The Optimality of the US and euro area Taylor Rule: Some Deterministic and Stochastic Experiments

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1- Introduction

The purpose of this paper is to examine the optimality of the reaction function in the two-area medium size model MARCOS (US and euro areas)[3]. The optimality of the monetary reaction function has been intensively investigated in many respect. First, the properties of a simple monetary rule, generally a Taylor rule, are compared to a more sophisticated one deduced from an optimisation framework (Rudebusch and Svensson [1999]). Second, the parameters and the horizon of inflation expectation of the Taylor rule are computed in order to minimise a loss function of the monetary authorities (Batini and Haldane [1999], Batini and Nelson [2000], Jondeau and Le Bihan [2000]). We retain the second approach to examine the optimality of the monetary reaction function.

For tractability purpose, the optimality of the monetary reaction function is usually studied in a simplify framework where the economy is described as a VAR or a small structural model. These models are generally composed of two or three equations: an IS curve, a Phillips curve and an UIP relation. The description of monetary policy channels is rather poor. Black, Macklem and Rose [1998], Drew and Hunt [1999], Yuong [2000] investigate the optimality of the Taylor rule in the context of a large scale macroeconomic model[4] (QPM and FPS). However this approach raises several difficulties. Because the model is non linear and all the state variables enter the optimal monetary policy rule, its computation becomes intractable for a large scale model. Furthermore, the optimality of the Taylor rule is assessed by the minimisation of a loss function under the constraint of the model. In the context of a large scale model, especially if it is calibrated, the task is rather tricky. To overcome this problem, Black, Macklem and Rose [1998] propose a stochastic simulation based method which has been applied to single-country macroeconomic models (Black, Macklem and Rose [1998], Drew and Hunt [1999], Yuong [2000]).

The aim of this paper is the optimality of the Taylor rule in the case of a two-area model. In a first step we determine the optimality of the rule conditional on a particular shock: a supply shock or a demand shock. For each shock, we run deterministic simulations for different values of the parameters of the Taylor rule and/or for different horizon. The optimal Taylor rule will be the one with the parameter set minimising the criterion composed of variances of output, inflation and interest rate. In a second step, the conditionality on the nature of the shock is relaxed and we define the optimality in a more general context. We suppose that the economy is stochastically hit by numerous shocks (supply, demand, monetary, exchange rate and world demand). For this purpose MARCOS is stochastically simulated. The optimality of the Taylor rule is examined with respect to either the parameters or the horizon using Black, Macklem and Rose [1998] methodology.

The first part of the paper is devoted to MARCOS presentation whereas the optimality of the Taylor rule is discussed in the second part. Deterministic simulations are presented in the third part. The last part deals with the results of stochastic simulations.

[3] See Jacquinot and Mihoubi [2000].

[4] Only the optimality of the coefficient of the Taylor are considered.

2- MARCOS at a glance

MARCOS (*Modèle à Anticipations Rationnelles de la Conjoncture Simulée*) is a yearly model designed for economic policy evaluation. It is calibrated and composed of two area-blocks: the Euro and US areas. The goal of MARCOS is to get a comprehensive and understandable tool to analyze economic policies. MARCOS is a medium-size model (around 100 equations for each area) with a coherent accounting framework and rational expectations.

The overall coherence of the model is ensured by a top down strategy (from theoretical structure to equations). A balanced growth path exists and explicitly comes from the short-term dynamics of the model. Parameters in equations are structural and invariant to economic policy shocks. They are directly derived from different agents optimising framework (households, unions, firms). The MARCOS's supply side homogeneity is thus completely guaranteed and the wage-setting follows a bargaining process. Forward looking expectations are model-path consistent. They appear in the real sphere: consumption, investment, fiscal-authority reaction function; as well as in the nominal sphere: Phillips curve, monetary-authority reaction function, Fisher equation, uncovered interest rate parity.

Recent works implementing this approach include Laffargue [1995], QPM (Black *et alii* [1994], Coletti *et alii* [1996]), QUEST II (Roeger and in't Veld [1997]), FPS (Black *et alii* [1997]), and MULTIMOD Mark III (Laxton *et alii* [1998]). MARCOS slightly differs from these models by its more general theoretical framework: we simultaneously assume monopolistic competition, wage bargaining and life cycle hypothesis. Five agents are retained in MARCOS: households, firms, public administration, rest of the world and unions.

2.1- MARCOS agents

Households

Consumption is split between workers and retired in a pay-as-you-go retirement scheme. In addition, two kinds of households are distinguished whether they are liquidity constrained or not. The neo-classical households, that are non constrained, hold treasury bonds and firms and determine their consumption by maximising their inter-temporal utility function. Following Gertler |1997], at each date working age households face a constant probability to become retire and retired households face a constant probability to die. Income is determined by real wages under the assumption of a life cycle bell-shaped (Faruqee, Laxton and Symansky [1997]). Wages are deduced from a right to manage model.

Firms

Employment, subject to adjustment cost, is thus determined by the labour demand, given the wage bargained. Furthermore, modelling the wage-bargaining process allows to compute an equilibrium unemployment rate consistent with both workers and firms objectives. In a profit optimizing framework, the labour demand equation cannot be distinguished from the value added price equation. The value added price is thus the implicit GDP price. In order to take account of nominal rigidities and pressures on the price setting, the demand price is modelled by a Phillips curve with model consistent expectations. The nominal block is composed of seven prices: demand price, value added price, consumption price, investment price, public expenditures price, import and export deflators.

The profit maximisation program including capital adjustment costs gives investment thus related to the Tobin's q[5]. The foreign trade equations are rather traditional with exports and imports respectively depending upon world demand, domestic demand and price competitiveness.

Government

The government raises direct and indirect taxes. The personal income tax rate is endogenous and adjusted by the government in order to reach a public debt target. The employer social contribution rate is endogenously determined in order to guarantee the long-term social budget equilibrium. In the short run the employer social contribution rate is exogenous and the government guaranties the equilibrium of social account.

The nominal rigidities

The Phillips curve describes the relationship between the rate of inflation (p_t) and the output gap where the potential output is equal to its steady state level given by the model $\left(\frac{Y_t}{\overline{Y}_t}-1\right)$. Due to nominal rigidity, an inflation-unemployment dilemma is quite possible in the short term.

$$p_t = z p_{t-1} + (1-z)p_t^a + y\left(\frac{Y_t}{\overline{Y}_t}-1\right)$$

With $y > 0$. Current inflation depends upon the past inflation (p_{t-1}) and the expected inflation for the next period ($p_t^a = E_t(p_{t+1})$). The expected rate of inflation (p_t^a) could be either auto-regressive or model coherent. The parameter z_a $(0 \le z_a \le 1)$ indicates the respective weights of naive and model-coherent expectations

$$p_t^a = z_a p_{t+1} + (1-z_a)p_t$$

Furthermore, expectations completely forward looking (*i.e.* when $z = 0$ and $z_a = 1$) are excluded. Otherwise, the monetary policy will be limited to the announcement of an inflation target that will be immediately verified by all agents expectations.

Interest Rates

Four interest rates are included in MARCOS: the one-year interest rate, the ten-year interest rate, the composite interest rate associated to the public debt (mix of the two previous ones) and the foreign short-term interest rate. Short term and long term interest rates are related by a yield curve with a constant term premium. Monetary authorities fix the short-term interest rate according to an inflation and output-gap targeting. These interest rates have three types of effects on the real sphere. First, they directly influence the neo-classical households consumption *via* wealth and saving-consumption substitution effects as well as investment *via* the optimal capital stock which equates the long-term capital productivity and the real interest rate. Second, they directly determine levels of public and external debts and thus the

[5]The stock market is supposed in perfect information situation.

households wealth. Third, they determine the exchange rate by the uncovered interest rate parity relationship and then modify the price competitiveness and the trade balance.

The Reaction Function

The reaction function is a Taylor rule (Taylor [1993]): monetary authorities control the nominal short-term interest rate ($\bar{r}_t$), reacting to shocks on inflation or deviations of output from its potential level:

$$\bar{r}_t = \bar{r}_{t-1} + \mu(\pi_{t+1} - \pi^*_{t+1}) + \tau\left(\frac{Y_t}{\bar{Y}_t} - 1\right)$$

with p^*_t as the inflation target, $\left(\frac{Y_t}{\bar{Y}_t} - 1\right)$ represents the output gap.

Area blocks linkage

The specifications of both euro area and US are identical, obviously calibrations are different. The linkage variables between the areas are interest rates, exchange rates, foreign demand and foreign prices. Hence, the Euro/US-dollar exchange rate is deduced from an uncovered interest rate parity. Exchange rates with the rest of the world currencies also follow an uncovered interest rate parity. The foreign demand for each area is composed of the US, the Euro-area and the rest of the world imports weighted by their respective shares in the area imports. For each area, the foreign price depends upon exports prices of the other areas.

The world demand of the euro area (US respectively) is the sum of the US (euro area) and the rest of the world imports weighted by the share of the euro area (US) and the rest of the world in the total imports.

$$DM_i = \sum_{j=1}^{3} q_{i,j} M_j$$

with *i=US,EA,RW* and *j=EA,US,RW* and DM_i: the world demand in the region *i* ; $q_{i,j}$: the share of the imports of region *j* from the region *i* with $q_{i,i} = 0$; M_j: the imports of the region *j*.

