shows strong evidence for a threshold behavior in the real interest rates. Asymmetries in interest rates changes to inflation shocks in Central Bank reaction function imply that monetary authorities are trying to run a credible anti-inflationary policy, reacting differently to positive and to negative inflation surprises.

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1 Introduction

The main contribution of this paper is to allow for asymmetries in Unit Root tests, as a bootstrap Lagrange Multiplier (LM) extension of the Self-Exciting Threshold AutoRegressive (SETAR) tests (cf (Enders and Granger 1998)), especially in the framework of a cointegration relationship between short term nominal interest rates and inflation rates. The motivations for investigating some asymmetries in the relationship between interest rates and inflation rates are severalfold. The growing interest in inflation targeting and the opportunistic behavior of the Central Bank in the context of rising inflation are some of the reasons for exploring asymmetries in real interest rates. According to the concept of inflation targeting, interest rate feedback rules imply that nominal interest rates should respond to increases in inflation with a more than one-to-one increase (cf (Clarida, Gali, and Gertler 1998)). The validation of a two-regime behavior will help to emphasize asymmetries in interest rates changes to inflation shocks in Central Bank reaction function, which imply that monetary authorities try to run a credible anti-inflationary policy, reacting more strongly to positive[1] than to negative inflation surprises. More precisely, in response to inflationary pressures, monetary authorities are quick to raise nominal interest rates which leads to a return of the real interest rates to their equilibrium value. On the other hand, in a falling inflation environment, the authorities may not be as quick to reduce the level of nominal interest rates.

Moreover, (Hamilton 1988), (Sola and Driffil 1994) and (Gray 1996) all find strong evidence for non-linear behavior in U.S. nominal interest rates, using Markov switching models. Furthermore, (Anderson 1997), (Enders and Granger 1998) and (Enders and Siklos 1999) find evidence of non-linearity in nominal yields using threshold autoregressions.

[1] which will correspond to a decrease of the real interest rates, nominal interest rates being kept constant.

Finally, (Pippinger and Goering 1993) and (Caner and Hansen 2001) have underlined the poor power of ADF tests in distinguishing between non-linearity and non-stationarity.

The innovation of the paper will be to coincide these asymmetries investigation with a cointegration test robust to structural change. We will proceed in two steps: as a first step we conduct cointegration tests by allowing a break in the cointegrating vector as well as a mean shift for the constant in the long-run equation, following (Gregory 1996) methodology.

The main justification for introducing structural breaks in the cointegration model is that, according to some measurements of inflation expectations, it is very likely that the inflation rate was under-expected during the oil crisis and over-expected during the deflation period. OCDE's forecasts with the help of econometric models confirm over-estimation of inflation rates due to systematic errors in the agents' anticipation during this period. (Bismut 1988) for instance argue that expectations differ a lot with realization in the deflation period of the early 80s. Thus, one could observe from the data that a decade of low real interest rates in the 70s gave way to a decade of high real rates in the 80s[2]. These stylized facts are interpreted as a consequence of the change of monetary policy during the Volcker presidency in 1979.

As a second step, we undertake Self-Exciting Threshold AutoRegressive (SETAR) tests for the real interest rate,as well as a test of non-linearity allowing a smooth transition from one regime to another. The null hypothesis is the unit root hypothesis while the alternative is the stationary Logistic Smooth Transition Autoregressive (LSTAR) model[3] so that the shifts between the two regimes are driven by a logistic transition function.

These recent econometric methods will help us to resolve the Fisher effect

[2] this was a worldwide phenomenon.
[3] with a possibility of a unit root in one of the regimes.

"puzzle". Many macroeconomists searched for a characterization of the statistical properties of the real interest rate but no consensus emerged about the statistical properties of the real rate of interest, despite intensive empirical studies. Furthermore, many macro-economic theory models (GMM, CAPM...) routinely assume that the real rate of interest is a stationary process while empirical works indicate that this is not so or at best holds only over short periods. Moreover, the fact that the real interest rate is a crucial determinant of investment, savings and indeed virtually all intertemporal decisions renders its characteristics of intrinsic interest, so that a potential nonstationarity of the ex ante real interest rate has important consequences concerning monetary policies effects but also for financial theory fields

The literature clearly indicates that the nominal interest rate is nonstationary ((Fama and Gibbons 1982) and (Mankiw and Miron 1986)). However, it has proven difficult to provide definitive evidence concerning the ex ante real interest rate, as it is inherently unobservable. (Rose 1988) tested for cointegration using the techniques suggested by (Engle and Granger 1987). At the annual frequency, none of the tests indicated cointegration at even the ten percent significance level. (Mishkin 1992) raised an interesting problem about the Fisher effect's lack of robustness depending on the period considered. Mishkin therefore conducts a reexamination of the Fisher effect in the postwar United States and finds that the evidence does not support a short-run relationship in which a change in expected inflation is associated with a change in interest rates. More recently, (Garcia and Perron 1996) reanalyzed data over the period 1961-1986 using Markov Switching (MS) methods and found support for a stable real rate of interest, subject to infrequent changes in the constant. Then these authors concluded that the ex ante real rate of interest was effectively stable, but subject to occasional mean shifts over 1961-1985. Three regimes were found over this time period and the conclusion does not seem unreasonable (at least for these data). However, if we have a look at the graphs for the ex post real interest rate series calculated in the same way over the longer period 1951-1999, it seems that a large number of mean shifts is needed to accommodate this approach and the results seem to be much less satisfying.

