APPLICATION OF THE PROPORTIONALITY-CALIBRATED AIDS MODEL TO PREDICTING POTENTIAL WELFARE EFFECTS OF MERGERS BETWEEN FERTILIZER SELLERS AS PART OF THE TURKISH PRIVATIZATION PROGRAM

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Abstract

Antitrust economists routinely utilize simulation models to predict the price effect of a transaction or agreement that involves collective pricing and/or collective profit maximization, such as a merger. Proportionality-Calibrated Almost Ideal Demand System (PCAIDS) model is a variant of the Almost Ideal Demand System (AIDS) model used to locally approximate a demand system for a differentiated-product market where the competitors are engaged in Bertrand conduct. PCAIDS model exploits the independence of irrelevant alternatives (IIA) assumption to simulate firms' (collective or unilateral) pricing conduct even where only a few pieces of market-level and firm-level information are available. We apply this model to actual and hypothetical proposed merger cases in Turkey, both as a market test and to predict the unilateral price increase effect of each transaction. We also calculate the minimum level of marginal cost savings that the merging firms would need, in order to maintain post-merger prices at pre-merger levels.

Keywords: calibration, fertilizer, mergers, PCAIDS, simulation, SSNIP, unilateral, welfare

JEL Classification Codes: L41 (Horizontal Anticompetitive Practices); L65 (Industry Studies: Chemicals)

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In Turkey, selective privatization of publicly owned assets has been on the agenda for several years. Turkish Privatization Administration (TPA) is a government body established to supervise the transfer of public assets to private buyers. Decisions of the Privatization Administration are subject to challenge by the Turkish Competition Authority (TCA).[2] Fertilizer producer *İstanbul Gübre Sanayii A.Ş.* (İGSAŞ) was selected to be privatized in 1998. In May 1998, TPA informed TCA of its decision to sell İGSAŞ to a competing fertilizer producer, *Toros Gübre* (Toros). TCA started an investigation of the sale and in March 2000, decided against it.

In this paper we use a stylized merger simulation model to estimate the proposed privatization's likely effect on the price of nitrogenous fertilizers used as an agricultural input. In evaluating the likely economic effects of the transaction, we consider "unilateral price effects" that would have resulted from the proposed privatization. We find economically significant price effects even in the absence of "coordinated effects."

Our predictions support TCA's market definition from the (economic) perspective of the *U.S. Horizontal Merger Guidelines* (*Guidelines*),[3] in that a merger between all nitrogenous fertilizer sellers would have raised the price of the product well in excess of 5%. Moreover, we find that a merger between Toros and İGSAŞ would have raised the price of nitrogenous fertilizer no less than 1%, and probably a higher amount. We also predict the offsetting marginal cost savings ("efficiencies") needed to reduce the privatization's price effect to zero. To place our findings in a comparative perspective, we also simulate hypothetical mergers between İGSAŞ and other Turkish nitrogenous fertilizer producers.

Our findings suggest that a combined İGSAŞ-Toros could have significantly raised the price of the product, relative to the current ("no privatization") state of the

[2] First, TPA informs TCA of its intent to privatize a given asset and requests TCA's view. Next, TPA holds a sale auction and determines at most three candidates as potential acquirers. TCA then decides whether any of these potential sales would violate the competition law. Finally, the Supreme Council for Privatization sells the asset to a candidate not vetoed by TCA. In İGSAŞ case, TCA's initial view claimed absence of jurisdiction over a potential İGSAŞ-TÜGSAŞ merger; it also determined that a potential sale neither to Gübretaş, nor to Bagfaş and/or Ege, would be problematic. Subsequently TPA determined Toros as the only potential acquirer, which TCA vetoed.
[3] United States Department of Justice and Federal Trade Commission, *Horizontal Merger Guidelines*, 1997.

world. Even more importantly, the combined İGSAŞ-Toros would have significantly raised prices relative to a merger with any of the smaller potential buyers.

Our estimated price increase is likely to be an underestimate of the true expected price increase for two independent reasons. First, insofar as the publicly owned İGSAŞ did not behave as a pure profit maximizer, and did not exploit its market power to the extent that a privately owned firm would have, predicted merger effects would be biased downward relative to true effects. Second, state subsidies (paid on a "sold unit" basis) to sellers (or purchasers) will dampen fertilizer users' responsiveness to a price increase. Further, this dampening effect increases with the ratio of the unit subsidy to the purchase price.[4] If this unit subsidy effect has not been netted out from fertilizer market demand elasticity estimates, our predictions will tend to be smaller than the true price effect of a merger.

The rest of the paper is organized as follows. In Section I, we give a brief overview of the Turkish Competition Law and more detailed accounts of the proposed transaction and TCA's opposing decision. In Section II, we discuss the place of simulation models in competition economics. Section III discloses model assumptions and results, Section IV concludes.

I. Background

Turkish Competition Law prohibits mergers and acquisitions which create or strengthen the dominant position of one or more enterprises as a result of which competition is significantly impeded in the market for goods and services in the whole or part of the territory of the country.

When appraising mergers and acquisitions, TCA particularly takes into account the need to maintain and develop effective competition within the country in view of the structure of the relevant market, and the actual or potential competition from firms located either within or outside the country. TCA also considers the market position of these firms, their economic and financial powers, the alternatives available to suppliers and users, their opportunities for access to sources of supply or entry into markets; any legal or other barriers to entry into the market; supply and demand trends for the relevant

[4] TCA (2000) reports that unit subsidies paid by the state amounted to 20-25% of the price at that time.

goods and services, the interests of the intermediate and ultimate consumers, developments in the technical and economic progress provided that they are to the advantage of consumers and do not form an obstacle to competition. TCA may also permit a merger or an acquisition on condition that appropriate measures are implemented, and certain obligations are complied with, so as to alleviate anticompetitive concerns.

In its investigation of the proposed İGSAŞ-Toros merger, TCA determined two separate relevant product markets: nitrogenous fertilizers and composite fertilizers. Nitrogenous fertilizers include only nitrogen as a nutrient. Almost half of all such fertilizers sold in Turkey consist of urea; other types are ammonium sulfate (AS), ammonium nitrate (AN) and calcium ammonium nitrate (CAN). Composite fertilizers, unlike nitrogenous fertilizers, contain more than one nutrient. The geographical market was determined to be the geographical area of the Turkish Republic. In its geographical market definition, TCA determined that sellers of each type of fertilizer do not price discriminate with respect to location or proximity. Almost 80 percent of İGSAŞ's total sales are nitrogenous fertilizers.

TCA decided that the proposed sale would only be problematic for competition in the nitrogenous fertilizer market; this is the market we focus on. In that market, Bagfaş, Gübretaş, TÜGSAŞ, and Ege are the other producers in addition to İGSAŞ and Toros. These producers also bring in approximately two-thirds of imports (potential acquirer Toros alone accounts for one-third); about ten independent firms import the remainder.[5] Imports make up between 40% and 50% of all sales. Nitrogenous fertilizers are also a byproduct of steel-iron industry; nearly 2% of all nitrogenous fertilizer volume is sold by Turkish steel-iron producers. The largest domestic producers of nitrogenous fertilizers are TÜGSAŞ (41% of total capacity), Toros (25% total of capacity), İGSAŞ (23% of total capacity), and Bagfaş (9% of total capacity), followed by steel-iron producers each with a 1% or less capacity share.

Farmer cooperatives like TKKMB, Tariş and Çukobirlik buy fertilizers at the wholesale level and resell them to their members. The rest of the sales are made directly

[5] In addition to entry barriers into production, TCA also stressed entry barriers as an importer, which was evident in that the dominant share of imports belonged to the producers.

to farmers. Gübretaş, which has no production capacity in nitrogenous fertilizers, is vertically integrated with TKKMB.

Market shares of the sellers can be calculated in two ways. One can present shares either in terms of physical sales, or in terms of the nitrogen amount contained in each firm's sales. For example, in 100 kilograms (kg) of urea, there is 46 kg nitrogen and 54 kg fill. A firm that physically sells 100 kg of urea and 50 kg of CAN (which contains 13 kg of nitrogen) is in fact selling 46+13 kg of nitrogen, and its sales volume can alternatively be stated as 59 kg of nitrogen. Market shares of nitrogenous fertilizer sellers for each of years 1997, 1998 and 1999 are presented in Table 1 below, reproduced from TCA (2000). In the table, columns labeled "including fill" show shares in terms of the physical sales volume and columns labeled "nitrogen content" show shares in terms of nutrient volume. Independent importers and firms producing fertilizer as a byproduct are included in the "other" category. In its decision, TCA used market shares based on nitrogen content.

Table 1 -- Percent shares in the nitrogenous fertilizers market

Year:	**1997**		**1998**		**1999**		**1997-99**
Firms	Share, including fill	Nitrogen content share	Share, including fill	Nitrogen content share	Share, including fill	Nitrogen content share	Average nitrogen share
Toros	24.24	21.41	26.34	24.17	32.64	31.46	25.68
TÜGSAŞ	21.32	17.51	24.45	19.81	23.10	18.84	18.72
İGSAŞ	18.40	26.34	17.83	24.59	10.87	14.76	21.90
Ege	1.17	0.95	2.13	2.04	2.72	2.71	1.90
Gübretaş	13.59	14.96	4.02	4.77	3.20	3.92	7.88
Bagfaş	5.56	3.83	6.46	5.52	5.61	4.94	4.76
Others	15.72	14.99	18.77	19.09	21.86	23.38	19.15
Total	100	100	100	100	100	100	100

TCA stated that the sale would amount to a merger between the two largest sellers of nitrogen, which would create a dominant market position. The combined firm would also have the largest production capacity of nitrogenous fertilizer; its share would approximate one half of all capacity. In its assessment, TCA emphasized the exclusive dealership practices of Toros; no other fertilizer producer had engaged in similar vertical restrictions. TCA argued that these vertical restraints would reinforce the dominant position of the combined firm. As a result, TCA in its final decision prohibited the acquisition of IGSAS by Toros.