The foreign price of the euro area (US) is a geometric means of the main competitors exports prices weighted by the share of the euro area (US respectively) in the US (euro area respectively) imports.

$$p^*_i = \prod_{j=1}^{3} \left(pm_j\right)^{q_{i,j}}$$

with p^*_i: the foreign price for the region *i*; $q_{i,j}$: the share of the imports of region *j* from the region *i* with $q_{i,i} = 0$; pm_j: the imports price in the region *j.*

The exchange rate between the euro area and the US is determined according to an uncovered interest rate parity:

$$\frac{e_{t+1}}{e_t} = \frac{1 + r^{CT}_{US,t}}{1 + r^{CT}_{EA,t}}$$

with e_t : the EUR-USD exchange rate; $r^{CT}_{US,t}$: the US short-term interest rate; $r^{CT}_{EA,t}$: the euro area short term interest rate.

In the long run, the real interest rates in the US and the euro area are identical.

2.2- Calibration

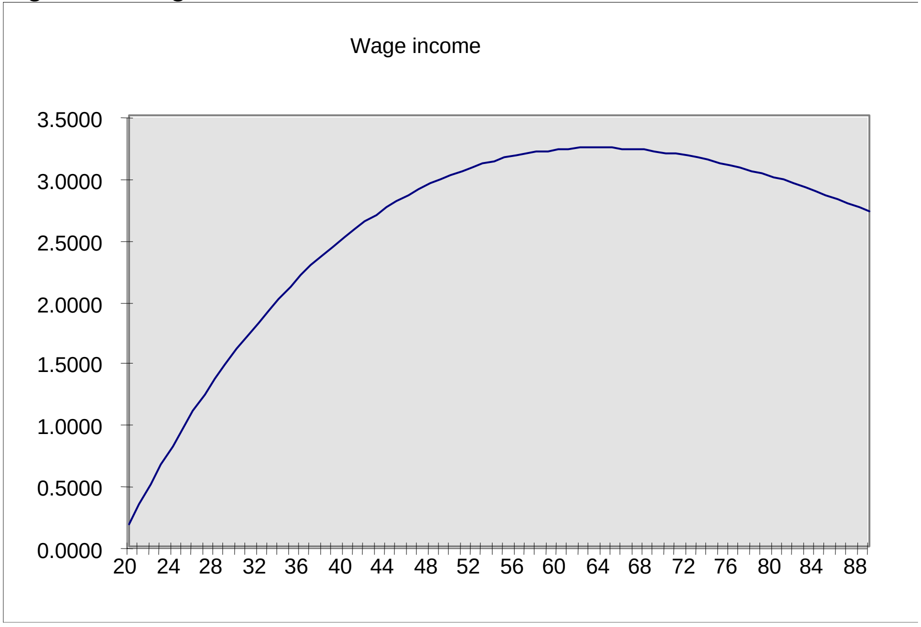
The euro area

The euro area model is calibrated using annual data provided by Eurostat and the ECB. Tables 2.1 to 2.6 in Appendix 2 report the coefficients values and the main features of the steady state for the euro area and the US.

The calibration relies on the assumption that the euro area economy was on average at its steady state during the period 1985-1997. Thus variables describing the steady state (g, n, p, r) are put to their 1985-1997 sample mean values. For unobserved parameters two cases could be considered: parameters considered as endogenous during the calibration – the model is inverted - (q, r, g_0) and parameters set to realistic values. Hence, the retire probability $(1-w)$ is 0.025 implying an expected working time of 40 years and the death probability (p) is equal to 0.05 corresponding to an expected adult life time of 60 years and the capital depreciation rate (d) is set to 4.5% in order to be in line with the investment rate at the steady state. We get a capital life time of 22 years. The bargaining power of the union (b) is set to 0.5 leading to a gain coming from the matching of a vacant job with an unemployed worker equally shared between the employer and the employee. In order to get a mark-up rate about 10%, the price elasticity of the good demand (h) is equal to 11. The adjustment cost on capital (m_K) is set to 6. Estimations of the adjustment cost are rather not robust. Using panel data, estimates are often not significantly different from zero and could even be negative. Using aggregate data, Bloch and Coeuré [1995] found values between 9 and 30 for the French economy. In an European comparison, Roeger and in't Velt [1997] found adjustment costs rather closed for each country with m_K equal to 5.99. We retain this value for the adjustment cost.

The coefficients a_1, a_2, a_1, a_2 et a_3 determining the path of the wage income during the adult life time (Table 2.3) are set such that the labour income has the usual life cycle pattern (Figure 1).

Figure 1 : Wage Income Profile



It is worth noting that consumption in MARCOS is in fact the aggregation of the households consumption and investment. The share of Keynesian household consumption (i) is endogenous during the calibration. Its value is deduced from the simulation of the overall model taking into account constraints on the household wealth and on their consumption.

During the calibration, variables in level (Y_t, PIB_t, $\bar{L}_t$) have been set to their 1997 values. Ratios and rates (shares of the different components of the demand in the GDP, ratios of the different debts to the GNP and taxes rates) are supposed to be equal to their mean on the 1985-1997 sample. At the steady state, the unemployment rate (which measures only the compensated unemployment) is equal to its estimated equilibrium value of 8.6%.

Parameters of the monetary policy reaction function are those proposed by Taylor [1993]: the parameter related to inflation m is equal to 1.5 and the parameter which measure the sensibility to the output gap is set to 0.5. Thus the central bank is more aggressive on inflation than on activity.

This calibration provides a steady state rather closed to actual values. So at the steady state, the capital coefficient is equal to 3.3 and the labour income share is equal to 56%.

2.3 Estimations

The Phillips curve

The parameters of the hybrid Phillips curve have been estimated using a full information maximum likelihood method[6]. The implementation of this kind of method requires a fully specify model. We have used a very simplify version of the model composed of a IS curve, a reaction function and a Phillips curve. The IS curve could be view as a reduced form of the

[6] Estimation have been carry out using the MatLab program of Fuhrer (1995).

demand side of MARCOS. The parameters of the reaction function have not been estimated but set to their values in MARCOS. The overall estimated model is the following:

$$\begin{cases} ygap_t &= b_1 ygap_{t-1} + b_2 rr_t + b_3 \\ r_t &= 1.5p_{t,t+1} + 0.5 ygap_t + m \\ p_t &= z p_{t-1} + (1-z) p_{t,t+1} + y . ygap_{t-1} + a \end{cases}$$

with $p_{t,t+1}$ the inflation expectation made at time t for t+1.

We thus get the following results:

Coefficients	
z	0.43 (22.2)
y	0.045 (1.90)
a	-0.0002 (-2.7)
Log-likelihood	354

Sample: 1972-1999

It is worth noting that current inflation is almost equally dependant on its past and future values (ζ=0,43).

External trade

Specification are rather usual. Exports are explained by world demand and competitiveness (defined as the export price over foreign price ratio). Imports depend on absorption and a competitiveness indicator (defined as the import price over value added price ratio)

$\Delta\ln(M_t) - \Delta\ln(ABS_t)$	Coefficients	$\Delta\ln(X_t) - \Delta\ln(DM_t)$	Coefficients
$\Delta\ln(pm_t) - \Delta\ln(p_t)$	-0.21 (-4.36)	$\Delta\ln(px_t) - \Delta\ln(p_t^*)$	-0.95 (-3.38)
$\ln(M_{t-1}) - \ln(ABS_{t-1})$	-0.21 (-2.60)	$\ln(X_{t-1}) - \ln(DM_{t-1})$	-0.20 (-2.00)
$\ln(pm_{t-1}) - \ln(p_{t-1})$	-0.16 (-3.80)	$\ln(px_{t-1}) - \ln(p_{t-1}^*)$	-0.11 (-0.42)
Trend	0.004 (3.17)	*Trend*	0.007 (1.04)
Intercept	-8.80 (-3.16)	*Intercept*	-12.14 (-0.96)
R^2	0.7	R^2	0.5
DW	2.4	DW	1.64

Sample: 1970-1997

The trend in the external trade equations is rather critical in the long run. In this case, the imports and the exports shares in GDP grow infinitely. However, the attempts to replace the linear trends by logistic functions lead to unreliable results. Furthermore, this specification is quite usual as far as external equation are concerned. Although not significant, the price elasticity parameter in the export equation has been kept considering its realistic value.

The external prices

The external prices equation are static. They are explained by the value added price and the foreign price expressed in domestic currency:

$$\ln(px_{ze,t}) = 0.75\ln(pva_{ze,t}) + 0.25\ln(p^*_{ze,t})$$
$$\ln(pm_{ze,t}) = 0.48\ln(pva_{ze,t}) + 0.52\ln(p^*_{ze,t})$$

The US

The calibration and the estimation for the US have been carried out in the same manner. The results are presented in Tables 2.4 to 2.6 in Appendix 2.