To summarize, the empirical evidences reviewed just before give a mixed picture about the statistical properties of the real rate of interest, and it is probably fair to say that the generating mechanism for the real rate is imperfectly understood.

The next section will describe cointegration testing procedures and Self-Exciting Threshold AutoRegressive (SETAR) models for unit root and linearity tests to deal with the Fisher effect as a changing regimes cointegration relationship in which we allow the constant and/or the cointegrating vector to shift in the long-run equation. Linearity will also be tested in the context of a Logistic Smooth Transition Autoregressive (LSTAR) model. The third section will display the main econometric results obtained from these tests applied to the US data which show strong evidence for a threshold cointegration relationship between interest rates and inflation.

2 Cointegration framework

2.1 The Fisher effect

Thanks to (Fisher 1896), it is recognized that expectations of inflation can affect interest rates determination[4]. Since that, this concept has played an important role in the formulation of a wide range of economic models.

The Fisher effect represents a relation of determination between the nominal interest rates and the expected inflation rates, the former reflecting at each time

[4] (Fisher 1930) seems to have been the first to conduct a sustained study and to explore the matter in serious empirical research.

the latter. So, Irving Fisher formulated the concept of the ex ante real rate of interest r_t^e, so as to provide a rate of interest which accounts for the value of loan repayments in real terms.

Thus, a nominal interest rate of i_t will assure an ex ante real rate of r_t^e when the anticipated price change expected by the agents is π_t^e, provided that $1 + i_t = (1 + r_t^e)(1 + \pi_t^e)$, thereby adjusting the compensation for the lender to the anticipated losses in purchasing power in the principal as well as in the interest. The Fisher equation is commonly simplified as:

$$i_t = r_t^e + \pi_t^e \tag{1}$$

This implies that if the inflation expectations are perfectly accurate, the interest rate will follow the inflation evolution.

We make the hypothesis that all the economic agents have rational expectations. Then, we define the expectation errors as a process which will be independent to the current information set available to the agents.

The forecast error ε_t represents the difference between ex ante inflation rates expected by the agents in the economy and the inflation rates really observed ex post:

$$\varepsilon_t = \pi_t^e - \pi_t \tag{2}$$

ε_t will be unforecastable given any information known at time t, under rational expectations. In most of the empirical works, the expectation errors have been assumed to be stationary in level and are considered therefore to be a martingale difference.

Under these assumptions, ex post real interest rates and ex ante ones differ only by a stationary component; therefore both these time series have the same long run properties. Hence, we can determine the relation between the ex ante and the ex post real interest rate:

$$r_t = i_t - \pi_t = r_t^e + (\pi_t^e - \pi_t) \equiv r_t^e + \varepsilon_t$$

$$\Rightarrow$$

$$r_t^e = r_t - \varepsilon_t \tag{3}$$

Using OLS methods, the Fisher equation suggests a regression link between i_t and π_t^e, depending on the properties of the real rate r_t^e. In particular, the Fisher effect asserts that the coefficient b should be equal (or very close) to one in a regression of the form :

$$i_t = a + b\pi_t^e + u_t \tag{4}$$

and the residuals u_t should be stationary.

The ex ante inflation rates being unobservable, we have to rely on ex post inflation rates to perform the tests, with the idea that the results will lead to the same interpretations as long as the rationality assumption is held.

Moreover, as the series studied (inflation rates and interest rates) appear to display non stationarity, it is necessary to undertake cointegration tests so as to underline the long run relationship between inflation rates and interest rates and to test the validity of the Fisher relation in the presence of a cointegrating vector.

2.2 Smooth transition in the regime-switching error correction model in a cointegration relation with time breaks

2.2.1 Non linearities in the long-run equation and in the mean reversion process

Since (Gregory 1996), it is possible to consider cointegration relationships in which the parameters are no longer time invariant. This means that the long-run relation holds over some period of time and shifts to a new long run equilibrium. It is then possible to treat the structural change as changes in the intercept and/or changes in the slope (i.e. the cointegration vector).

(Perron 1989), (Zivot and Andrews 1992), (Gregory 1996) and (Perron 1997) all argue that standard unit root tests are biased towards the null of non-stationarity (and the null of no cointegration in the case of residual-based cointegration tests) in the presence of unanticipated structural breaks or regime changes.

Basically, two types of model will be relevant for our analysis.

In the level shift model, the equilibrium equation shifts in a parallel fashion as only the intercept changes.

Level shift model (model S):

$$i_t = a_1 + (a_2 - a_1)DU_t + b\pi_t + Z_t \tag{5}$$

with $DU_t = \begin{cases} 0 & if\, t < Tb \\ 1 & if\, t \geq Tb \end{cases}$

In the regime shift model, we allow a change in the coefficient of the long run equilibrium in addition to a level shift.

Regime shift model (model C/S):

$$i_t = a_1 + (a_2 - a_1)DU_t + b_1\pi t + (b_2 - b_1)\pi_t DU_t + Z_t \tag{6}$$

So as to best define the error correction mechanism, as the last step, we use a TAR model (see ($Van\ Dijk$ 2002) for a detailed review) for the bivariate time series $Y_t = \begin{bmatrix} y_t \\ x_t \end{bmatrix}$, whose components y_t, x_t each contain a unit root.

If both components have a joint stochastic trend B_t ($y_t + \beta x_t = B_t$ where $B_t = B_{t-1} + \eta_t$) , then it is possible to find a stationary linear combination of these two integrated variables:

$y_t + \alpha x_t = Z_t$, where $Z_t = \rho Z_{t-1} + \varepsilon_t$ with $|\rho| < 1$.