II. Competition Economics and Merger Simulation

The last ten years have witnessed substantial developments in applied economics of competition. Particularly important developments have taken place in the field of predicting the welfare effects of horizontal mergers and acquisitions. In the U.S., the competition agencies as well as the interested parties now commonly use empirical models for estimation and simulation of unilateral price effects in horizontal merger cases (Weiskopf, 2002).[6]

When analyzing potential economic effects of a horizontal merger, competition agencies first determine the extent of the relevant market, over two dimensions: product space and geographic space. *Guidelines* suggest a test known as the "small but non-transitory increase in price" (SSNIP). SSNIP test asks the following question: could a "hypothetical monopolist" of a group of products produced and/or sold in a geographic area impose a small but non-transitory in price relative to the competitive situation? The relevant antitrust market is defined to be the smallest group of products and the smallest geographic area that include the merging products and satisfy the SSNIP test.[7]

To understand the probable effects of the merger on consumer welfare, competition agencies might ask two related but separate questions. The first question they usually ask is, "would the merger cerate or enhance market power, or facilitate its use, by increasing market concentration?" The underlying presumption is that in relatively concentrated markets where only a few firms account for most of the sales of a product, those firms can exercise market power by coordinating their actions (explicitly or implicitly). Competition economists usually approach market concentration by calculating the market shares of the products in the relevant market. Using these shares, they then calculate the Herfindahl-Hirschman Index (HHI), the sum of squared market

[6] See Hausman et al. (1994), Nevo (2000), Werden (1997) and (2000), Werden and Froeb (2000). For a critical view, see Muris (2003).
[7] See U.S. Department of Justice and Federal Trade Commission (1992). A SSNIP threshold of 5% is usually assumed in practice.

shares (in percentage points) of all sellers in the relevant market.[8] Markets with a post-merger HHI of less than 1000 are considered "unconcentrated," and mergers in such markets unlikely to have adverse competitive effects. Markets with a post-merger HHI between 1000 and 1800 are labeled "moderately concentrated"; a merger that increases the HHI by less than 100 points in such a market is deemed unlikely to have adverse competitive consequences. Markets with a post-merger HHI of greater than 1800 are considered "highly concentrated." Even in highly concentrated markets, a merger that increases the HHI by less than 50 points is seen as relatively benign; a merger that increases the HHI by more than 100 points is seen as likely to create or enhance market power or facilitate its exercise.[9]

Absent a simulation approach that enables the analyzer to predict post-merger equilibrium prices and market shares directly, post-merger concentration level is usually approximated by using pre-merger shares. As we demonstrate below, when a merger would lead merging parties to raise price, this approach would overestimate post-merger concentration relative to the simulated equilibrium approach.

The second question that the agencies might ask is, "can the merging firms exercise market power through unilateral or non-coordinated conduct?" *Guidelines* define unilateral conduct as "conduct the success of which does not rely on the concurrence of other firms in the market or on coordinated responses by those firms." Generally, an informed analysis of unilateral effects would consist of an estimation stage, followed by calibration and simulation stages. Estimation stage involves calculation of parameter values for a demand system that characterizes the behavior of purchasers of the relevant products before the merger. Calibration involves "fitting" the estimated demand parameters to pre-merger market shares and profit margins; under short-term profit maximizing behavior, demand parameters and observed market shares imply a profit margin for each product.[10] At the simulation stage, estimated demand parameters and

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[8] The least concentrated market has many sellers with very small shares; in such a market the HHI is close to zero. At the other extreme, the most concentrated market has only one seller; the HHI for a monopolist is $100^2 = 10000$. HHI values for all possible market configurations fall between these two extremes. For example, the HHI for a market of three sellers with shares of 50%, 30%, and 20% is $50^2 + 30^2 + 20^2 = 3800$.

[9] U.S. Department of Justice and Federal Trade Commission (1992).

[10] The relevant profit margin is the "gross margin," also known as the Lerner Index, defined as one minus the ratio of marginal cost to price = 1 – c/p.

profit margins are used to predict the likely price increase resulting from exercise of incremental market power by the merging products. When solving for the model's equilibrium, all producers are assumed to maximize their short-term profits after, as well as before, the merger.

The estimation approach may be relatively empirical, or relatively Bayesian. In a relatively empirical approach, econometric techniques are used to estimate demand parameters using disaggregated (e.g., consumer-level) transactions data, with few a priori restrictions placed on the magnitudes and signs of the parameters to be estimated.[11] Although this approach can be highly informative, obtaining and econometrically processing large amounts of data is typically expensive and time-consuming. For example, estimating a demand system for ten products would involve simultaneous estimation of at least 100 demand parameters.[12] Assuming that useful data can be found and afforded, adequately estimating such a system would require many hours of labor by econometricians working with specialized computer software. Even then, there is no guarantee that the resulting econometric estimates would be economically reasonable or statistically significant, or that they would represent a fair approximation to the true parameter values.[13]

An alternative to the empirical approach is the Bayesian approach, in which demand parameters are estimated from a relatively few pieces of data that can be easily and inexpensively obtained.[14] In comparison with the empirical approach, the main

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[11] See, *inter alia*, Capps et al. (2002), Hosken et al. (2002), SCHEFFMAN AND COLEMAN, Rill (1997). When parameters are empirically estimated, any a priori restrictions placed on the model can be tested; see Capps (2002).

[12] A typical demand system would specify the quantity demanded of each product as a function of all the product prices. With ten products, the demand system may be written as $\mathbf{D}(\mathbf{q}, \mathbf{p}, \mathbf{X}; \mathbf{b}, \mathbf{g}) = \mathbf{0}$, where **D** is a vector of implicit functions whose i'th element is the demand equation for the i'th product, **q** and **p** are arrays of product quantities and prices, respectively, **X** is an array of additional variables, **b** and **g** are arrays of parameters to be estimated. For example, the linear form is $q_i = a_i + b_{i,1}\times p_1 + \ldots + b_{i,10}\times p_{10} + \mathbf{g}_i\cdot\mathbf{X}_i$, the constant elasticity form is $\text{Log}(q_i) = a_i + b_{i,1}\times \text{Log}(p_1) + \ldots + b_{i,10}\times \text{Log}(p_{10}) + \mathbf{g}_i\cdot\mathbf{X}_i$, and the Almost Ideal demand System (AIDS) form is $p_i q_i/R = a_i + b_{i,1}\times \text{Log}(p_1) + \ldots + b_{i,10}\times \text{Log}(p_{10}) + \mathbf{g}_i\cdot\mathbf{X}_i$, where $R = \sum_{j=1}^{10} p_j q_j$, for product i = 1, …, 10. See Hosken et al. (2002), Capps et al. (2002), Crooke et al. (1999).

[13] For example, Capps et al. (2002) note convergence problems encountered during empirical estimation of the AIDS model. They also highlight the potential tradeoff between statistical bias and statistical efficiency when estimating alternative demand systems, and suggest that mean-squared error is the appropriate selection criteria.

[14] See Epstein and Rubinfeld (2001) and (2003), Werden and Froeb (2000), Froeb and Tschantz (2000).

disadvantage of a Bayesian model is the relatively high number of a priori restrictions placed on demand parameters in exchange for reduced data requirements; see Epstein and Rubinfeld (2001) and (2003).

To simulate proposed or hypothetical mergers between fertilizer sellers, we use Proportionality-Calibrated Almost Ideal Demand System (PCAIDS), first introduced in Epstein and Rubinfeld (2001). PCAIDS is a variant of the more generalized Almost Ideal Demand System (AIDS) model used to locally approximate a demand system where firms set prices to maximize short-run profits; PCAIDS exemplifies the type of simulation model that we call Bayesian. Appendix I discusses the model structure and inputs. Appendix II contains a sample simulation output and variable definitions.

III. Model Inputs and Results

Before simulating the effects of mergers, we tested TCA's definition of the relevant market as nitrogenous fertilizer sellers. Under our most conservative elasticity and margin assumptions, and based on market shares at a national level, we predict that a merger between the largest three producers Toros, TÜGSAŞ and İGSAŞ would enable these three firms to raise their prices in excess of 5%, which we assumed to be the SSNIP threshold that a competition agency would apply. This suggests that the three largest firms alone satisfy the SSNIP test of the *Guidelines*. By implication, any market that includes these three firms would satisfy the same test.[15] Table 2 shows the results of this simulation.

Table 2 – Price effect of a hypothetical merger between Toros, TÜGSAŞ and İGSAŞ

Model inputs		Merging firms' unilateral price increase	Assumed SSNIP threshold for market delineation
Market elasticity	TOROS margin		
1.60	0.10	5.4%	5.0%

In our market definition (SSNIP) test, we assume that nitrogenous fertilizer sellers cannot price discriminate between different buyers depending on buyer characteristics

[15] When we simulated the SSNIP test, we assumed a market elasticity of 1.6 for nitrogenous fertilizers and a profit margin of 0.1 for Toros. As we explain below, these two assumptions correspond to the most conservative case among those we consider in our simulations. The three sellers will always satisfy the SSNIP test under a lower market elasticity and/or a higher profit margin.