3- Optimal Taylor rule

The optimality of the monetary policy rule is defined as the suitable calibration of the Taylor rule. We mean by suitable, the values of the coefficients of the Taylor rule that minimise a weighted sum of variances of output, inflation and interest rate conditional on the model. Formally the program is:

$$\begin{cases} \underset{m,t}{Min} \sum_{t=1}^{T} b^t \left[l_y.V(y_t - y_t^*) + l_p.V(p_t - p_t^*) + l_r.V(\Delta r_t) \right] \\ r_t = r_{t-1} + m(p_{t+1} - p_{t+1}^*) + t\,(y_t - y_t^*) + e_t^r \qquad (1) \\ F(Z_t, X_t) = e_t^Z \qquad (2) \end{cases}$$

Where y_t is the output, y_t^* its potential value, p_t the inflation, p_t^* the inflation target, and r_t the nominal interest rate. Equation (1) is the usual Taylor rule whereas equation (2) corresponds to the overall model (the monetary policy rule excepted) with Z_t and X_t the endogenous (determining y_t, p_t and y_t^*) and exogenous (p_t^*) variables respectively. e_t^r and e_t^Z are the innovations of the Taylor rule and of the rest of the model. The coefficient b is the discount factor.

The model (2) is usually (Ball[1997], Jondeau Le Bihan [2000]) composed of an IS curve and a Phillips curve. The problem could be rewritten as:

$$\begin{cases} \underset{m,t}{Min} \sum_{t=1}^{T} b^t \left[l_y.V(y_t - y_t^*) + l_p.V(p_t - p_t^*) + l_r.V(\Delta r_t) \right] \\ r_t = r_{t-1} + m(p_{t+1} - p_{t+1}^*) + t\,(y_t - y_t^*) + e_t^r \\ y_t = r y_{t-1} - a_1 (r_t - p_t) + a_2 + e_t^y \\ p_t = A(L) p_{t-1} + b\left(y_{t-1} - y_{t-1}^*\right) + e_t^p \end{cases}$$

with $A(1) = 1$ to verify the long-run verticality of the Phillips curve. Due to the linearity of the model, the analytical solution is then straightforward. It can be shown (Svensson [1998]) that the model admits the following AR(1) form:

$$\tilde{Z}_t = B\tilde{Z}_{t-1} + e_t$$

with $\tilde{Z}_t = \left(r_t, Z_t'\right)'$.

With B a matrix depending upon m and t
The whole system could then be rewritten as:

$$\begin{pmatrix} V(\Delta r_t) \\ V(y_t - y_t^*) \\ V(p_t - p_t^*) \end{pmatrix} = [I - (B \otimes B)]^{-1} vec[V(e_t)]$$

the optimal coefficients of the Taylor rule are deduced form the minimisation of

$$\sum_{t=1}^{T} b^t \begin{pmatrix} l_y & l_p & l_r \end{pmatrix} \begin{pmatrix} V(\Delta r_t) \\ V(y_t - y_t^*) \\ V(p_t - p_t^*) \end{pmatrix} = \frac{1}{1-b} \begin{pmatrix} l_y & l_p & l_r \end{pmatrix} [I - D]^{-1} vec[\Omega]$$

with $V(e_t) = \Omega$ and $D = B \otimes B$ a quadratic form of the model coefficients including m and t .

Applying this method to MARCOS raises several difficulties. MARCOS presents strong non-linearity. Its linearization around the steady state is a cumbersome task and leads to an approximated rather than an exact solution. The other solution consisting in simulating stochastically the model for a set of coefficients rather than solving it analytically has to be considered despite its difficult implementation. In this case, each shock in MARCOS has to be uncorrelated with the contemporaneous endogenous variables. Due to the forward-lookingness of MARCOS, at each period the model is solved taking into account the path formed by all the future periods. If we simulate stochastically at once a full path, the residuals of future periods will be correlated with the current endogenous. To avoid such a problem, we have to simulate the model as follow. At each date we introduce a shock at the current period and we set it to zero for all the following periods and we simulate the entire path. This procedure should be repeated date after date to get a complete path. So to simulate a complete path of T periods we have to run T forward looking simulations of the model.

MARCOS is calibrated around its equilibrium steady state rather than estimated. It means that we do not dispose of a residuals covariance matrix. Thus, the matrix Ω in the minimisation problem is unknown and furthermore applying stochastic simulations is impossible since we do not know the residual distribution.

Before proceeding to stochastic simulations, we examine the optimality of the Taylor rule conditional on a specific shock. Hence, we have simulated deterministically the model for two types of shocks: a demand shock which could be viewed as a shock on the IS curve (e_t^y) and a supply shock which could be analysed as a shock on inflation (e_t^p). Actually, the supply and the demand shocks are respectively defined as a labour productivity shock (+1%) and as a government expenditure shock.

The deterministic simulations allow to compute simultaneously optimal coefficients and horizons. We have in this case to compute a total number of 900 forward looking simulations for each area (and each shock in the deterministic case) corresponding to the product of the 30 nodes of the grid for the different horizons by the 30 nodes of the mesh for the different coefficients. This task becomes rather cumbersome with stochastic simulation (the entire simulation time amounts to 20 days with a pentium IV of 1.7Ghz!!).

4- Deterministic simulations

Two simulation exercises are run, representing respectively a demand shock and a supply shock. The first simulation presented is a permanent increase in the Government consumption by 1% of GDP. For this exercise, the Government reaction function is "disconnected" during the first five years in order to increase the Government consumption without increasing the direct tax rate and let fluctuate the ratio of public debt over GDP. From the sixth year to the final period, the government reaction function is "reconnected" and the direct tax rate is endogenous again with the debt ratio target increased by 5 points. This simulation provides a description of the adjustment mechanism after the government spending shock. The shock leads to an expansion in the short term. The second simulation, emphasising the effects of a supply shock in MARCOS, is a permanent increase in the productivity that raises output by 1%. This increase in the output is obtained by an equivalent shock on the labour technical progress.

5.1- Simulations results

A demand shocks in the US

The effect on the US

In the long run, the 5 point increase in the public debt over GDP ratio implies a rise of income tax rate of about 2 points to finance the greater public debt burden (Figure 1). This direct tax increase induces a proportional decrease in the household consumption. However, the public debt expansion does not involve a complete Ricardian equivalence effect for at least two reasons. First, because the household life time is finite whereas the government life time is infinite, classical households do not buy all government bonds and do not increase equivalently their saving. They expect the Government levies taxes after their death. Second, Keynesian households are subject to a liquidity constrain and cannot increase their saving. Consequently, the household saving does not increase enough to finance the public debt, implying an external debt rise. Almost all the public debt increase is financed abroad. The balance of payment equilibrium is obtained by a weakening of the real exchange rate and thus an improvement in the trade balance because of the uncovered interest rate parity assumption. In the long run, imports decline by 1.5% and exports increase by 1.7% (both becoming stable after about twenty years). The trade balance is positive to compensate the capital outflows due to the external debt payment.

In the short term, the Government expenditure shock induces a disposable income increase and consequently a higher consumption level for Keynesian households. Because of the strong decrease in the wealth, the classical consumption plunges immediately. The wealth evolution comes from the contraction of the household financial asset. The global effect on the overall consumption is slightly negative. The public debt increase involves higher interest rate and lower private investment (about 4%). The interest rate increase also implies a small real exchange rate appreciation because of the uncovered interest rate parity assumption. This real exchange rate movement implies a decrease in exports and a rise in imports. Although investment and consumption decrease, the GDP grows due to the shock on the Government expenditures.

In the medium run, the higher direct taxes rate reduces the household consumption and the GDP and brings back the economy to its long-term steady state path.

The effect on the euro area

The effects are rather weak on the euro area economy (Figure 2). In the long run, the modification of relatives prices (valued added price over the demand price) leads to an increase in the real capital cost (defined with respect to the value added price) and thus to a small capital reduction. Combined with the consumption contraction implied by direct tax raise and the wealth contraction, the GDP slightly decreases.

In the short run, the increase of US imports stimulates the euro area exports. However the substantial decrease in the US imports in the medium run is attenuated by the Euro competitiveness improvement. The increase in the public debt burden involves a higher direct tax rate and then a reduction in consumption. In the medium run the consumption contraction explains the small negative impact on output and thus on investment.

A demand shocks in the Euro Zone

The US and the euro area models have an identical structure. Consequently, results are qualitatively close but responses are smaller than in the previous case (Figure 3). The mechanisms previously described still apply here. The feedback effects are negligible (Figure 4). Results for the US are nearly five time smaller than for the euro area in the previous simulation.

A Supply shocks in the US

The effect on the US

This positive productivity shock has two effects. First, in short run, it increases wages and has no effect on employment (box 1). The rise in wages and hence in the human wealth increases instantaneously the Keynesian household's consumption and more progressively the consumption of classical households. The aggregate consumption grows by 1.4% the first year and increases by 1.25% 8 years after. Second, the labour efficiency increase induces a capital productivity improvement and thus a rise in the desired capital stock. Thus the investment increases in the short run and the production too. However, because in the short run the unemployment is slightly above its equilibrium level, very weak deflationary pressures appear (the inflation rate decreases by about 0.05 points after 6 years). Moreover, the improvement of the external competitiveness leads to an increase in the exports (0.8%) and a reduction in the foreign trade deficit. In the long run, the trade balance deficit induces a reduction in the external debt.