In the standard Error Correction Model (ECM), the short run adjustment towards long run equilibrium is supposed to be always present and time-invariant.

The parameters which measure the mean reverting intensity towards equilibrium are considered as fixed. Nevertheless, the movements towards equilibrium value do not always appear. So as to take into account the possible non-linearities in the adjustment dynamics towards equilibrium, we introduce two regimes in the dynamics of the error term Z_t.

We have then two different adjustment procedures towards the equilibrium relationship, according to which regime belongs the Error Correction variable. Given that the coefficients γ_i are defined according to the autoregressive coefficient ρ, the coefficients in the ECM also depends of the regime $i = \{1, 2\}$:

$\Delta y_t = \mu_i + \gamma_{1,i}.Z_{t-1} + A_{1,i}(L)\Delta Y_t + v_{1,t}^i$

$\Delta x_t = \mu_i + \gamma_{2,i}.Z_{t-1} + A_{2,i}(L)\Delta Y_t + v_{2,t}^i$

with $\gamma_{1,i} = -(1-\rho_i)\beta/\left(\beta - \alpha\right)$ and $\gamma_{2,i} = -(1-\rho_i)/\left(\beta - \alpha\right)$

This model allows us to specify a time-varying adjustment mechanism: there will be mean-reversion as soon as γ_i will be negative and significant (and none if it is positive or non significant). Here the components of Y_t are linked by a long-run equilibrium relationship, whereas the adjustment towards this equilibrium is nonlinear and can be characterized as regime switching, with the regimes determined by the size and/or sign of the deviation from equilibrium.

2.2.2 Asymmetries and smooth transition between the regimes in the Error Correction Model

To detect any asymmetry, we undertake a simple Self Extracting Threshold AutoRegressive (SETAR) test as well as a Momentum-Threshold AutoRegressive (M-SETAR) test for size (amplitude) and sign asymmetries respectively. In the SETAR model, the dynamics of the variable studied depends on the level of this variable. In the M-SETAR model, the variable of interest has a distinct adjustment according to whether this variable increases or decreases from a given threshold.

Following (Balke and Fomby 1997), we estimate the cointegration relationship (6) in which the residuals Z_t follow a SETAR process: $\Delta Z_t = \phi_1 Z_{t-1}(1 - I_t) + \phi_2 Z_{t-1} I_t + u_t$ where I_t is the heavyside function : $I_t = \left\{ \begin{array}{ll} 1 & \text{if } Z_{t-l} < \tau \\ 0 & \text{if } Z_{t-l} \geq \tau \end{array} \right\}$, or an M-SETAR process if the heavyside function depends on the difference of the residuals : $I_t = \left\{ \begin{array}{ll} 1 & \text{if } \Delta Z_{t-l} < \tau \\ 0 & \text{if } \Delta Z_{t-l} \geq \tau \end{array} \right\}$.

Here the threshold will correspond to the attractor as soon as $\tau = 0$.

In testing whether the TAR is statistically significant relative to a linear $AR(p)$ one faces the problem that the threshold parameter is not identified under the null hypothesis. However, (Hansen 1996) shows that given a set of possible threshold values $\lambda \in \Lambda = [\lambda_1, \lambda_2]$ along with the least squares threshold estimate $\hat{\lambda}$, one can perform a sequence of Wald tests over the values in this set. Evidence for the null hypothesis of linearity can be assessed using:

$$W_T = W_T\left(\hat{\lambda}\right) = \sup_{\lambda \in \Lambda} W_T(\lambda)$$

where $W_T\left(\hat{\lambda}\right)$ is the Wald test of the null hypothesis. The asymptotic null distribution of W_T is non-standard. Appropriate critical values can be found by bootstrapping the data. (Caner and Hansen 2001) perform a Monte-Carlo experiment to explore the size and power properties of the Bootstrap W_T test. The evidence suggests that the test is free from size distortions and that the power of the test increases with the magnitude of the threshold effect.

Such a nonlinear extension incorporates the smooth transition mechanism in an ECM to allow for nonlinear or asymmetric adjustment.

Again, we define the error correction term as Z_t, the deviation from the long run equilibrium relationship defined as $\beta' X_t$ where X_t includes a $(K \times 1)$ vector Y of k I(1) variables (and $K - k$ deterministic as well as dummy variables in the case of the Gregory & Hansen methodology) and β a $(K \times 1)$ vector. Here, it is interesting to introduce smoothness in the transition function by using a two regime Vector Smooth Transition Auto Regressive (VSTAR) model. We have then the Smooth Transition Error-Correction Model [STECM]:

$$\begin{aligned}\Delta Y_t &= (\Phi_{1,0} + \alpha_1 Z_{t-1} + \sum_{j=1}^{p-1} \Phi_{1,j} \Delta Y_{t-j}) G(Z_{t-1}, c, \gamma) &(7)\\ &\quad + (\Phi_{2,0} + \alpha_2 Z_{t-1} + \sum_{j=1}^{p-1} \Phi_{2,j} \Delta Y_{t-j})[1 - G(Z_{t-1}, c, \gamma)] + \varepsilon_t &(8)\end{aligned}$$

where $\Phi_i = \left[\phi_{i,0}, \alpha_i, \phi_{i,1}, ..., \phi_{i,p-1}\right]$ is a $(P + 1 \times k)$ vector of parameters.