(for example, buyer size, location or proximity, history of purchasing preferences, or type of agricultural crop). If sellers are able to discriminate between buyers, then they may be able to collectively raise prices to a subgroup of buyers independent of other subgroups. In that case, the relevant market may be even smaller. For example, if sellers are able to discriminate between geographic locations, then the relevant market test may need to be applied separately to more than one geographical region and fertilizer sellers that sell predominantly in each region.[16]

Next, we simulated the unilateral price effects of the proposed merger between Toros and İGSAŞ, the results are presented in Table 3.

Table 3 – Unilateral price effects of the proposed Toros- İGSAŞ merger under alternative model inputs

Model inputs		Unilateral price effects of the merger			
Market elasticity	TOROS margin	Merging firms	Non-merging firms	Industry	Offsetting efficiencies
1.60	0.50	5.5%	0.5%	2.8%	5.8%
1.60	0.10	1.8%	0.3%	1.0%	3.0%
1.00	0.90	30.3%	0.9%	14.3%	24.3%
1.00	0.50	12.2%	1.3%	6.2%	14.7%
1.00	0.10	1.9%	0.3%	1.0%	3.3%
0.15	0.90	125.4%	10.9%	58.2%	> 100%
0.15	0.50	17.4%	2.3%	8.8%	28.4%
0.15	0.10	2.0%	0.3%	1.1%	3.6%

At the calibration stage, we used an arc elasticity of 1.6, referenced in Cakmak (1997), as our high estimate for the point elasticity of demand facing all nitrogenous fertilizer sellers.[17] We interpret this elasticity value as the long-run elasticity for nitrogenous fertilizer demand; this value is consistent with the implied range of long-run demand elasticity values for nitrogenous fertilizer reported by Hansen (2001). The first three rows of Table 3 show simulation results based on this elasticity value.

We also simulate the price effect under two alternative assumptions for nitrogenous fertilizer demand: a unit elasticity, and a derived agricultural input demand elasticity of 0.15. In Table 3, the middle three rows show simulation results based on a unit elasticity of demand facing all nitrogenous fertilizer sellers, which we interpret as a

[16] TCA (2000) did not find geographical price discrimination likely.
[17] We state only the magnitude (absolute value) of all demand elasticities.

medium-term demand elasticity estimate.[18] The last three rows show results based on a demand elasticity of 0.15, which we interpret as a short-term elasticity estimate.

To derive the agricultural input demand elasticity that we interpret as a short-term elasticity, we first assumed that the short-run demand elasticity facing all agricultural producers who use nitrogenous fertilizer is unity, which is very likely an overestimate.[19] (This would imply that a small change in the price of all agricultural products would not change total expenditures on these products.) We also assumed that the short-run elasticity of substitution between nitrogenous fertilizers and all other agricultural inputs is zero. Under these assumptions, the short-run derived agricultural input demand for nitrogenous fertilizer would be approximately equal to the share of nitrogenous fertilizer as an agricultural input.[20] Republic of Turkey State Planning Organization (SPO) (2000) reports an agricultural input share of 0.10 to 0.15 for all fertilizers; we used the higher of these two figures as (an upper bound on) the agricultural input share of nitrogenous fertilizer.[21]

There are two pieces of information that indirectly support our 0.15 estimate of the short-run demand point elasticity for nitrogenous fertilizer. First, our estimate is well within the range of short-run demand point elasticity estimates reported by Hansen (2001) across countries; it is also within two standard deviations of Hansen's mean estimate of short-run elasticity for his panel data of Danish farms. Second, SPO (2000)

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[18] While we interpret alternative elasticity values as long-run or medium-run (as opposed to short-run), we are implicitly assuming that the market share distribution does not change much between time periods. To test the sensitivity of our calibration to market shares, we replaced the 1999 market share distribution with the 1997-1999 average market distribution for a subsample of our cases; we found that the results did not change substantively. If anything, merger effects became somewhat stronger because İGSAŞ's 1997-1999 average share (approximately 22%) is higher than its 1999 share (approximately 15%). We discuss model inputs generally in Appendix I.

[19] For example, Peterson et al. (1999) assume that the U.S. demand elasticity for agricultural products is between 0.2 and 0.5. WALKENHORST assumes a demand elasticity of 0.3 for food products in a "stylized agricultural economy." One might surmise that the implied upper bound for the derived elasticity of demand for agricultural products that are used as inputs in food production should be approximately 0.3. Since the demand elasticity for all agricultural products cannot be higher than the elasticity of demand for a subgroup of them, 0.3 would also constitute an upper bound for the elasticity of demand facing all agricultural products. From this perspective, our assumption of a unit elasticity of short-run demand facing all agricultural products seems very conservative.

[20] See Warren-Boulton (1974).

[21] While the true substitution elasticity between nitrogenous fertilizers and all other inputs may be nonzero, it is likely to be small. We believe that this downward bias is more than offset by the two upward biases we deliberately introduce by using an overestimate of the demand elasticity for agricultural products and an overestimate of the input share of nitrogenous fertilizers in agricultural production.

reports a 20% fall in fertilizer purchases when the price of nutrition content relative to the price of agricultural crops increased from 1 (or 1.5) to 6 in 1994. This suggests a demand elasticity of roughly 0.05 for all fertilizers in Turkey,[22] which we interpret as a lower bound on nitrogenous fertilizer short-run demand elasticity.

Another input we used to calibrate the model is the profit margin; we considered alternative profit margin values of 0.1, 0.5, and 0.9 for Toros. A profit margin of 0.1 would imply that marginal costs are 90% of price, which we think is a rather extreme assumption.[23] Under this assumption, the price effect of the merger is close to 2% for the merger parties, and 1% for the whole industry. Under a less extreme assumption of a 0.5 profit margin, the merger's price effect ranged from about 6% to 17% for the parties, and about 3% to 9% for the industry. At the other extreme, we predicted even higher price increases when we set the value of the margin to 0.9.

An alternative to simulating the price increase from a merger is to calculate the marginal cost savings that the merging firms need to achieve for the post-merger prices (and elasticities) remain at their pre-merger levels; see Appendix I. In Table 3, the column labeled "Offsetting efficiencies" shows the percent reduction in marginal costs of the merged firm that would leave prices (and elasticities) unchanged. Under relatively extreme assumptions, no amount of reduction in marginal costs would be sufficient to compensate for the merger's price effect. For example, with a market elasticity of 0.15 and a margin of 0.9, even the maximum possible amount of cost savings (100%) would not have been sufficient to completely constrain the price effect.

Although the predicted price increases and cost savings in Table 3 generally decrease with the market elasticity and generally increase with the margin, monotonicity with respect to model inputs is not an unchanging property of the PCAIDS model. Predicted price increases for the non-merging firms illustrate the point. With an assumed market elasticity of 1.0, the non-merging parties increase their prices less when the margin is 0.9 than when the margin is 0.5. What drives the non-merging firms to change

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[22] Assuming a pre-1994 relative price of 1, one calculates an approximate demand elasticity of 0.040. Assuming a pre-1994 relative price of 1.5, one calculates an approximate demand elasticity of 0.067. Their arithmetic average is 0.053.

[23] Under this assumption, the firm cannot earn a profit unless it has minimal fixed costs (approximately less than 10% of its revenues).

their prices at all is the increase in the merging parties' prices.[24] Moreover, the merging parties increase prices more when the margin is 0.9 (implying own-elasticities of around 1/0.9 = 1.1), than when the margin is 0.5 (implying own-elasticities of around 1/0.5 = 2.0). Based on these premises, one might expect the non-merging firms to raise their prices in direct relationship with the merging parties (and they do in a majority of the cases displayed in Table 3). Missing from this intuition is the strength of the relationship between the parties' prices and the non-parties' quantities; that is, the magnitude of the non-parties' cross-price elasticities of demand with respect to the parties' prices. With a market elasticity of 1, when own-price elasticities are relatively low (around 1.1), formal assumptions that underlie PCAIDS calibration imply low cross-price elasticities (0.02 to 0.05). But when own-price elasticities are approximately doubled (around 2.0), cross-price elasticities increase nearly ten-fold (0.22 to 0.46), greatly enhancing the non-parties' responsiveness to a change in the parties' prices. Among all the cases we consider, this is the only case in which the cross-price effect dominates the own-price effect.

We observe that the size of the market elasticity matters only when firm margins are relatively high, that is, when own price elasticities are relatively low. As margins approach zero and own elasticities approach infinity, the magnitude of price increases approach zero regardless of market elasticity.[25]

In our price simulations, we assumed that the marginal costs would not change due to the merger. Neither of TCA's initial view nor final decision mentions potential efficiencies (marginal cost savings). This may mean that neither the potential acquirers nor the TPA claimed any efficiencies; it may also mean that, as a matter of principle, TCA does not consider efficiencies to be a merger defense. As improvement of economic efficiency must be the strongest reason for any privatization program, either possibility strikes us as strange.[26] Marginal cost savings would have reduced merger's net price effect on fertilizer purchasers. For each combination of the market elasticity and the profit margin, one can approximate the merger's net price effect by using our

[24] In other words, since the non-parties are always reacting to the parties, theirs are only a second order reaction.
[25] Technically, the sensitivity of the price increase to market elasticity becomes lower order.
[26] A third possibility is that TCA did not credit the efficiencies claimed for this specific case.

calculation of "offsetting efficiencies." As an example, when market elasticity is 1.0 and profit margin is 0.5, we predict that the merging parties would have raised their prices 12% if the merger did not affect their marginal costs; we also calculate a minimum necessary marginal cost reduction of nearly 15% that would completely offset this 12% price effect. Interpolating between these two points, one can calculate that a 5% merger-related marginal cost reduction would have reduced the price effect from 12% to approximately $\frac{15-5}{15} \times 12 = 8\%$; a 7.5% marginal cost reduction would have approximately halved the price increase to 6%.