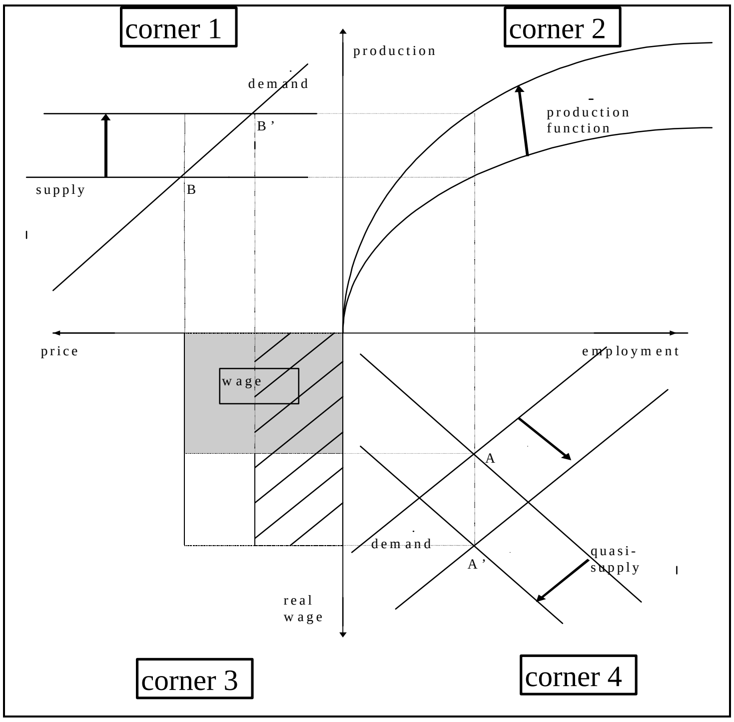
The effect in the euro area

Two short-run effects have to be considered. First, the positive effect on US imports improves the euro area exports whilst imports remain almost the same. Second, the higher US short term interest rate increases the euro area short term interest rate and reduces the investment. The net effect on the euro area GDP is slightly positive. In the long run, the terms of trade improvement lowers very weakly the real capital cost and increases the capital, the output and the employment.

A supply shock in the Euro Zone

The responses are generally very close to those previously commented for the US productivity shock, but with 3 times lower magnitudes. However, some differences could be observed for imports. Over the 1985-97 period used to calibrate the model, imports prices are more sensitive to valued added prices in the euro area than in the US. Instead of decreasing as in the US productivity shock, the euro area imports remain stable. As pointed out before, the weakness of feedback explains the quasi absence of effect for the US.

Box 1 : The effect of a productivity shock in Marcos



An increase in labour efficiency moves upward the production function. For the same amount of labour a bigger quantity is produced (corner 2). The labour efficiency improvement increases also the apparent labour productivity implying a unit labour cost decrease. In the long run, prices, deduced from a constant mark-up over the unit labour cost, decrease. The long-term labour demand, relating the labour productivity to the real labour cost, moves to bottom right corner because of the labour productivity improvement. However, unions take it into account (unemployment benefits are indexed to the real wage and to labour productivity). The labour supply moves with the same magnitude to the bottom left corner. In practice unions claim for higher wages. As a result, the equilibrium real wage increases, but the employment and the unemployment rate remain unchanged (corner 4). The real wage increase comes from the price decrease and thus the nominal wage does not change (the size of the "slice" area measuring the nominal wage after the shock is equal to the size of the grey area measuring the nominal wage before the productivity shock – corner 3)

Figure 1: The impact of an US public expenditure shock (1% during 5 years) and a 5% increase in the public debt 5 years later in the US [7]

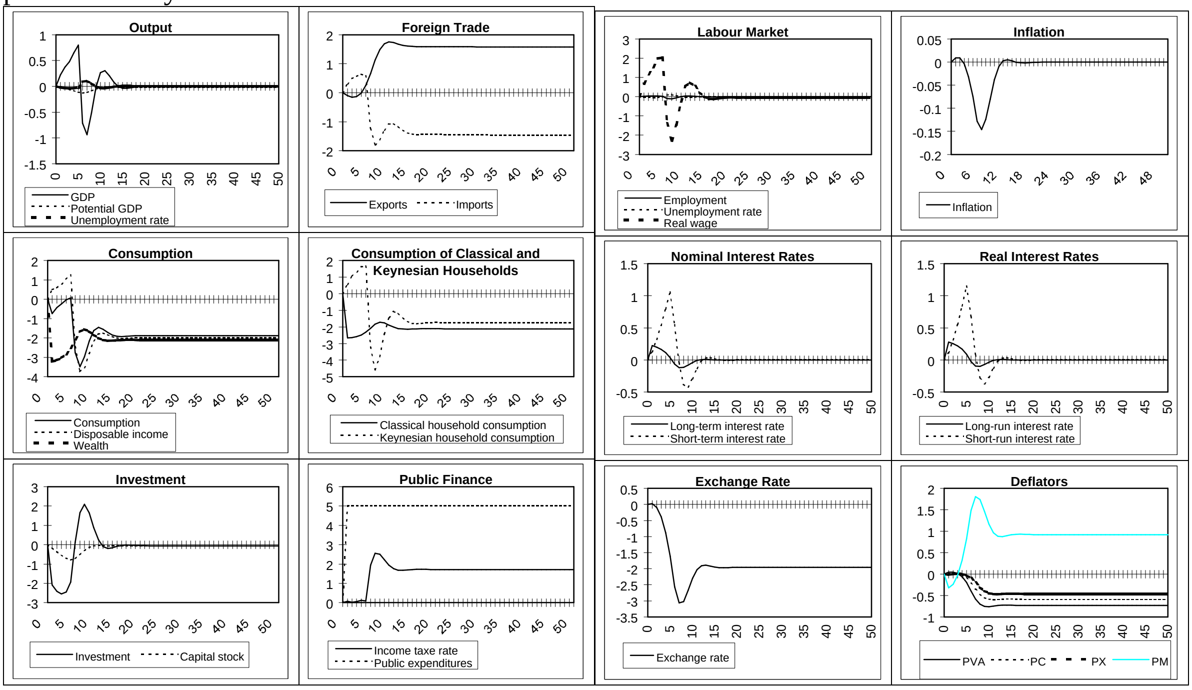
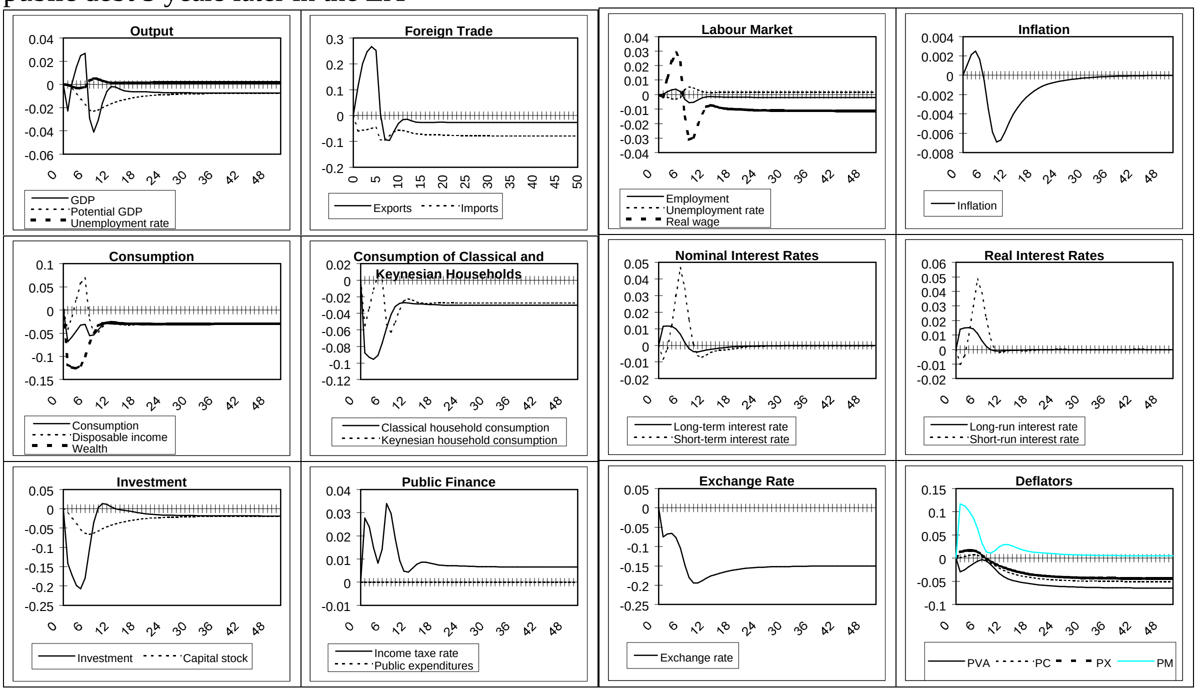


Figure 2: The impact of an US public expenditure shock (1% during 5 years) and a 5% increase of the public debt 5 years later in the EA



[7] For those results and for the next one the variables in level are expressed in relative deviation from their steady state value (in percent) and the variables corresponding to ratio are expressed in absolute deviation from their steady state value (in points).