Here, we choose the transition function $G(Z_{t-1}, c, \gamma)$ to be the first order logistic function $[1 + \exp(-\gamma_i (z_{t-1} - c_i))]^{-1}$ for $\gamma > 0$ and $c > 0$, so as to detect asymmetric behavior for small and large equilibrium errors. This results in gradually changing strength of adjustment for larger (both positive and negative) deviations from equilibrium.

Intuitively, market frictions often suggest that the degree of error correction is function of the size of the deviation from the equilibrium.

The transition function goes monotonically from zero to one as Z_{t-1} increases, being equal to 0.5 for $Z_{t-1} = c$. Consequently, the parameter c may be viewed as the threshold between two regimes. The parameter γ governs the smoothness of the transition between regimes. An advantage of the logistic function is that for $\gamma \to 0$, the function collapses to a constant (equal to 0,5). Hence, the model becomes linear when $\gamma = 0$ and the LSTAR model does nest a two-regime SETAR model as a special case.

In the test for linearity, according to (Luukkonen and Terasvirta 1988), we replace the transition function G by a suitable first order Taylor approximation. In the reparametrized equation, the identification parameter is no longer present so that the linearity can be tested by means of a Lagrange Multiplier (LM) statistic with a standard asymptotic χ^2 distribution under the null hypothesis of linearity: $LM \xrightarrow{H_0} \chi^2 (p + 1)$.

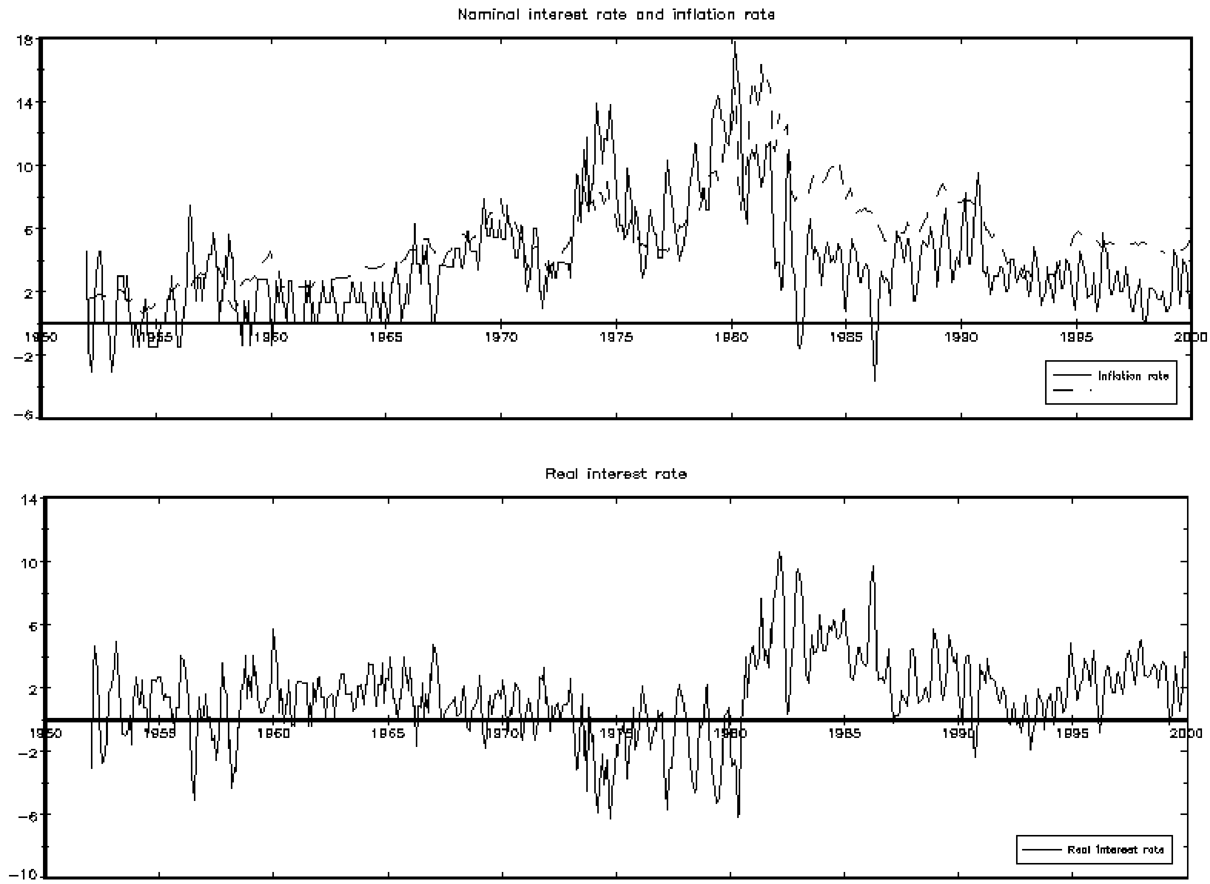


Figure 1: Real interest rates as the difference between nominal interest rates and inflation rates

3 Empirical results

The data used here for interest rates are monthly measures of the Treasury Bill Rate of 3 months to maturity and spans from 1951.1 to 1999.12. The inflation rates are calculated from monthly values of the urban CPI[5] (see figure 1).