Table 4 below shows the *equilibrium* HHI statistic before and after the proposed transaction, and the change. We emphasize the word *equilibrium*; since the merging firms lose share when they raise prices to maximize their post-merger joint profits, the resulting change in the equilibrium concentration is lower than increase that would be predicted using the pre-merger shares. For example, if the market elasticity is 1.6 and Toros's margin is 0.5, the change in *equilibrium* HHI, 862, is lower than the increase that the pre-merger shares would predict: 2(31.5)(14.8) = 928. The same effect is also evident in the share of the merged firm in Table 4; the simulated share is always less than the sum of the pre-merger shares: 31.5 + 14.8 = 46.3%.

Table 4 – Market concentration effects of the proposed Toros-İGSAŞ merger under alternative model inputs

Model inputs		*Equilibrium* concentration measures		
Market elasticity	TOROS margin	Post-merger HHI	Change in HHI	Merged firm's share
1.60	0.50	2471	862	45%
1.60	0.10	2292	683	43%
1.00	0.90	2494	885	46%
1.00	0.50	2362	753	44%
1.00	0.10	2278	669	43%
0.15	0.90	2071	461	40%
0.15	0.50	2233	624	42%
0.15	0.10	2260	650	42%

TCA report aggregates all producers and/or importers of nitrogenous fertilizer under the label "Others." Since we do not have market shares for individual firms in this category, we take the most conservative position and assume that they are many, each with a very small share of the market. As such, their concentration is assumed zero both before and after a merger.[27] Even with this conservative assumption, the anticompetitive concerns that underlie TCA's opposing decision against the merger proposal between Toros and İGSAŞ are clearly well justified from an *equilibrium* concentration perspective.

To put our simulation predictions for the Toros-İGSAŞ merger into perspective, we also simulated hypothetical mergers between İGSAŞ and two smaller producers of nitrogenous fertilizer, TÜGSAŞ and Bagfaş. Table 5 displays the results of these simulations (along with results for Toros) under alternative values for market elasticity. All simulations in Table 5 assume a 0.5 profit margin.

Table 5 – Unilateral and concentration effects of proposed or hypothetical mergers between İGSAŞ and potential acquirers

Model assumptions		Unilateral effects			*Equilibrium* concentration		
Potential acquirer	Market elasticity	Merging firms	Industry	Offsetting efficiencies	Post-merger HHI	Change in HHI	Merged firm's share
Toros	1.60	5.5%	2.8%	5.8%	2471	862	45%
Toros	1.00	12.2%	6.2%	14.7%	2362	753	44%
Toros	0.15	17.4%	8.8%	28.4%	2233	624	42%
TÜGSAŞ	1.60	2.7%	1.4%	4.5%	2163	553	34%
TÜGSAŞ	1.00	5.4%	3.0%	10.1%	2154	545	34%
TÜGSAŞ	0.15	7.2%	4.3%	16.5%	2138	528	34%
Bagfaş	1.60	1.4%	0.3%	3.0%	1754	145	20%
Bagfaş	1.00	2.6%	0.7%	6.0%	1752	142	20%
Bagfaş	0.15	3.4%	0.9%	8.4%	1746	137	20%

Table 5 shows that a merger with Bagfaş, with a post-merger *equilibrium* HHI under 1800 and a change less than 150, would have been close to what would be seen as an unproblematic merger under the *Guidelines*. By implication, a merger with a smaller competitor (e.g. Gübretaş or Ege) would have been even closer to, and possibly within,

[27] This assumption is conservative in that it minimizes concentration *levels* both before and after a merger. TCA (2000) makes the same assumption. In addition, we assume that the fringe sellers do not raise price after the merger. Since in our simulations all the large firms raise price, the aggregate market share of the fringe sellers increases due to the merger. Contrary to this, when we calculate the concentration levels, we assume that the fringe's aggregate share is constant. Consequently, our predicted *change* in concentration (HHI) is also biased downward relative to the true HHI change (this bias has a second-order magnitude). Again, since it results in a smaller change in concentration relative to the true change, this assumption is conservative; that is, it tends to minimize the effect of a merger.

the safe harbors.[28] Table 5 also indicates that the potential industry price increase under the proposed İGSAŞ-Toros merger is nearly double the potential increase under a hypothetical İGSAŞ- TÜGSAŞ merger, and almost ten times the increase under a hypothetical İGSAŞ- Bagfaş merger.

IV. Conclusions

A comparison of Table 3 with Table 4 is clear evidence of how important elasticity and margin assumptions can be in merger analysis. Although *equilibrium* concentration statistics in Table 4 differ between cases with different elasticity and margin assumptions, a Toros-İGSAŞ merger always falls outside of the *Guidelines* safe harbors. Table 3 displays much wider differences between these same cases. For example, if the margin is believed to be approximately 0.5, the industry price increase would be predicted as low as 3% or as high as 9% depending on the magnitude of the market elasticity! If the competition agency were to adopt as its operational threshold a 3% “tolerable” price increase (perhaps due to presumed efficiencies), then the decision whether to allow a Toros-İGSAŞ merger would hinge on whether the market elasticity is less than or greater than 1.6.

The value of the demand elasticity, in turn, depends on the length of the time allowed between a change in price and the resulting change in quantity demanded. If the agency adopted a relatively long-term view, then it would be logical to consider relatively high values of the demand elasticity. But if the agency adopts a relatively short-term view, then it would be logical to consider relatively low values of the demand elasticity. A similar observation applies to profit margins. Since more costs items are marginal in the long run than in the short run, the long-run value of the relevant profit margin (the Lerner index) is lower than its short-run value.

Presumably, the most stringent threshold to consider is a “tolerable” price increase of zero. Under this assumption, the agency would oppose all mergers predicted to raise price even by the smallest amount. In this case, if the market elasticity was believed to be near 1.6 (alternatively, if the agency adopted a long-term view), then it would be logical to expect a showing of 6% marginal cost savings to dispel any

[28] Our reference point for market shares is the 1999 nitrogen content share distribution.

anticompetitive concern. In contrast, if the agency believed the market elasticity to be 0.15 (alternatively, if it adopted a short-term view), then it would logically demand a marginal cost saving in excess of 28%. Of course, a 6% marginal cost reduction in the long-term (when more costs are marginal, and there is more time for cutting them) sounds much more credible than a 28% short-term reduction that the parties would have to demonstrate in order to meet the stringent threshold.

Our simulation results suggest that with a 0.5 gross margin, under a "tolerable" price increase threshold of 3%, a TÜGSAŞ- İGSAŞ merger would have been allowable only if the market demand is at least unitary elastic,[29] which corresponds to a medium- to long-term perspective in the context we adopted. On the other hand, a merger with Bagfaş (or a smaller competitor) would have been allowable under a much lower threshold (e.g. 1%) and a much less elastic demand (e.g., one with elasticity of 0.15), which corresponds to a much shorter-term perspective.

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Appendix I
A Brief Description of the PCAIDS Merger Simulation Model

The acronym "PCAIDS" means Proportionality-Calibrated Almost Ideal Demand System, introduced by Epstein and Rubinfeld (2001). The PCAIDS model is a variant of the more generalized Almost Ideal Demand System (AIDS) model used to locally approximate a demand system with no assumed functional form for a differentiated-product market where the competitors are engaged in Bertrand conduct. Antitrust economists utilize the simulation of a PCAIDS model to predict the price effect of a transaction or agreement that involves collective/joint pricing or collective/joint profit maximization. The PCAIDS model can be calibrated to simulate market conduct, e.g pricing, even where only a few pieces of market-level and firm-level information are available. Those inputs are (1) the pre-transaction shares of all different brands, (2) the market elasticity of demand facing all the firms, and (3) a firm-specific elasticity of demand. A variant of the PCAIDS model can be used to estimate the net price effect of an assumed transaction given expected or realized marginal cost savings. This variant requires the analyst to assume percent reductions in merging firms' marginal costs as additional inputs to the model.

An Almost Ideal Demand System (AIDS) is a system of equations that relate firms' revenue shares to the logarithmic prices ("log-prices") of all the firms in an assumed market. The typical AIDS model consists of two or more levels of equations. In an AIDS model with two levels of equations, the "lower" level might be called the firm level, and the "upper" level is typically termed the market level. At the firm level, a number of firm-specific equations are formulated, each one describing a firm's share of all firms' revenues as a linear function of the individual log-prices of all firms, and a constant term. Each firm's revenue share is expected to be negatively correlated with the level of its own price and positively correlated with the levels of other firms' prices. To illustrate, with only two firms, firm 1's equation would appear as: $s_1 = a_1 + b_{11}\text{Log}(p_1) + b_{12}\text{Log}(p_2)$ where s_1 is the revenue share of firm 1, and p_i is firm i's price (i=1,2). Similarly for firm 2, $s_2 = a_2 + b_{21}\text{Log}(p_1) + b_{22}\text{Log}(p_2)$ where s_2 is the revenue share of firm 2.