Figure 3: The impact of an EA public expenditure shock (1% during 5 years) and a 5% increase of the public debt 5 years later in the EA

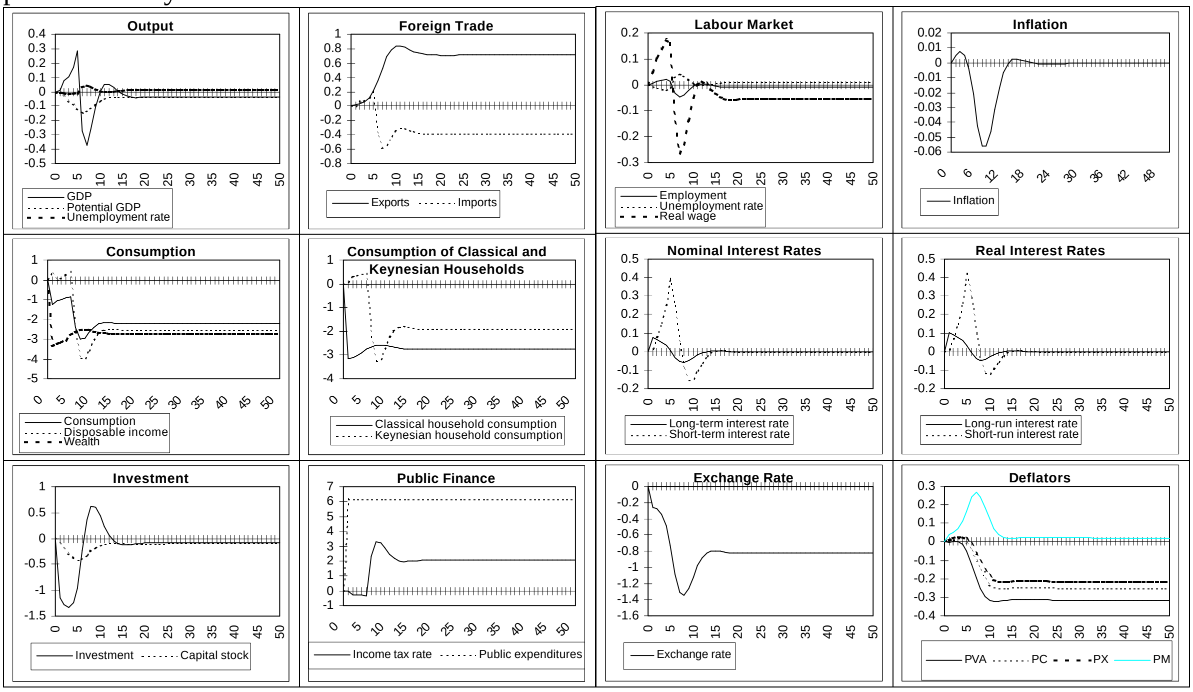


Figure 4: The impact of an EA public expenditure shock (1% during 5 years) and a 5% increase of the public debt 5 years later in the US

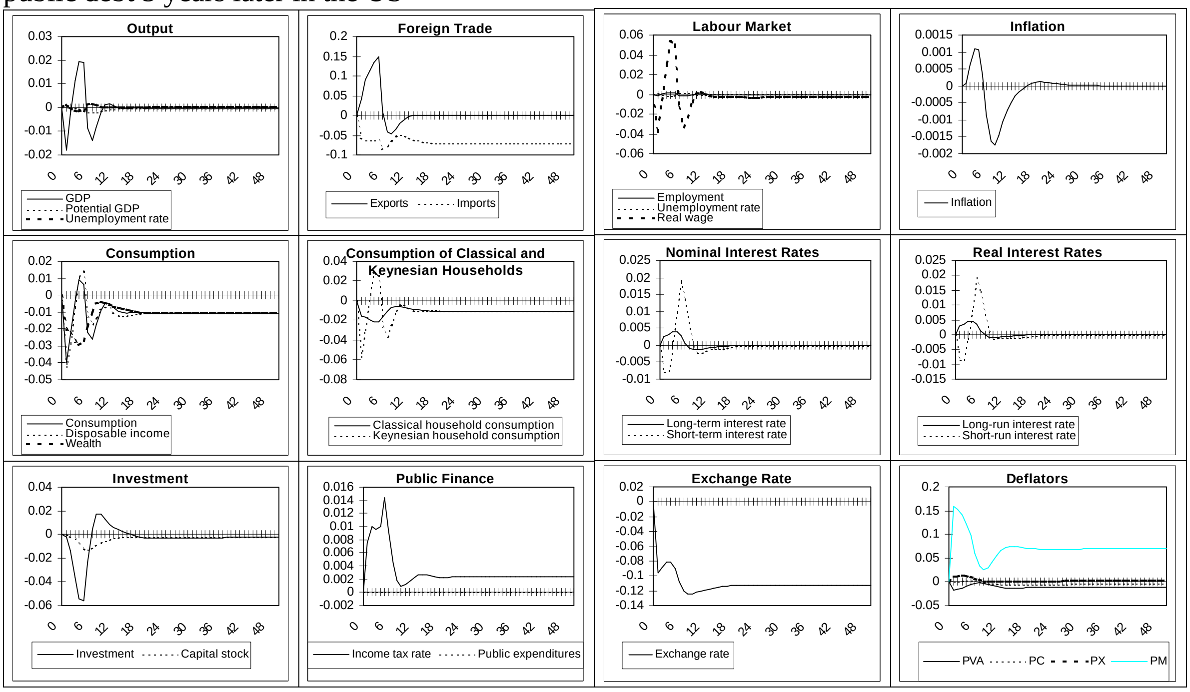


Figure 5: US impact of a US productivity shock (+1 point on labour technical progress)

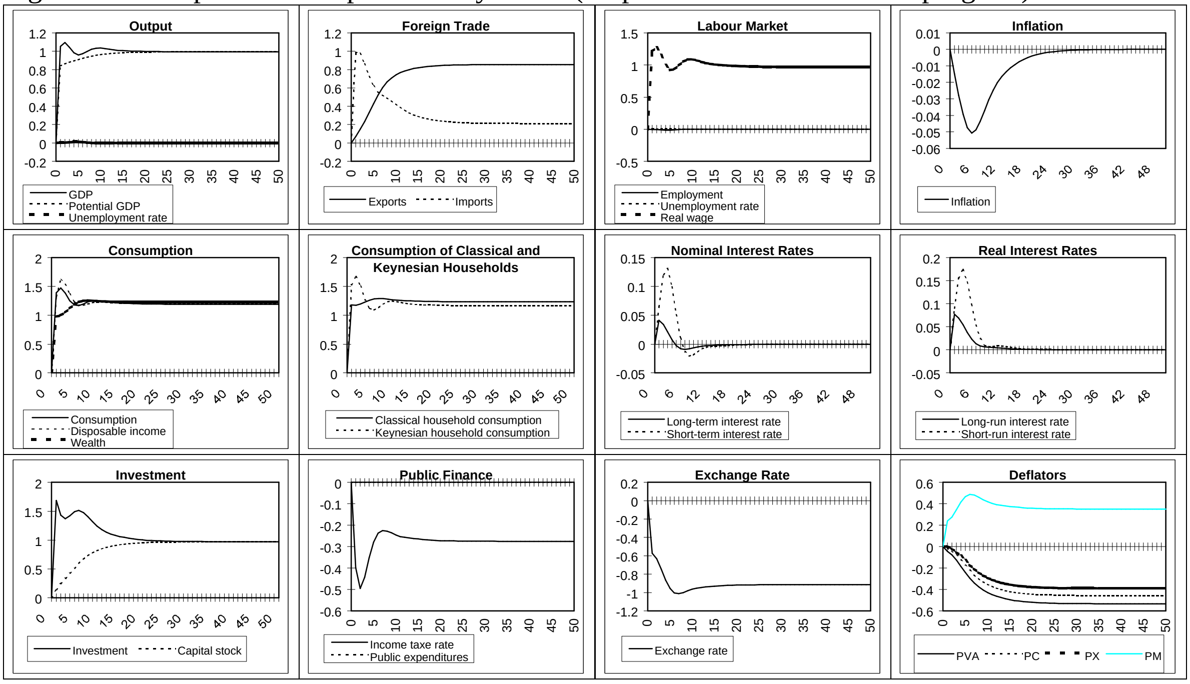


Figure 6: EA impact of an US productivity shock (+1 point on labour technical progress)