If we run ADF (Augmented Dickey Fuller) tests (see (Dickey and Fuller 1981)

[5]The choice of 1951 for the beginning of the data could be explained by the fact that tests for periods prior to 1951 would be meaningless. During World War II and up to the Treasury-Federal Reserve Accord of 1951, interest rates on Treasury Bill were pegged by the government with the result that Treasury Bill rates did not adjust to predictable changes in inflation rates.

and (Said and Dickey 1984)) with or without the modifications of ERS ((Elliott, Rothenberg, and Stock 1996)), we can conclude that the driving process of each series is a random walk with no drift. All the $t - stat$ for the null hypothesis of unit root are not significant at the 5% level (see table 1).

We will distinguish two cases: the first one corresponds to the case where we will apply Threshold AutoRegressive (TAR) tests to the residuals of the long run cointegration relation with breaks. On the other hand, a full Fisher effect can also be assumed and therefore TAR tests are applied directly to the real interest rate. This is the second case.

3.1 First step: the long run relationship between inflation and interest rates

3.1.1 Cointegration tests

We will follow both the Engle & Granger (EG) (Engle and Granger 1987) and the Johansen procedures to test for a unit root in the residuals of the long run relationship between inflation and nominal interest rates.

The standard method of EG to test the null hypothesis of no cointegration are residual-based. The candidate cointegrating relation is estimated by OLS and a unit root test is applied to the regression errors.

$$i_t = a + b\pi_t + v_t \tag{9}$$

In the framework of a cointegration relationship, we have the following long run equation:

$i_t = 1.03 + 1.15\pi_t + Z_t$

The results are contradictory, as the Johansen procedure is supporting the hypothesis of cointegration while the residual-based one is not (cf appendix).

If we expect any structural breaks to occur in the sample studied, then it is preferable to rely on the residual based cointegration tests of (Gregory 1996).

This will help us to specify correctly any sudden and exogenous change in the process, such as the different post oil-crisis monetary policy conducted after the nomination of Volcker as the chairman of the FED in 1979.

We propose to test the Fisher effect for both models, level shift model (model S) and the regime shift model (model C/S), described in the previous section.

In all cases, the time break is treated as unknown and is estimated with a data dependent method which corresponds to the minimum of the t-stats computed on a trimmed sample. Here, the results lead us to introduce a structural break in July 1979.

From both tests (Engle&Granger-ADF and Engle&Granger-PP), we have some uncertainty about the existence of a cointegration relationship between nominal interest rate and inflation rates, according to which test is used (Phillips-Perron test or ADF test). However, it is possible to reject the null hypothesis of no cointegration when allowing a break in the cointegrating vector.

3.1.2 SETAR tests on the residuals of the cointegration relationship

We then apply the TAR and M-TAR unit root tests on the residuals Z_t. We choose to begin by the M-SETAR tests because one restricting condition on TAR tests in general is the stationarity of the threshold variable. In the case of a M-TAR test, this restriction is avoided. So, if it is possible to reject the unit root hypothesis in the case of the M-TAR test, we are allowed to continue with the TAR test since we have demonstrated the stationarity of the threshold variable in non linear context.

We have two different adjustment procedures towards the equilibrium relationship, according to which regime belongs the Error Correction variable Z_t in the following VECM:

$$\Delta Y_t = \mu_i + \gamma_i Z_{t-1} + A_i(L)\Delta Y_t + v_t^i$$

Firstly, we run a M-TAR model (see the results at the end of the article).

The Wald statistic is maximum for a long difference of 10, estimated by the program for delays between 1 to 12. Moreover, unit root tests run for each of the regime suggest that the regime 1 is the most stationary between the two and most of the observations which belong to the regime 2 (which represent in total 20% of the observations) are located after 1979, which is consistent with the change in regime of the monetary policy occurred by the arrival of Volcker at the head of the FED.

And in the case of a TAR model, we have the two following regimes:

$$\Delta Z_t = \left\{ \begin{array}{c} \underset{(0.0383)}{-0.0405} - \underset{(0.0318)}{0.0751}.Z_{t-1} + \underset{(0.0548)}{0.125}.\Delta Z_{t-2} + ... - \underset{(0.0506)}{0.112}.\Delta Z_{t-10} \\ \text{when } Z_{t-4} \geq 1.75 \\ \underset{(0.179)}{0.393} - \underset{(0.0473)}{0.113}.Z_{t-1} + \underset{(0.0954)}{0.316}.\Delta Z_{t-1} + ... - \underset{(0.0803)}{0.205}.\Delta Z_{t-12} \\ \text{when } Z_{t-4} < 1.75 \end{array} \right\}$$

Note that $Z_t \geq x$ is equivalent to $i_t - 1.15\pi_t \geq x + 1.03$ and $Z_t < x$ to $i_t - 1.15\pi_t < x + 1.03$.

We have then the following two regime ECM (with the respective p-value in parenthesis)

$$\Delta i_t = \left\{ \begin{array}{c} \underset{(7\%)}{0.03}.Z_{t-1} + 0.53\Delta i_{t-1} + ... - 0.15.\Delta i_{t-9} - 0.047\Delta\pi_{t-2} \\ \text{when } Z_{t-4} < 1.75 \\ \underset{(0\%)}{0.375} - \underset{(0\%)}{0.014}.Z_{t-1} + 0.18\Delta i_{t-1} + ... + 0.27.\Delta i_{t-9} - 0.19\Delta\pi_{t-2} - ... - 0.18\Delta\pi_{t-8} \\ \text{when } Z_{t-4} \geq 1.75 \end{array} \right\}$$

The Wald statistic is maximum for a delay of 3, estimated by the program for delays between 1 to 12.

According to the tests results (see in the end of the article), we find evidence of stationarity and asymmetry in the residuals of the long run cointegration relationship between inflation and interest rates.