At the upper, or market level, the logarithm of all firms' revenues is formulated as a constant multiple of a logarithmic market price index (the "log-price index"), defined as a weighted average of all the firm-specific log-prices. "Total firm revenues" equal the sum of all firms' revenues. The top level expression is therefore a constant-elasticity equation between total market quantity and a price index (P) derived from a weighted-average of all firms' log-prices. To illustrate, $\mathrm{Log}(R/P) = \epsilon\,\mathrm{Log}(P)$, where $R = p_1q_1 + p_2q_2$ and $\mathrm{Log}(P) = w_1\mathrm{Log}(p_1) + w_2\mathrm{Log}(p_2)$. The coefficient ϵ is the constant elasticity of total (market) quantity with respect to the price index P.

The lower and the upper levels are doubly linked through total firm revenues and the log-price index. Specifically, at the firm level, each firm's revenue share equals the ratio of that firm's revenues to all firms' revenues: $s_1 = p_1q_1/R$ and $s_2 = p_2q_2/R$. And, the upper-level log-price index is a weighed average of the log-prices at the firm level, $\mathrm{Log}(P) = w_1\mathrm{Log}(p_1) + w_2\mathrm{Log}(p_2)$.

In general, calibrating an AIDS model to a market with multiple firms producing N products requires information or data on (1) the revenue shares of all brands, (2) the constant revenue elasticity coefficient (ϵ), and (3) N^2 coefficients that link product-specific log-prices to revenue shares (the b_{ij} coefficients in the lower level equations). For example, for a market with 5 products, formulation of an AIDS model requires estimation of 26 coefficients.

The PCAIDS model simplifies the estimation complexity of the general AIDS model by assuming that the b_{ij} coefficients are proportional to firms' pre-transaction shares.[1] Because of this simplifying assumption, all b_{ij} coefficients can be calculated if only one b_{ij} coefficient is known a priori. The N^2-1 b_{ij} coefficients that are not known a priori can be obtained by multiplying ("scaling") the known b_{ij} coefficient by a proportionality factor derived from firms' shares. For example, in a market with five firms, if only one b_{ij} coefficient is known (say, b_{11}), the remaining coefficients can be computed as:

$$b21 = -\frac{s_2}{1-s_1}b_{11},\ b31 = -\frac{s_3}{1-s_1}b_{11},\ b41 = -\frac{s_4}{1-s_1}b_{11},\ b_{51} = -(b_{11}+b_{21}+b_{31}+b_{41}),$$

[1]This is identical to the independence of irrelevant alternatives (IIA) assumption implied in the Logit demand models; see Werden, Froeb and Tardiff (1996).

$$b22 = -\frac{1-s_2}{s_1} b_{21},\ b33 = -\frac{1-s_3}{s_1} b_{31},\ b44 = -\frac{1-s_4}{s_1} b_{41},\ b55 = -\frac{1-s_5}{s_1} b_{51},$$

$$b23 = -\frac{s_2}{1-s_3} b_{33},\ b24 = -\frac{s_2}{1-s_4} b_{44},\ b25 = -\frac{s_2}{1-s_5} b_{55};$$

$$b32 = -\frac{s_3}{1-s_2} b_{22},\ b34 = -\frac{s_3}{1-s_4} b_{44},\ b35 = -\frac{s_3}{1-s_5} b_{55};$$

$$b42 = -\frac{s_4}{1-s_2} b_{22},\ b43 = -\frac{s_4}{1-s_3} b_{33},\ b45 = -\frac{s_4}{1-s_5} b_{55};$$

$$b52 = -\frac{s_5}{1-s_2} b_{22},\ b53 = -\frac{s_5}{1-s_3} b_{33},\ b54 = -\frac{s_5}{1-s_4} b_{44};$$

$$b_{12} = -(b_{22} + b_{32} + b_{42} + b_{52}),\ b_{13} = -(b_{23} + b_{33} + b_{43} + b_{53}),\ b_{14} = -(b_{24} + b_{34} + b_{44} + b_{54}),$$

$$b_{15} = -(b_{25} + b_{35} + b_{45} + b_{55}).$$

The coefficient b_{11}, in turn, can be calculated as $s_1(1 + e_{11} - s_1(\epsilon + 1))$ where e_{11} is the demand elasticity of product 1's quantity with respect to its own price, and s_1 is firm 1's share.

To summarize, the PCAIDS model can be calibrated to represent a market with an arbitrary number of brands if only two parameters, the top-level elasticity ϵ, and one of the brand-level elasticities in the system are known.

A PCAIDS model calibrated to relative shares has the following properties:

1. Positivity: the implied product-level quantity elasticities with respect to each product's own price will be greater than unity, implying a positive gross positive margin for each product;
2. Homogeneity, or "adding up property": coefficients in each product's equation add up to zero, implying that the product revenue shares will stay the same if all prices change by the same percentage amount;
3. Symmetry: $b_{ij} = b_{ji}$ for any two brands i and j.

To calculate a merger's anticipated price increase, the three basic inputs we used are: (1) the pre-transaction share distribution of the products, (2) market elasticity of demand facing all producers, and (3) the gross profit margin of Toros.

For the first input, we used the1999 "nitrogen content" share distribution of fertilizer producers calculated by TCA during its investigation.[2] Since we did not have revenue shares, we used the physical shares to calibrate the model. This is equivalent to assuming a common nitrogen content price for all products in the pre-merger period.[3] We assumed that the firms aggregated under the "Other" category constitute a "competitive fringe" and do not change their prices in response to a change in the prices of the larger firms.[4]

Second, we obtained an arc elasticity of market demand facing all nitrogen fertilizer producers, from Dr. Erol Çakmak of the Middle East Technical University, which we used as an upper bound on the possible range of this elasticity (1.6). As a lower bound, we made our own "back of the envelope" calculation (0.15), which we discuss in the text.

Finally, we assumed that the pre-merger gross profit margin for Toros (m_1) is between the extremes of 10% and 90%. In our simulations we used these two extreme values, as well as their midpoint, 50%. When simulating hypothetical mergers between İGSAŞ and alternative potential buyers, we used the midpoint value to calibrate the demand system.[5]

In estimating the profit-maximizing price increase that would result from a merger, the key price of information is the extent of the decrease in the magnitude of the merging firm's own-

[2]To test sensitivity with respect to share distribution, we re-ran the model using the 1997-1999 average share distribution for a subset of our elasticity and margin assumptions. Other things constant, we found more severe potential anti-competitive effects under this alternative calibration, mainly because the 1997-1999 average share of İGSAŞ (22%) is larger than its 1999 share (15%). However, we also found that none of our results changed in a substantive sense.

[3]An implication is that all simulated price increases (or cost savings) are per unit nitrogen content. If one assumes that fill material is costless, then the actual price increase (or cost savings) for each brand will be the same as the simulated increase (or savings). In general, the actual change in price (or cost) for product X will be proportional to the simulated change for X, and X's nitrogen content relative to that of Ege, whose 1999 market share of nitrogen content is practically identical to its market share when fill materials are included.

[4]If this assumption does not hold, then the true price increase would even be greater than our predicted price increase.

[5]To derive e_{11} we use the profit maximization condition $e_{11} = -1/m_1$.

price elasticity of demand from the pre-merger period to the post-merger period. While pre-transaction first-order conditions (FOC) for profit maximization factor in only own-price elasticities of merging firms, post-transaction FOC are based on the premise that each merger partner takes into account its cross-price elasticity with respect to the other merger partner. Hence, merged firm's post-transaction FOC involve these cross-price elasticities in addition to own-price elasticities. This formulation reflects merged firm's post-transaction profit calculus which takes into account extra profits that it would earn from the consumers who purchased from one partner pre-transaction, but would switch to the other partner if the price of the first one increased. Internalizing these "lost" customers provides merged firm with an incentive (and power) to raise its profit margins for both products. If marginal costs do not change as a result of the merger, higher margins imply higher prices. The formula for determining the magnitude of the transaction-related profit-maximizing price increase is displayed in Appendix II ("Post-transaction FOC Expression").[6]

An alternative to computing the expected price increase is to compute the marginal cost savings required for the post-transaction prices to remain at pre-transaction levels. In this approach, prices are assumed not to change as a result of the merger, therefore higher margins imply lower marginal costs. In this calculation, post-merger FOC are solved for the decrease in marginal costs that would keep the post-merger prices at their pre-merger levels, even as the merged firm's profit margin increases as a result of its increased market power.[7]

[6]Appendix II displays the PCAIDS module that we used in our simulations.

[7]Werden (1996) generally discusses the desirability of simulating marginal cost savings as an alternative to price simulation.

Appendix II

Predicting the Profit Maximizing Merger Price Increase Using the PCAIDS Model

Variable definitions are at the end.