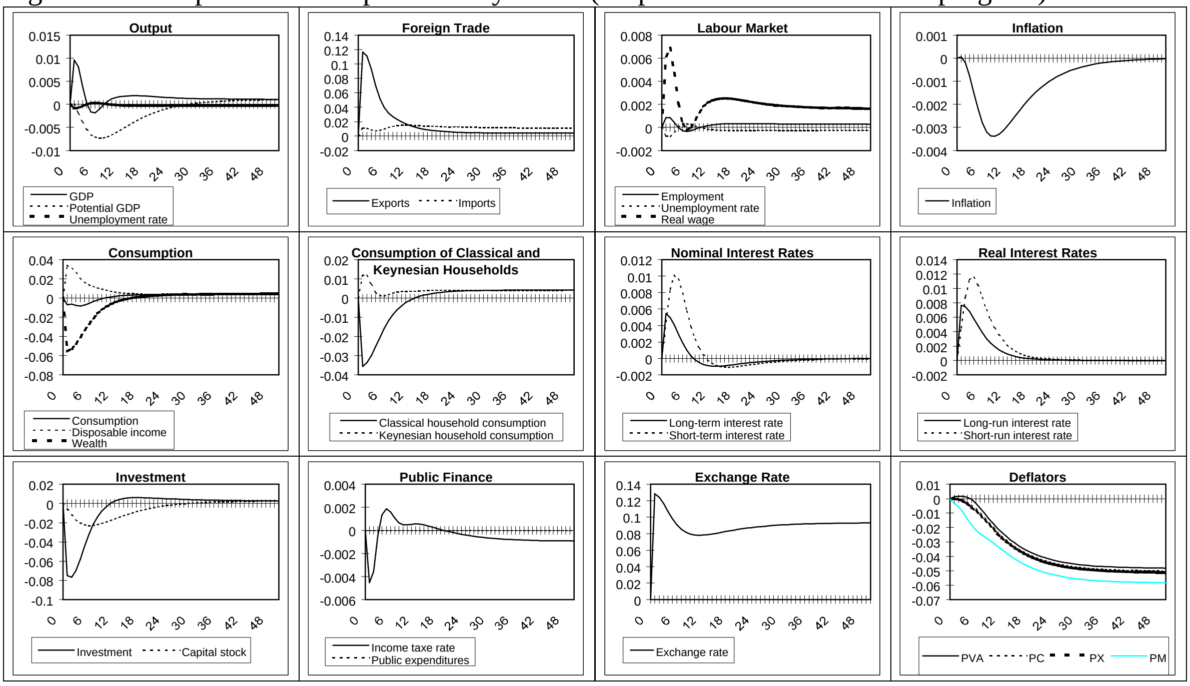


Figure 7: impact of an EA productivity shock (+1 point on labour technical progress) in the EA

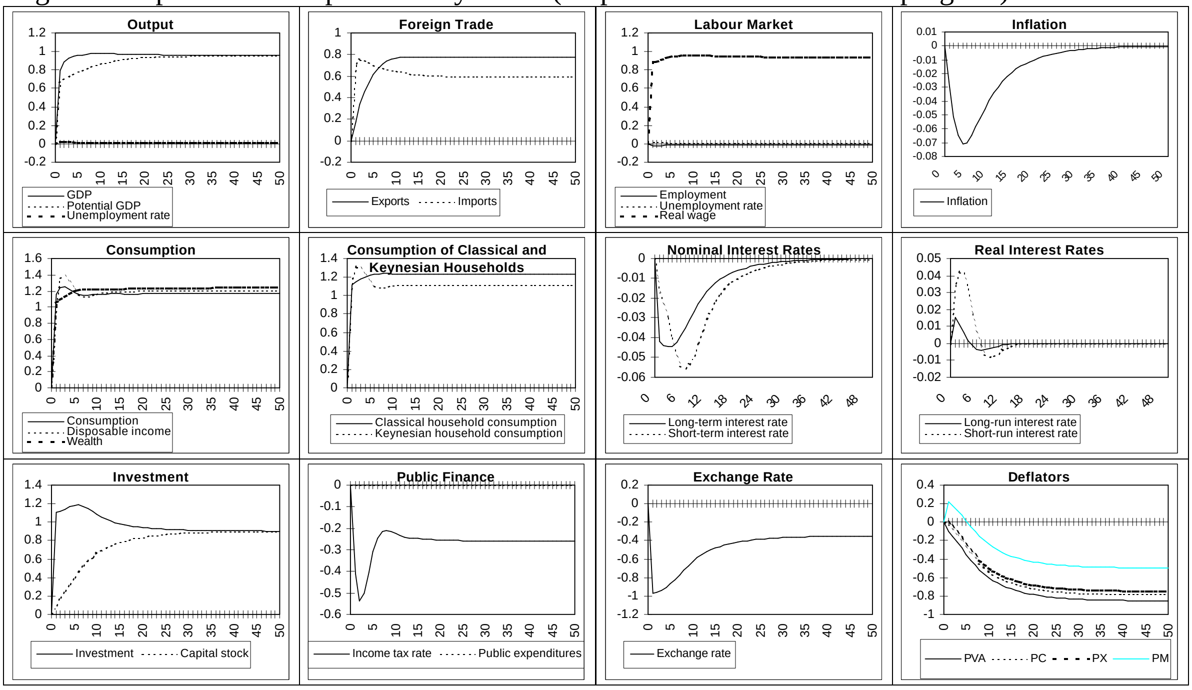
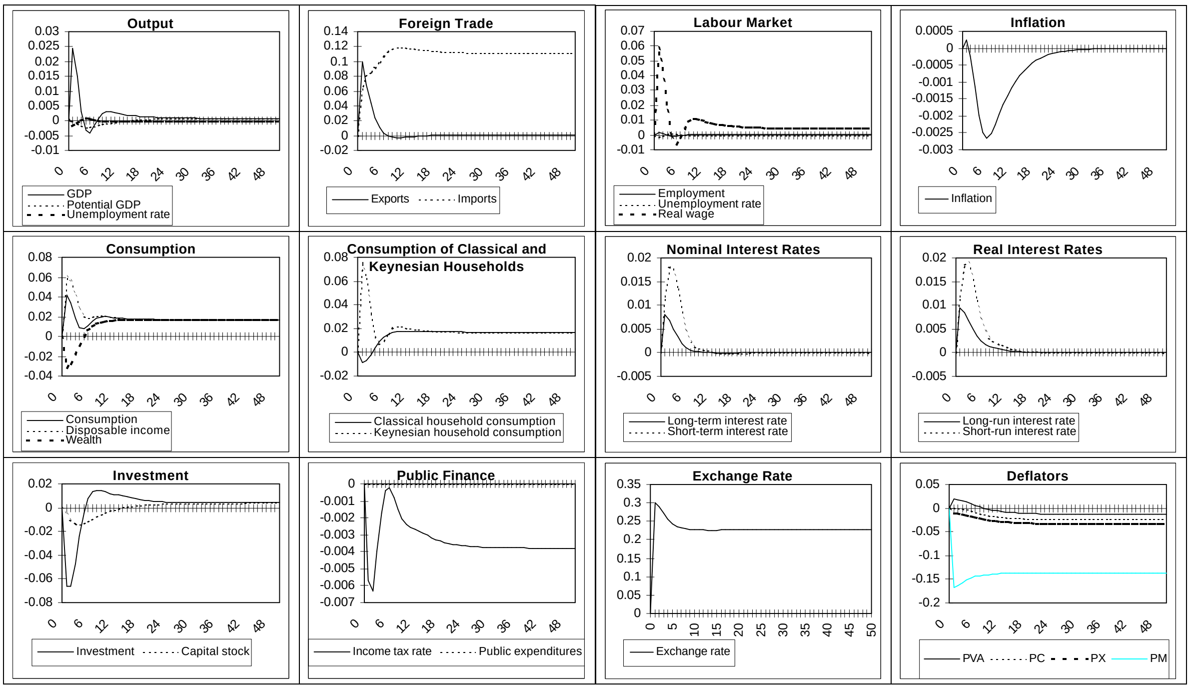


Figure 8: impact of an EA productivity shock (+1 point on labour technical progress) in the US



5.2- Optimal rules in a deterministic environment

Optimal Taylor rule coefficients

Theses simulations are carried out for each area (Euro and US) on a mesh composed of 30 nodes ($m = 0.4$, 0.8, 1.2, 1.6, 2 and $t = 0$, 0.2, 0.4, 0.6, 0.8, 1). For each shock, we compute the Taylor rule coefficients minimising the overall program criterion or specifically the variance of the output-gap, the variance of the inflation gap or the variance of interest rate changes. For the program criterion the weighting parameters l_y, l_p, l_r, b have been set equal respectively to 1, 1, 0.5 and $1/1.04$. For one shock in one area we report the coefficients which minimise the different criteria for each area (for the other area the Taylor rule coefficients are: $m = 1.5$ and $t = 0.5$). For example, for a productivity shock in the US, we compute the coefficients of the US Taylor rule which minimise the different criteria for the US. For the induced effect on the euro area, we also compute the coefficients of the euro area Taylor rule minimising the different criteria for the euro area.

It is worth noting that Blanchard and Kahn conditions depend on m and t values. For some range of values the Blanchard and Kahn conditions could not be met. For example, $m = 0$ leads to indeterminacy (the number of non redundant lead variables exceeds the number of eigenvalues greater than one) and this case has been ruled out.