3.2 Second step: the full Fisher effect

According to the results of unit root tests versus an alternative SETAR or M-SETAR applied to the residuals of the long run relationship between inflation rates and interest rates, there is a strong evidence for non linearity and we are then able to reject the null hypothesis of no cointegration in a non-linear context. So it is possible to retain a non linear Fisher effect. But what about assuming the hypothesis of a pure non linear Fisher effect, which means that the residuals of the cointegration relationship will be the real interest rates series?

3.2.1 SETAR tests on the real interest rates

More specifically, we have the following results for the M-SETAR model applied on the real interest rates:

Regime 1

$$\Delta r_t = \underset{(0.0613)}{0.0361} - \underset{(0.0247)}{0.0498} r_{t-1} + 0.1117.\Delta r_{t-2} + ... - 0.131.\Delta r_{t-11}$$
$$\text{if } r_{t-1} - r_{t-11} < 3.14$$

Regime 2

$$\Delta r_t = \underset{(0.571)}{-1.87} - \underset{(0.0763)}{0.126}\, r_{t-1} + 0.825.\Delta r_{t-1} + 0.656.\Delta r_{t-2} + ... - 0.381.\Delta r_{t-10}$$
$$\text{if } r_{t-1} - r_{t-11} \geq 3.14$$

Again, through Wald tests, it is in the first regime that the series appear to be the most stationary. And among the 10% of all the observations that belong to the regime 2, most of them are located after 1979.

In the context of a SETAR model, we have the following:

Regime 1

$$\Delta r_t = \underset{(0.0577)}{0.0787} - \underset{(0.0315)}{0.0625} r_{t-1} + 0.122.\Delta r_{t-2} + ... - 0.0839.\Delta r_{t-12}$$
$$\text{if } r_{t-3} < 3.39$$

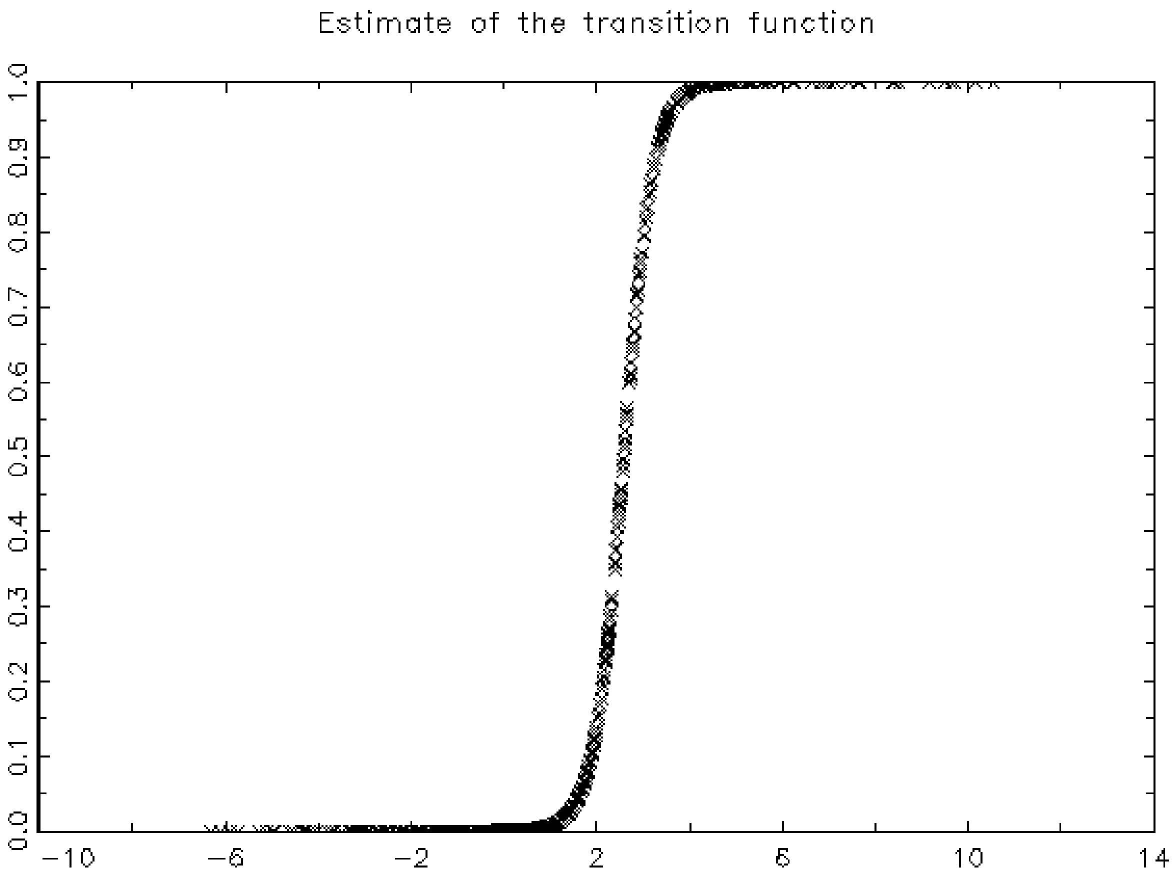
Regime 2

$$\Delta r_t = \underset{(0.328)}{0.432} - \underset{(0.069)}{0.0836} r_{t-1} + 0.425 \Delta r_{t-1} + 0.0984.\Delta r_{t-2} + ... - 0.0186.\Delta r_{t-12}$$
$$\text{if } r_{t-3} \geq 3.39$$

The Wald statistic is maximum for a delay of 3, estimated by the program for delays between 1 to 12. Moreover, unit root tests run for each of the regime suggest that the regime 1 is the most stationary between the two and most of the observations which belong to the regime 2 (which represent in total 20% of the observations) are located after 1979.