Pre-merger

```
Off[General::spell1];
Off[General::spell];
nbrands = 7;
Brands = {TorosGübre,
TÜGSAS,
IGSAS,
EgeGübre,
Gübretas,
Bagfas,
Fringe}

{TorosGübre, TÜGSAS, IGSAS, EgeGübre, Gübretas, Bagfas, Fringe}
```

■ Input revenues

```
R = {31.46,
18.84,
14.76,
2.71,
3.92,
4.94,
23.38};(*1999 BBM*)
RSum = Sum[R[[i]], {i, 1, nbrands}];
r = {r1, r2, r3, r4, r5, r6, r7, r8, r9, r0};
Do[r[[i]] = R[[i]]/RSum, {i, 1, nbrands}];
Do[r[[j]] = 0, {j, nbrands + 1, 10}];
s1 = r[[1]]; s2 = r[[2]]; s3 = r[[3]]; s4 = r[[4]]; s5 = r[[5]];
s6 = r[[6]]; s7 = r[[7]]; s8 = r[[8]]; s9 = r[[9]]; s0 = r[[10]];
s = {s1, s2, s3, s4, s5, s6, s7, s8, s9, s0};

S = DiagonalMatrix[s];

B = ( b11 b12 b13 b14 b15 b16 b17 b18 b19 b10
      b21 b22 b23 b24 b25 b26 b27 b28 b29 b20
      b31 b32 b33 b34 b35 b36 b37 b38 b39 b30
      b41 b42 b43 b44 b45 b46 b47 b48 b49 b40
      b51 b52 b53 b54 b55 b56 b57 b58 b59 b50
      b61 b62 b63 b64 b65 b66 b67 b68 b69 b60
      b71 b72 b73 b74 b75 b76 b77 b78 b79 b70
      b81 b82 b83 b84 b85 b86 b87 b88 b89 b80
      b91 b92 b93 b94 b95 b96 b97 b98 b99 b90
      b01 b02 b03 b04 b05 b06 b07 b08 b09 b00 );

X = ( x11 x12 x13 x14 x15 x16 x17 x18 x19 x10
      x21 x22 x23 x24 x25 x26 x27 x28 x29 x20
      x31 x32 x33 x34 x35 x36 x37 x38 x39 x30
      x41 x42 x43 x44 x45 x46 x47 x48 x49 x40
      x51 x52 x53 x54 x55 x56 x57 x58 x59 x50
      x61 x62 x63 x64 x65 x66 x67 x68 x69 x60
      x71 x72 x73 x74 x75 x76 x77 x78 x79 x70
      x81 x82 x83 x84 x85 x86 x87 x88 x89 x80
      x91 x92 x93 x94 x95 x96 x97 x98 x99 x90
      x01 x02 x03 x04 x05 x06 x07 x08 x09 x00 );

b11 = s1 (e11 + 1 - s1 (ϵ + 1));

{b12, b13, b14, b15, b16, b17, b18, b19, b10} = Table[-Sum[B[[i, j]], {i, 2, 10}], {j, 2, 10}];

b21 = -b11 - Sum[B[[i, 1]], {i, 3, 10}];

{b22, b33, b44, b55, b66, b77, b88, b99, b00} = Table[-(1 - s[[j]])/s1 B[[j, 1]], {j, 2, 10}];
```

junk21	junk2	b23	b24	b25	b26	b27	b28	b29	b20	
b31	b32	junk3	b34	b35	b36	b37	b38	b39	b30	
b41	b42	b43	junk4	b45	b46	b47	b48	b49	b40	
b51	b52	b53	b54	junk5	b56	b57	b58	b59	b50	
b61	b62	b63	b64	b65	junk6	b67	b68	b69	b60	=
b71	b72	b73	b74	b75	b76	junk7	b78	b79	b70	
b81	b82	b83	b84	b85	b86	b87	junk8	b89	b80	
b91	b92	b93	b94	b95	b96	b97	b98	junk9	b90	
b01	b02	b03	b04	b05	b06	b07	b08	b09	junk0	

```
Table[Table[-s[[i]]/(1 - s[[j]]) B[[j, j]], {j, 1, 10}], {i, 2, 10}];
```

■ Input inpatient market and brand demand elasticities

```
ϵ = -1.6;
e11 = -1 / 0.5;

{x11, x22, x33, x44, x55, x66, x77} = Table[-1 + B[[j, j]]/s[[j]] + s[[j]] (ϵ + 1), {j, 1, nbrands}];
{x88, x99, x00} = Table[-1, {i, nbrands + 1, 10}];
z = {z1, z2, z3, z4, z5, z6, z7, z8, z9, z0};
```

Junk1	x12	x13	x14	x15	x16	x17	x18	x19	x10	
x21	Junk2	x23	x24	x25	x26	x27	x28	x29	x20	
x31	x32	Junk3	x34	x35	x36	x37	x38	x39	x30	
x41	x42	x43	Junk4	x45	x46	x47	x48	x49	x40	
x51	x52	x53	x54	Junk5	x56	x57	x58	x59	x50	=
x61	x62	x63	x64	x65	Junk6	x67	x68	x69	x60	
x71	x72	x73	x74	x75	x76	Junk7	x78	x79	x70	
x81	x82	x83	x84	x85	x86	x87	Junk8	x89	x80	
x91	x92	x93	x94	x95	x96	x97	x98	Junk9	x90	
x01	x02	x03	x04	x05	x06	x07	x08	x09	Junk0	

```
Table[Table[B[[i, j]]/z[[i]] + z[[j]] (ϵ + 1), {j, 1, 10}], {i, 1, 10}];
```

```
junk1  e12  e13  e14  e15  e16  e17  e18  e19  e10
 e21   e22  e23  e24  e25  e26  e27  e28  e29  e20
 e31   e32  e33  e34  e35  e36  e37  e38  e39  e30
 e41   e42  e43  e44  e45  e46  e47  e48  e49  e40
 e51   e52  e53  e54  e55  e56  e57  e58  e59  e50
 e61   e62  e63  e64  e65  e66  e67  e68  e69  e60   =
 e71   e72  e73  e74  e75  e76  e77  e78  e79  e70
 e81   e82  e83  e84  e85  e86  e87  e88  e89  e80
 e91   e92  e93  e94  e95  e96  e97  e98  e99  e90
 e01   e02  e03  e04  e05  e06  e07  e08  e09  e00

junk1  x12  x13  x14  x15  x16  x17  x18  x19  x10
 x21   x22  x23  x24  x25  x26  x27  x28  x29  x20
 x31   x32  x33  x34  x35  x36  x37  x38  x39  x30
 x41   x42  x43  x44  x45  x46  x47  x48  x49  x40
 x51   x52  x53  x54  x55  x56  x57  x58  x59  x50
 x61   x62  x63  x64  x65  x66  x67  x68  x69  x60   /.
 x71   x72  x73  x74  x75  x76  x77  x78  x79  x70
 x81   x82  x83  x84  x85  x86  x87  x88  x89  x80
 x91   x92  x93  x94  x95  x96  x97  x98  x99  x90
 x01   x02  x03  x04  x05  x06  x07  x08  x09  x00

{z1 → s1, z2 → s2, z3 → s3, z4 → s4, z5 → s5, z6 → s6, z7 → s7, z8 → s8, z9 → s9, z0 → s0};
```

```
    e11  e12  e13  e14  e15  e16  e17  e18  e19  e10
    e21  e22  e23  e24  e25  e26  e27  e28  e29  e20
    e31  e32  e33  e34  e35  e36  e37  e38  e39  e30
    e41  e42  e43  e44  e45  e46  e47  e48  e49  e40
    e51  e52  e53  e54  e55  e56  e57  e58  e59  e50
L = e61  e62  e63  e64  e65  e66  e67  e68  e69  e60 ;
    e71  e72  e73  e74  e75  e76  e77  e78  e79  e70
    e81  e82  e83  e84  e85  e86  e87  e88  e89  e80
    e91  e92  e93  e94  e95  e96  e97  e98  e99  e90
    e01  e02  e03  e04  e05  e06  e07  e08  e09  e00
```

```
       e11   0    0    0    0    0    0    0    0    0
        0   e22   0    0    0    0    0    0    0    0
        0    0   e33   0    0    0    0    0    0    0
        0    0    0   e44   0    0    0    0    0    0
PreE =  0    0    0    0   e55   0    0    0    0    0  ;
        0    0    0    0    0   e66   0    0    0    0
        0    0    0    0    0    0   e77   0    0    0
        0    0    0    0    0    0    0   e88   0    0
        0    0    0    0    0    0    0    0   e99   0
        0    0    0    0    0    0    0    0    0   e00
```

■ Derive the pre-merger margins from the pre-merger FOC Expression, s+PreE·S·m = 0.

```
m = {m1, m2, m3, m4, m5, m6, m7, m8, m9, m0} = LinearSolve[PreE.S, -s];
Print["Brands = ", Brands];
Print["Pre-merger shares =", s];
Print["Share sum = ", Sum[s[[t]], {t, 1, nbrands}]];
Print["Pre-merger margins=", m];
Print["B = ", MatrixForm[B]];
Print["Pre-merger elasticities ="];
Print[MatrixForm[L]];
Print["Weighted sum of elasticities = ",
  Sum[s[[i]] L[[i, j]], {i, 1, 10}, {j, 1, 10}]];
```

Brands = {TorosGübre, TÜGSAS, IGSAS, EgeGübre, Gübretas, Bagfas, Fringe}

Pre-merger shares =
{0.314569, 0.188381, 0.147585, 0.0270973, 0.0391961, 0.0493951, 0.233777, 0, 0, 0}

Share sum = 1.