Results relative to the variance of interest rates change (Table 1) are quite obvious. For any shock or area, the variance of interest rates change is minimised if the central bank does not react i.e. m and t are equal to 0.

As expected, when the monetary authority has a unique inflation variance target (respectively output variance), it achieves its goal by being the more aggressive on the inflation-gap (output-gap). But as we suppose that the coefficients of Taylor rule should lie in some reasonable range (here $0 \le t \le 1$ and $0.4 \le m \le 2$), when the constraint is bound the adjustment operates on the output-gap (inflation-gap).

Demand shock

For a demand shock, effects on inflation and output are identical. Thus to minimise the output variances, whatever the area considered, m and t coefficients should be higher as possible. The minimisation of inflation variance is obtained for t equal to 0.4 and m greater than 2 for inflation.

The other area will import inflation and then face inflation pressure. The indirect effects on the other area are also identical, except for the US where the minimisation of the inflation variance is obtained with a slightly more aggressive attitude of the central bank with respect to output-gap ($t = 0.8$ against $t = 0.2$) due to the higher fluctuation of imported inflation in the US.

Supply shock

A productivity shock induces opposed effects on inflation and output, combining a positive permanent increase of output and an inflation reduction. Consequently, the minimisation of one variance provokes an increase in the other one. The reduction of the variance of inflation (respectively output) leads the monetary authority to be aggressive on inflation: m high and t low (respectively m low and t high). There is a true trade-off between stabilising inflation-gap or output gap.

Since for the other area, the productivity shock also implies an increase in the output-gap and a disinflation movement, the optimal coefficient for the Taylor rule will vary in the same way.

Optimal Taylor rule horizon

The monetary policy affects the economy with some delay. The monetary authority has to take into account the transmission delay to conduct its policy. And in presence of transmission lag, it will be sub-optimal to target the current inflation rate rather than its future value. The issue here is: what is the optimal horizon of monetary policy? In other words, what are the leads in the Taylor rule for inflation rate target as well as output-gap that minimise the loss criterion? This question of the optimal horizon could be related to Batini and Nelson [2000] OFH definition: “the optimal feedback horizon (‘OFH’) [is] the best point in the future for which the authorities should form the inflation forecast that enter their policy rule”. However Batini and Nelson [2000] consider only inflation targeting. We extend the optimal horizon to the output-gap target.

The transmission delay depends upon the openness of the area. For an open economy the exchange rate channel operates faster than the output gap channel. Nevertheless, even in open countries the transmission delay is still significant. In MARCOS, both areas present a weak degree of openness.

Concretely, we minimise the loss function respect to k_μ and k_τ

$$\bar{r}_t = \bar{r}_{t-1} + \mu(\pi_{t+k_\mu} - \pi^*_{t+k_\mu}) + \tau\left(\frac{Y_{t+k_\tau}}{\bar{Y}_{t+k_\tau}} - 1\right)$$

To compute the optimal horizon, we proceed as previously by searching on a grid formed of 30 nodes ($k_\mu = (0,1,2,3,4)$ and $k_\tau = (0,1,2,3,4,5)$).

As for optimal coefficient of the Taylor rule, the minimisation of the variance of interest rate changes implies higher lead values. This result has a simple interpretation. In order to get the lowest variance on interest rate changes, the monetary authorities have to target the variables the closest to their steady state values, in our case the more forward variables.

Demand shock

In the case of euro area the optimal attitude consists in fast reaction and involves short leads on targets. However, for the US we observe a surprising long optimal horizon for output-gap when output variance is targeted and for inflation when inflation variance is target.

The induced effects on the other area are quite similar, with a very short horizon on both areas as far as inflation variance and output-gap variance are concerned.

Productivity shock

The optimal horizon for inflation is systematically high (greater than 4 years). Conversely, when inflation is targeted, the optimal leads on output-gap display a much higher dispersion across areas: longer leads in the US than in the euro area. This difference could be related to the higher degree of output-gap persistence in the Euro.

The indirect effects are the same: k_m is high whereas k_t is low. Because the induced effects are analogue to those of a productivity shock (output-gap increase with a inflation decrease), the optimal attitude of the monetary authorities is to quickly react to output gap and have a longer horizon on inflation.

In Tables 3 to 6, optimality is computed with respect to both parameters and leads. Results confirm our previous observations concerning the optimal parameters and leads. However, when the focus is the indirect effect, some differences appear for the optimal coefficients for an output-gap variance objective. When the optimisation is carried out jointly on coefficients and leads, the parameter associated to output-gap is larger (greater or equal to 2 instead of 0.4).

Table 1: Optimal coefficients in the Taylor rule

	US productivity shock				US demand shock			
	US		EA		US		EA	
	m	t	m	t	m	t	m	t
$V(Y)$	≤ 0.4	≥ 1	≤ 0.4	≥ 1	≥ 2	≥ 1	≤ 0.4	≥ 1
$V(p)$	≥ 2	0.2	≥ 2	0	≥ 2	0.4	≥ 2	0.2
$V(dr)$	≤ 0.4	0.6	≤ 0.4	0	≤ 0.4	0	≤ 0.4	0
Criterion	≤ 0.4	0.6	≥ 2	0.6	≥ 2	≥ 1	≥ 2	≥ 1
	EA productivity shock				EA demand shock			
	EA		US		EA		US	
	m	t	m	t	m	t	m	t
$V(Y)$	≤ 0.4	≥ 1	≤ 0.4	≥ 1	≥ 2	≥ 1	≤ 0.4	≥ 1
$V(p)$	≥ 2	0	≥ 2	0.2	≥ 2	0.4	≥ 2	0.8
$V(dr)$	≤ 0.4	0	≤ 0.4	0	≤ 0.4	0	≤ 0.4	0
Criterion	1.6	0.8	1.2	≥ 1	≤ 0.4	0	≤ 0.4	≥ 1

Table 2: Optimal leads in the Taylor rule

	US productivity shock				US demand shock			
	US		EA		US		EA	
	km	kt	km	kt	km	kt	km	kt
$V(Y)$	4	1	4	0	0	5	0	0
$V(p)$	4	3	4	0	4	1	0	0
$V(dr)$	4	5	4	5	0	3	1	5
Criterion	4	1	4	0	1	5	0	0
	EA productivity shock				EA demand shock			
	EA		US		EA		US	
	km	kt	km	kt	km	kt	km	kt
$V(Y)$	4	0	4	0	0	0	0	0
$V(p)$	4	0	4	0	1	1	0	1
$V(dr)$	4	5	3	5	1	5	1	5
Criterion	4	0	4	0	0	0	0	0

Table 3: Optimal coefficients and leads in the Taylor rule for a US productivity shock

	inflation coefficient : m	Output-gap coefficient : t	Lead on Inflation : km	Lead on output gap : kt
$V(Y_{US})$	0.4	0.8	4	0
$V(p_{US})$	2	0.2	4	2
$V(dr_{US})$	0.8	0.2	4	5
$Criterion_{US}$	0.8	0.6	4	0
$V(Y_{EA})$	2	0.8	4	0
$V(p_{EA})$	2	0.2	4	0
$V(dr_{EA})$	0.4	0	4	2
$Criterion_{EA}$	2	0.8	4	0

Table 4: Optimal coefficients and leads in the Taylor rule for a US demand shock

	inflation coefficient : m	Output-gap coefficient : t	Lead on Inflation : km	Lead on output gap : kt
$V(Y_{US})$	2	0.8	0	0
$V(p_{US})$	2	0.4	0	1
$V(dr_{US})$	0.4	0	0	4
$Criterion_{US}$	2	0.8	0	0
$V(Y_{EA})$	2	0.8	0	0
$V(p_{EA})$	2	0.2	1	2
$V(dr_{EA})$	0.4	0.2	0	5
$Criterion_{EA}$	2	0.8	0	0

Table 5: Optimal coefficients and leads in the Taylor rule for an EA productivity shock

	inflation coefficient : m	Output-gap coefficient : t	Lead on Inflation : km	Lead on output gap : kt
$V(Y_{US})$	2	0.8	4	0
$V(p_{US})$	1.2	0.4	4	5
$V(dr_{US})$	0.4	0	4	3
$Criterion_{US}$	2	0.8	4	0
$V(Y_{EA})$	0.8	0.8	4	0
$V(p_{EA})$	2	0	0	3
$V(dr_{EA})$	0.4	0	4	1
$Criterion_{EA}$	2	0.6	4	0

Table 6: Optimal coefficients and leads in the Taylor rule for an EA demand shock

	inflation coefficient : m	Output-gap coefficient : t	Lead on Inflation : km	Lead on output gap : kt
$V(Y_{US})$	2	0.8	0	0
$V(p_{US})$	2	0.4	0	1
$V(dr_{US})$	2	0	0	2
$Criterion_{US}$	2	0.8	0	0
$V(Y_{EA})$	2	0.8	0	0
$V(p_{EA})$	2	0.2	0	1
$V(dr_{EA})$	0.4	0.2	0	5
$Criterion_{EA}$	2	0.8	0	0

5- Stochastic simulations

In the previous section, we have supposed that shocks are known and we have computed the optimal coefficients and horizons of the Taylor rule conditionally to those shocks. Here, we want to relax this restriction by simulating stochastically MARCOS.