So the real interest rate process display size (and sign) asymmetries.

3.2.2 Special case: Smooth transition between the regimes



Logistic transition function versus the threshold variable (real interest rates)

Finally, we choose to model the real interest rate in the error correction model by a Logistic Smooth Transition AutoRegressive (LSTAR) process[6]. This kind of nonlinear behavior may result from non-synchronous interventions, heterogeneous agents and some intervention costs.

We have then two regimes in the STAR model for the real interest rate series (see table of results at the end of the article).

The t-stat of the first order autoregressive coefficient of Δr_t equals to 13.06 in the higher regime, meaning that it is significantly different from zero[7] so that we reject the null hypothesis of a unit root in this regime. Intuitively, as r_t is stationary in the higher regime, the system will be globally stationary.

Here our test statistic for linearity is equal to 20.53 which corresponds to a p-value less than 0.01%, thus allowing us to reject the linearity hypothesis for the real interest rates. The transition speed between the two regimes is equal to 1.23.

We have then the following two regime ECM (with the p-value for the significancy of the respective parameters below in parenthesis):

$$\Delta i_t = \left\{ \begin{array}{c} \underset{(0\%)}{0.375} - \underset{(0\%)}{0.143}.r_{t-1} + \underset{(1.1\%)}{0.18\Delta i_{t-1}} + ... + \underset{(0\%)}{0.27.\Delta i_{t-9}} - \underset{(0\%)}{0.19\Delta \pi_{t-2}} - ... \underset{(0\%)}{-0.18\Delta \pi_{t-8}} \\ if \qquad r_{t-1} < 2,58 \\ \underset{(6.9\%)}{-0.026.r_{t-1}} + \underset{(0\%)}{0.53.\Delta i_{t-1}} + ... - \underset{(0.3\%)}{0.15.\Delta i_{t-6}} - \underset{(0\%)}{0.05\Delta \pi_{t-2}} \\ if \qquad not \end{array} \right\}$$

Here, the model clearly displays a regime of mean reversion in the lower regime where the relative coefficient for mean reversion is negative and significant while in the high regime the coefficient is less important in absolute value while being less significant. This would mean that the series in this regime display weaker mean-reversion effects then being less likely to return to the long

[6] Usually, two interpretations of the STAR model are possible. On the one hand, the STAR model can be thought of as a regime-switching model that allows for two regimes, associated with the extreme values of the transition function (i.e. 0 and 1) where the transition from one regime to the other is smooth. On the other hand, the STAR model can be said to allow for a 'continuum' of regimes, each associated with a different value of $G(Z_{t-1}, c, \gamma)$ between 0 and 1. In this paper we will use the 'two regime' interpretation.

[7] We use the usual critical values since the full Fisher Effect has been validated in a previous work (cf Million(2002)), meaning that the nominal interest rates and the inflation rates are cointegrated with a cointegration vector of $[1, -1]$.