Pre-merger margins={0.5, 0.482244, 0.47677, 0.461306, 0.462813, 0.464091, 0.488484, 0., 0., 0.}

B =

-0.255197	0.0701372	0.0549482	0.0100887	0.0145933	0.0183905	0.0870386	0	0	0
0.0701372	-0.180961	0.0329061	0.00604169	0.00873928	0.0110133	0.0521235	0	0	0
0.0549482	0.0329061	-0.148898	0.0047333	0.0068467	0.00862823	0.0408356	0	0	0
0.0100887	0.00604169	0.0047333	-0.0312026	0.00125708	0.00158418	0.0074976	0	0	0
0.0145933	0.00873928	0.0068467	0.00125708	-0.0445731	0.00229151	0.0108452	0	0	0
0.0183905	0.0110133	0.00862823	0.00158418	0.00229151	-0.0555749	0.0136672	0	0	0
0.0870386	0.0521235	0.0408356	0.0074976	0.0108452	0.0136672	-0.212008	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

Pre-merger elasticities =

-2.	0.109934	0.0861269	0.0158133	0.0228738	0.0288257	0.136426	0	0	0
0.183574	-2.07364	0.0861269	0.0158133	0.0228738	0.0288257	0.136426	0	0	0
0.183574	0.109934	-2.09745	0.0158133	0.0228738	0.0288257	0.136426	0	0	0
0.183574	0.109934	0.0861269	-2.16776	0.0228738	0.0288257	0.136426	0	0	0
0.183574	0.109934	0.0861269	0.0158133	-2.1607	0.0288257	0.136426	0	0	0
0.183574	0.109934	0.0861269	0.0158133	0.0228738	-2.15475	0.136426	0	0	0
0.183574	0.109934	0.0861269	0.0158133	0.0228738	0.0288257	-2.04715	0	0	0
-0.188741	-0.113029	-0.0885511	-0.0162584	-0.0235176	-0.029637	-0.140266	-1	0	0
-0.188741	-0.113029	-0.0885511	-0.0162584	-0.0235176	-0.029637	-0.140266	0	-1	0
-0.188741	-0.113029	-0.0885511	-0.0162584	-0.0235176	-0.029637	-0.140266	0	0	-1

Weighted sum of elasticities = -1.6

■ Merging firm's post-merger demand elasticity at pre-merger prices

$$\frac{1}{s1+s3}\left((L[[1,1]]+L[[1,3]])\,s1+(L[[3,1]]+L[[3,3]])\,s3\right)$$

```
-1.91387
```

Post-merger

```
τ = {τ1, τ2, τ3, τ4, τ5, τ6, τ7, τ8, τ9, τ0};
(*** τ stands for δ* in Epstein & Rubinfeld ***)

δ = {δ1, δ2, δ3, δ4, δ5, δ6, δ7, δ8, δ9, δ0} = Log[τ + 1];

Δ = DiagonalMatrix[e^δ];

σ = {σ1, σ2, σ3, σ4, σ5, σ6, σ7, σ8, σ9, σ0} = s + B.δ;

Σ = DiagonalMatrix[σ];
```

$$\{\xi 11,\ \xi 22,\ \xi 33,\ \xi 44,\ \xi 55,\ \xi 66,\ \xi 77\} = \text{Table}\left[-1+\frac{B[[i,i]]}{\sigma[[i]]}+\sigma[[i]]\,(\epsilon+1),\ \{i,1,\text{nbrands}\}\right];$$

```
{ξ88, ξ99, ξ00} = Table[-1, {i, nbrands + 1, 10}];

Clear[X, Ξ];
```

X =

ξ11	x12	x13	x14	x15	x16	x17	x18	x19	x10
x21	ξ22	x23	x24	x25	x26	x27	x28	x29	x20
x31	x32	ξ33	x34	x35	x36	x37	x38	x39	x30
x41	x42	x43	ξ44	x45	x46	x47	x48	x49	x40
x51	x52	x53	x54	ξ55	x56	x57	x58	x59	x50
x61	x62	x63	x64	x65	ξ66	x67	x68	x69	x60
x71	x72	x73	x74	x75	x76	ξ77	x78	x79	x70
x81	x82	x83	x84	x85	x86	x87	ξ88	x89	x80
x91	x92	x93	x94	x95	x96	x97	x98	ξ99	x90
x01	x02	x03	x04	x05	x06	x07	x08	x09	ξ00

;

Λ =

ϵ11	ϵ12	ϵ13	ϵ14	ϵ15	ϵ16	ϵ17	ϵ18	ϵ19	ϵ10
ϵ21	ϵ22	ϵ23	ϵ24	ϵ25	ϵ26	ϵ27	ϵ28	ϵ29	ϵ20
ϵ31	ϵ32	ϵ33	ϵ34	ϵ35	ϵ36	ϵ37	ϵ38	ϵ39	ϵ30
ϵ41	ϵ42	ϵ43	ϵ44	ϵ45	ϵ46	ϵ47	ϵ48	ϵ49	ϵ40
ϵ51	ϵ52	ϵ53	ϵ54	ϵ55	ϵ56	ϵ57	ϵ58	ϵ59	ϵ50
ϵ61	ϵ62	ϵ63	ϵ64	ϵ65	ϵ66	ϵ67	ϵ68	ϵ69	ϵ60
ϵ71	ϵ72	ϵ73	ϵ74	ϵ75	ϵ76	ϵ77	ϵ78	ϵ79	ϵ70
ϵ81	ϵ82	ϵ83	ϵ84	ϵ85	ϵ86	ϵ87	ϵ88	ϵ89	ϵ80
ϵ91	ϵ92	ϵ93	ϵ94	ϵ95	ϵ96	ϵ97	ϵ98	ϵ99	ϵ90
ϵ01	ϵ02	ϵ03	ϵ04	ϵ05	ϵ06	ϵ07	ϵ08	ϵ09	ϵ00

=

```
X /. {z1 → σ1, z2 → σ2, z3 → σ3, z4 → σ4, z5 → σ5, z6 → σ6, z7 → σ7, z8 → σ8, z9 → σ9, z0 → σ0};
```

■ Merger

```
(*** The following is ⎋E⎋, not E ***)

      ϵ11   0    ϵ31   0     0     0     0     0     0     0
       0   ϵ22    0    0     0     0     0     0     0     0
      ϵ13   0    ϵ33   0     0     0     0     0     0     0
       0    0     0   ϵ44    0     0     0     0     0     0
       0    0     0    0    ϵ55    0     0     0     0     0
E =    0    0     0    0     0    ϵ66    0     0     0     0     ;
       0    0     0    0     0     0    ϵ77    0     0     0
       0    0     0    0     0     0     0    ϵ88    0     0
       0    0     0    0     0     0     0     0    ϵ99    0
       0    0     0    0     0     0     0     0     0    ϵ00
```

■ Percent changes in marginal costs

■ Merger without efficiencies

```
γ = {γ1, γ2, γ3, γ4, γ5, γ6, γ7, γ8, γ9, γ0} = Table[0, {i, 1, 10}];
Γ = DiagonalMatrix[1 + γ];
```

■ Reductions in marginal cost required to maintain prices at pre-merger levels

```
ω = {ω1, ω2, ω3, ω4, ω5, ω6, ω7, ω8, ω9, ω0};
Ω = DiagonalMatrix[1 + ω];
```

■ Merger

```
      e11   0    e31   0     0     0     0     0     0     0
       0   e22    0    0     0     0     0     0     0     0
      e13   0    e33   0     0     0     0     0     0     0
       0    0     0   e44    0     0     0     0     0     0
       0    0     0    0    e55    0     0     0     0     0
Φ =    0    0     0    0     0    e66    0     0     0     0     ;
       0    0     0    0     0     0    e77    0     0     0
       0    0     0    0     0     0     0    e88    0     0
       0    0     0    0     0     0     0     0    e99    0
       0    0     0    0     0     0     0     0     0    e00
```

Effect of the merger

■ Post-merger FOC Expression

```
A := Simplify[σ + Ε.Σ.(1 - Γ.Inverse[Δ].(1 - m))];
```

■ Partial market (only merging brands raise price)

```
τpart = {τpart1, τpart2, τpart3, τpart4, τpart5, τpart6, τpart7, τpart8, τpart9, τpart0} =
   {τ1, τ2, τ3, τ4, τ5, τ6, τ7, τ8, τ9, τ0} /. FindRoot[
     {A[[1]] == 0, τ[[2]] == 0, A[[3]] == 0, τ[[4]] == 0, τ[[5]] == 0,
      τ[[6]] == 0, τ[[7]] == 0, τ[[8]] == 0, τ[[9]] == 0, τ[[10]] == 0},
     {τ1, 0.001}, {τ2, 0.001}, {τ3, 0.001}, {τ4, 0.001}, {τ5, 0.001},
     {τ6, 0.001}, {τ7, 0.001}, {τ8, 0.001}, {τ9, 0.001}, {τ0, 0.001}
    ];
PartEquilibrium = {τ1 → τpart1, τ2 → τpart2, τ3 → τpart3, τ4 → τpart4,
  τ5 → τpart5, τ6 → τpart6, τ7 → τpart7, τ8 → τpart8, τ9 → τpart9, τ0 → τpart0}

{τ1 → 0.0436998, τ2 → 0., τ3 → 0.0782841,
 τ4 → 0., τ5 → 0., τ6 → 0., τ7 → 0., τ8 → 0., τ9 → 0., τ0 → 0.}
```

■ Market w/ Fringe constrained

```
τstar = {τstar1, τstar2, τstar3, τstar4, τstar5, τstar6, τstar7, τstar8, τstar9, τstar0} =
   {τ1, τ2, τ3, τ4, τ5, τ6, τ7, τ8, τ9, τ0} /. FindRoot[
     {A[[1]] == 0, A[[2]] == 0, A[[3]] == 0, A[[4]] == 0, A[[5]] == 0,
      A[[6]] == 0, τ[[7]] == 0, τ[[8]] == 0, τ[[9]] == 0, τ[[10]] == 0},
     {τ1, τpart1}, {τ2, 0.001}, {τ3, τpart3}, {τ4, 0.001}, {τ5, 0.001},
     {τ6, 0.001}, {τ7, 0.001}, {τ8, 0.001}, {τ9, 0.001}, {τ0, 0.001}
    ];
FullEquilibrium = {τ1 → τstar1, τ2 → τstar2, τ3 → τstar3, τ4 → τstar4,
  τ5 → τstar5, τ6 → τstar6, τ7 → τstar7, τ8 → τstar8, τ9 → τstar9, τ0 → τstar0}

{τ1 → 0.0443476, τ2 → 0.00852654, τ3 → 0.0789827, τ4 → 0.00956684,
 τ5 → 0.00950242, τ6 → 0.00944664, τ7 → 0., τ8 → 0., τ9 → 0., τ0 → 0.}
```