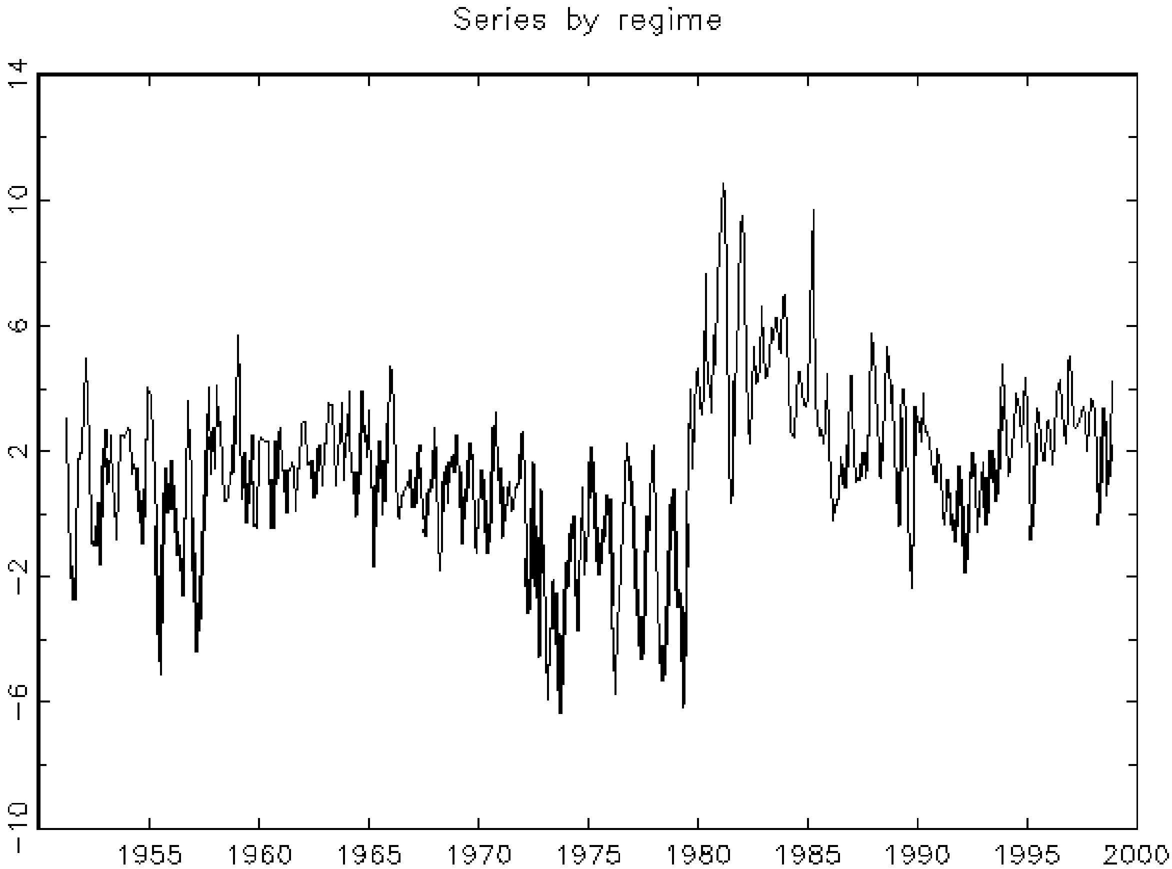


Figure 2: High and low regimes for the real interest rates

run equilibrium value around the threshold value of the real interest rate of 2,58. This would help us to explain why real interest rates were kept so high in the 80s in the United States.

4 Interpretations and conclusion.

This paper examined the long-run relationship between nominal interest rates and inflation with an application to the US data, and showed strong evidence for a threshold behavior in the real interest rates series.

Through SETAR & M-SETAR tests, we underlined the existence of two kinds of asymmetries: size asymmetries and sign ones. We explain size asymmetries differential adjustments to small and large deviations from the long run equilibrium caused by factors such as non convex adjustment costs for the Central Bank failing to activate the restrictive monetary policy mechanisms when the deviations are small. On the other hand, sign asymmetries which are differential adjustments to positive and negative deviations from long run equilibrium occur when a process exhibits hysteretic dynamics on one side of the attractor. Moreover, we found strong evidence for a smooth transition between the regimes of high inflation and low inflation.

Furthermore, this could be interpreted as an opportunistic behavior of the Central Bank, meaning that the policy maker (still pursuing an objective of price stability) will change his behavior depending on the level of inflation. Whenever the inflation rate falls on a band of tolerable inflation, the policy maker will be more reluctant to conduct an active policy (by decreasing nominal rates for instance), but merely engage in a policy of watchful waiting (which is consistent with a stance of inflation targeting). However, in a context of high inflation (which will correspond to our low regime where real interest rates are decreasing everything else remaining equal), the monetary authorities will change nominal interest rates so that inflation rates will go back to acceptable values.

This evidence should resolve the puzzle of why the Fisher effect appears to be strong in some periods but not in others. Just as this analysis predicts, a long-run Fisher effect appears to be strong in the periods when interest rates and inflation exhibit stochastic trends: these two series will trend together and thus there will be a strong correlation between inflation and interest rates. On the other hand, as soon as those variables do not exhibit stochastic trends simultaneously, a strong correlation between interest rates and inflation will not appear if there is no short-run Fisher effect. Thus, the presence of a long-run but not a short-run Fisher effect predicts that a Fisher effect will not be detectable during periods when interest rates and inflation do not have trends. It is exactly in these periods that Mishkin was unable to detect any evidence for a Fisher effect. Recognition that the level of inflation and interest rates may contain stochastic trends suggests that the apparent ability of short-term interest rates to forecast inflation in the postwar United States is spurious. Indeed, according to Mishkin, the findings here are more consistent with the views expressed in (Fisher 1930) than with the standard characterization of the so-called Fisher effect in the last twenty years. The evidence in this paper thus supports a return to Irving Fisher's original characterization of the inflation interest rate relationship.

Our results are consistent with those of (Clarida, Gali, and Gertler 1998) who found adjustment of nominal interest rates by the Central Bank more than one-to-one with future expected inflation rates as an optimal policy rule, calling into question the standard Fisher effect hypothesis.

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Type of Test	i_t	π_t	r_t	5% *c.v.*
ADF	-2.52	-2.5	-2.7	-2.88
DF-GLS	-0.66	-1.26	-1.83	-1.95
KPSS	2.07	/	/	0.463

Table 1: Unit root test results

Type of test	*Stat*	5% *c.v.*
EG-ADF	-2.49	-3.36
EG-PP	-10.43	-3.36
Johansen (rk=0)	24.09*	20.0
Johansen (rk6 1)	5.99	9.2
GH-C	-5.33	-4.61
GH-C/S	-3.35	-4.95

Table 2: Cointegration test results

5 Appendix

Type of Test	M-SETAR1	SETAR1	M-SETAR2	SETAR2	LSTAR
number of lags	12	12	12	12	12
μ_1 (low regime)	-0.016	-0.4	0.036	0.078	0.3*
ρ_1 (low regime)	-0.05	-0.075	-0.049	-0.0625	0.77*
μ_2 (high regime)	-0.6	0.393	-1.87	0.432	-0.06
ρ_2 (high regime)	+0.127	-0.113	-0.126	-0.084	0.9*
UR statistic test p-value	0%	0%	4%	0%	0%
Delay order	10	4	10	3	1
Linearity test statistic	85.5	64.8	42.9	55.5	20.53
Statistic p-value	0%	1%	3%	0%	0.2%
Threshold estimate	1.83	1.75	3.14	3.39	2.58

Table 3: SETAR tests results