■ Revenue shares

```
Brands

{TorosGübre, TÜGSAS, IGSAS, EgeGübre, Gübretas, Bagfas, Fringe}
```

■ Pre-merger

```
s
```

{0.314569, 0.188381, 0.147585, 0.0270973, 0.0391961, 0.0493951, 0.233777, 0, 0, 0}

■ Post-merger

```
σ /. FullEquilibrium
```

{0.308675, 0.192633, 0.139121, 0.0276759, 0.040036, 0.0504567, 0.241403, 0, 0, 0}

■ Mid-point shares

```
(*mps =s;*)
mps = ((s + σ)/2) /. FullEquilibrium;
```

■ Post-merger share-weighted price increase

■ By MERGER

```
Sum[(τstar[[i]] × mps[[i]] / Sum[mps[[j]], {j, 1, 3, 2}]) /. FullEquilibrium, {i, 1, 3, 2}]
```

0.0552604

■ By NON-MERGER

```
(τstar[[2]] × mps[[2]] / (mps[[2]] + Sum[mps[[j]], {j, 4, 7}]) /. FullEquilibrium) +
 Sum[(τstar[[i]] × mps[[i]] / (mps[[2]] + Sum[mps[[j]], {j, 4, 7}])) /. FullEquilibrium, {i, 4, 7}]
```

0.00501711

■ OVERALL

```
Sum[(τstar[[i]] × mps[[i]] / Sum[mps[[j]], {j, 1, nbrands}]) /. FullEquilibrium, {i, 1, nbrands}]
```

0.0278765

■ Reductions in merging firms' marginal cost required to maintain prices at pre-merger levels

```
ϒ := Simplify[s + Φ.S.(1 - Ω.Inverse[Δ].(1 - m))];
ωpart = {ωpart1, ωpart2, ωpart3, ωpart4, ωpart5, ωpart6, ωpart7,
    ωpart8, ωpart9, ωpart0} = {ω1, ω2, ω3, ω4, ω5, ω6, ω7, ω8, ω9, ω0} /.
    Block[{τ1 = τ2 = τ3 = τ4 = τ5 = τ6 = τ7 = τ8 = τ9 = τ0 = 0},
     FindRoot[
      {ϒ[[1]] == 0, ω[[2]] == 0, ϒ[[3]] == 0, ω[[4]] == 0,
       ω[[5]] == 0, ω[[6]] == 0, ω[[7]] == 0, ω[[8]] == 0, ω[[9]] == 0, ω[[10]] == 0},
      {ω1, 0.001}, {ω2, 0.001}, {ω3, 0.001}, {ω4, 0.001}, {ω5, 0.001},
      {ω6, 0.001}, {ω7, 0.001}, {ω8, 0.001}, {ω9, 0.001}, {ω0, 0.001}
     ]
    ];
OffsettingEfficiencies = {ω1 → ωpart1, ω2 → ωpart2, ω3 → ωpart3, ω4 → ωpart4,
  ω5 → ωpart5, ω6 → ωpart6, ω7 → ωpart7, ω8 → ωpart8, ω9 → ωpart9, ω0 → ωpart0,
  τ1 → 0, τ2 → 0, τ3 → 0, τ4 → 0, τ5 → 0, τ6 → 0, τ7 → 0, τ8 → 0, τ9 → 0, τ0 → 0}

{ω1 → -0.0450014, ω2 → 0., ω3 → -0.0874006, ω4 → 0., ω5 → 0., ω6 → 0., ω7 → 0., ω8 → 0.,
 ω9 → 0., ω0 → 0., τ1 → 0, 0 → 0, 0 → 0, 0 → 0, 0 → 0, 0 → 0, 0 → 0, 0 → 0, 0 → 0, 0 → 0}
```

■ Overall merger revenue share-weighted reductions in marginal cost required to maintain prices at pre-merger levels

■ By MERGER

$$\text{Sum}\left[\left(\frac{\omega[[i]] \times \text{mps}[[i]]}{\text{Sum}[\text{mps}[[j]], \{j, 1, 3, 2\}]}\right) \text{/. OffsettingEfficiencies}, \{i, 1, 3, 2\}\right]$$

```
-0.0583605
```

■ Concentration

■ Pre-merger HHI

$$\text{PreMergerHHI} = 10000\, \text{Sum}[s[[i]]^2, \{i, 1, 6\}]$$

```
1609.33
```

■ Post-merger HHI

```
PostMergerHHI = 10000 (((σ[[2]]^2 + (σ[[1]] + σ[[3]])^2) /. FullEquilibrium) +
    Sum[(σ[[i]]^2 /. FullEquilibrium), {i, 4, 6}])
```

2471.37

■ Change in HHI

```
PostMergerHHI - PreMergerHHI
```

862.047

■ Merging firm post-merger share

```
(σ[[1]] + σ[[3]]) /. FullEquilibrium
```

0.453465

Definitions

R = Vector of brands' pre-merger revenues, R_i.
r = Generic vector array to define pre-merger revenue shares.
s = Vector of brands' pre-merger revenue shares, s_i.
S = Diagonal matrix of brands' pre-merger revenue shares, s_i.
B = Matrix of own- and cross-partial derivatives of brand i's share with respect to brand j's log price: $b_{ij} = \frac{\partial s_i}{\partial \mathrm{Log}[p_j]}$.
X = Generic matrix array to define pre- and post-merger elasticities, x_{ij}.
z = Generic vector array to define brands' pre- and post-merger revenue shares, z_i.
L = Matrix of pre-merger own and cross price elasticities: $L_{ij} \equiv e_{ij} = \frac{\partial q_i}{\partial p_j} \frac{p_j}{q_i}$.
PreE = Diagonal matrix of pre-merger own price elasticities, e_{ii}.
ϵ = Market demand elasticity.
e1 = Merging firm's post-merger demand elasticity at pre-merger prices.
m = Vector of pre-merger price-cost margins, $m_i = \frac{p_i - c_i}{p_i}$.

σ = Vector of brands' post-merger revenue shares, σ_i.
Σ = Diagonal matrix of brands' post-merger revenue shares, σ_i.
Λ = Matrix of post-merger own and cross price elasticities: $\Lambda_{ij} \equiv \epsilon_{ij} = \left(\frac{\partial q_i}{\partial p_j}\right)^* \frac{p_j^*}{q_i^*}$, where "*" denotes "post-merger value."
E = Block-diagonal matrix of (transposed) post-merger own and cross price elasticities among brands commonly owned after the merger:

$$E_{ij} = \begin{cases} \epsilon_{ji} & \text{if } i = j, \text{ and/or brands } i \text{ and } j \text{ are commonly owned post–merger} \\ 0 & \text{otherwise.} \end{cases}$$

τ = Vector of fractional price increases due to the merger, τ_i.
δ = Vector of log-ratios of post-merger prices to pre-merger prices, Log(1+τ_i).
Δ = Diagonal matrix of post-merger prices relative to pre-merger prices, $1 + \tau_i$.
ξ_{ii} = Post-merger own price elasticity for brand i calibrated to its post-merger revenue share.
γ = Vector of percentage change in brand marginal costs due to the merger ("efficiencies"), γ_i. A percent decrease (increase) in pre-merger marginal costs of brand i due to the merger is expressed as γ_i<0 (>0).
Γ = Diagonal matrix of marginal costs after merger-related efficiencies relative to the pre-merger marginal costs, $1 + \gamma_i$.
μ = Vector of post-merger price-cost margins, $\mu_i = \frac{p_i^* - c_i^*}{p_i^*} = \frac{(1+\tau_i)\, p_i - (1+\gamma_i)\, c_i}{(1+\tau_i)\, p_i}$, where "*" denotes "post-merger value."
ω = Vector of percent cost savings necessary to maintain prices at the pre-merger levels ("offsetting efficiencies"), ω_i. Cost savings are expressed as negative changes in pre-merger marginal costs: $\omega_i \le 0$.
Ω = Diagonal matrix of marginal costs after offsetting efficiencies relative to marginal costs before offsetting efficiencies, $1 + \omega_i$.
Φ = Block-diagonal matrix of (transposed) pre-merger own and cross price elasticities among producers commonly owned after the merger:

$$\Phi_{ij} = \begin{cases} e_{ji} & \text{if } i = j, \text{ and / or brands } i \text{ and } j \text{ are commonly owned post–merger} \\ 0 & \text{otherwise.} \end{cases}$$

A = Post-merger system of first-order profit maximization conditions at the post-merger elasticities.

τstar = Vector of profit-maximizing percent price increases due to the merger, τstar_i.

τpart = Vector of profit-maximizing percent price increases due to the merger when non-merging brands are constrained not to change their prices, τpart_i. $\tau\text{part}_i \neq 0$ (=0) if brand i is (is not) a party to the merger.

Υ = Post-merger system of first-order profit maximization conditions at the pre-merger elasticities.

ωpart = Vector of equilibrium offsetting efficiencies, ωpart_i. $\omega\text{part}_i < 0$ (=0) if brand i is (is not) a party to the merger.