The stochastic simulation strategy

The main issue here is how to run stochastic simulations in MARCOS whereas it is mainly calibrated. For estimated models, the exercise is quite easy: shocks are simply drawn from the distribution of estimated residuals. In our case, we suppose (as Black et al. [1997] for QPM and Drew and Hunt [1998] for FPS) that the economy could be approximated by a reduced form core model and the estimation of the core model will give the distribution of shocks required by the stochastic simulation. The VAR methodology is the most appropriated to get such a core model. According to the VAR literature, the economy is hit by independent innovations and impulse responses are run in order to identify them. Each residual could finally be expressed as combination of these innovations. In order to proceed to stochastic simulations, residual terms are added to some behavioural equations of MARCOS and defined such that the model could mimic the impulse responses given by the VAR.

Two VAR are estimated: one for each area. This strategy of estimating two different VAR for a two-country model can be first justified by the weakness of the links between areas (in the model). Furthermore, each VAR should be large enough to capture the main shocks the US and Euro zones are supposed to face. But estimating a too large VAR (with annual data) could raise some degree of freedom problems[8]. For both zones, the VAR is composed of the five following variables: world demand, world demand deflator, consumption, demand deflator and yield curve. Volumes and the yield curve are in level, prices are computed in growth rate. The order of appearance of the variables gives the causal ordering of the VAR and consequently its identification scheme. The priority of foreign variables with respect to domestic variables indicates the top position of the foreign sector in the causal hierarchy of the model. The last position of interest rates signifies that the monetary authority reacts to all the previous information. The interpretation of innovations associated to each equation is quite standard. The first two shocks are respectively the world demand shock and the terms of trade shock whereas the shock to the consumption can be viewed as a demand shock and the shock to the demand price as a supply shock on the Phillips curve. The shock to the yield curve is interpreted as a monetary shock.

The VAR is estimated over the 1970-2000 period for the euro area and the 1974-1997 period for the US. The number of lags, equal to one, has been determined by AICC and Schwarz criteria. Impulses are responses to an one-standard-deviation shock on each innovation.

The VAR gives an estimate of the response for the five variables to each innovation and the problem is how to design MARCOS to exactly replicate the impulse responses function (IRF) of the VAR over the first period (one year), i.e. before any effect of economic policy. The aim is here to catch the purely exogenous shocks hitting the economy and for this reason the period should be free of any policy effect. On the one hand, the VAR identifies the innovations, their standard deviations, and also produces a precise picture of the dynamics of the economy. On the other hand, and by construction, MARCOS has no residuals. The strategy will be to use the information given by the VAR to introduce the appropriate residual terms in behavioural equations of MARCOS. These terms are added to the level of behavioural equations of MARCOS whose economic definition is the nearest to the one of the VAR. Their role is to give the deviation from the steady state that will permit the replication of the IRF. These residuals will be a combination of the innovations. Once weights determined, the stochastic simulations can finally be implemented from a normal distribution N(0,1).

Contributions of innovations to MARCOS residuals are computed as follow. For example in the case of a world demand shock in the Euro zone, first the IRF of the VAR to a world demand shock (responses of the five variables to this shock) are retrieved. Second, add-factors are introduced in MARCOS not only in the corresponding five equations of the euro area but also in the same five equations of the US area. They are introduced in both areas in order to take account of inter-relationships. Third, the model is simulated over 50 periods (the time to be sure that all variables reach their steady state) with add-factors as endogenous and behavioural variables as exogenous. The monetary reaction function is switched off to assure the independence of computed residual terms from the structural form of the reaction function. The weights of the first innovation to MARCOS residuals are then retrieved. The contributions of the world demand innovation to the one-period residuals are obtained.

[8] A VAR for both areas will contain 10 variables. With two lags, the number of parameters to be estimated for each equation is equal to 20! That is particularly expensive for annual data.

Fourth, the procedure is applied for all IRF and fifth, re-iterated for the next period. MARCOS could now be stochastically simulated.

As noticed in section 3, the forward-lookingness induces a stochastic simulation run date by date to get a complete path. These simulations have to be repeated for the number of replications. The simulation protocol retained is of 30 replications over 50 periods. As pointed out for FPS by Drew and Hunt [1998], for less than 30 replications standard deviation of output display instability[9].

Given the estimated reduced form:

$$X_t = A(L)\ X_{t-1} + v_t$$

where $V(v_t) = \Sigma$, the associated structural VAR is:

$$X_t = A(L)\ X_{t-1} + Be_t$$

where X_t is the vector of the five dependent variables, *A(L)* the lag polynomial matrix, *L* the lag operator. The shocks e_t are iid N(0, I) with *I* the identity matrix. *B* is a matrix such that $B'B = \Sigma$. e_t has five independent components e_t^j where a single-period unitary shock on e_t^j produces the IRF *j*. Since $v_t = Be_t$ we can write:

$$v_t = \sum_{j=1}^{5} Bi^j\ e_t^j$$

where i^j is a selector vector of zeros excepted the *j*th row equal to one. Weights of the sum (ε_t^j) are N(0, 1) random numbers.

Now let us construct the residual terms that MARCOS needs to replicate the IRF. To the equation *i* is associated the residual term u_t^i (with $i = 1,\ldots,10$ in order to catch direct as well as indirect effects, 5 for each area). As noticed before, the random number e_t^j represents the innovation associated to the variable *j* of the VAR. Each simulation gives the numerical value $a_{i,t}^j$ for the effect of shock *j* on variable *i* at date *t*. $t = 1$ since the IRF are replicated for one period only. Finally,

$$u_1^i = \sum_{j=1}^{10} \alpha_{i,1}^j\ \varepsilon_1^j$$

whereas $u_t^i = 0$ for $t \neq 1$.

It is worth noting, that in addition of cross correlation, Drew and Hunt [1998] allow for serial correlation among the structural model residuals (u_t^i). In this case residuals of the model will be:

$$u_t^i = \sum_{k=0}^{K} \sum_{j=1}^{10} a_{i,1+k}^j e_{t-k}^j$$

9 For more than 30 replications the marginal change of standard deviation is less than 1%.

with *K* the order of autocorrelation correction. For MARCOS *K=0*.

In order to get comparable results, our different stochastic simulations innovations have to be identical for all experiments. For this reason we maintained the same seed to the random generator. We also have skip the 20 first observations of each stochastic simulations in order to have results unaffected by initial conditions (here deterministic).

Results

Results reported in Tables 7 and 8 could hardly be compared with deterministic simulations. In the later case results refer to permanent shocks and are conditional on the type of shock (supply shock or demand shock). Here, five different kinds of temporary shocks hit stochastically the economy at each date. Thus results are not conditional on a particular shock.

There are analogies between the two areas. Hence, the optimal coefficients are nearly identical except for the variance of the criteria. This difference has to be related to the fact that the variance of the output-gap is larger in the US than in the euro area. The same remark applies also to the optimal horizon. When an objective of output-gap stability is tracked, the optimal behaviour of the monetary authorities is to react to immediate deviations of output and inflation from their target. As noted previously, when the stability of the interest rate is targeted the central bank has to retain the longer horizon as possible to avoid large movement in the interest rate.

These results could be compared to those of Batini and Nelson [2000] despite their different specification of the reaction function (only inflation and an auto-regressive term on interest rate enter). With a small forward-looking model they found an optimal horizon inferior to one year (2 quarters) and a coefficient related to inflation in the Taylor rule equal to 1.2. In our case, with the same criterion (l_y, l_p, l_r and b set equal to 1, 1, 0.5 and $1/1.04$), we found an optimal lag of 0 year on both inflation and output-gap. However, on the inflation coefficient we have contrasting results. The US present a reaction function less aggressive to inflation than the euro area (less than 0.4 for the US against more than 2 for the euro area). But according to our findings, the Fed seems to pay more attention to the output-gap than the ECB. This point could probably be related to the greater fluctuations of output in the US than in Europe.

It is noticeable that output-gap coefficient is, whatever the objective of the central bank, always strictly positive (contrary to the Batini and Nelson assumption). The coefficient t is always greater than 0.2 (at the exception of the particular interest-rate variance criterion) implying a reaction function containing the output-gap.

Efficient frontiers for the euro area and the US are exhibited in Figures 9 and 10. The main difference relies on the larger variance of output-gap in the US than in the euro area, at the opposite of what could be observed for inflation. This fact could be originated from a greater magnitude of the shock governing out-gap in the US.

Table 7: Optimal coefficients in the Taylor rule (stochastic case)

	EA		US	
	m	t	m	t
$V(Y)$	≤ 0.4	≥ 1	≤ 0.4	≥ 1

$V(p)$	≥ 2	0.2	≥ 2	0.4
$V(dr)$	≤ 0.4	0	≤ 0.4	0
V(criteria)	≥ 2	0.4	≤ 0.4	≥ 1

Table 8: Optimal leads in the Taylor rule (stochastic case)

	EA		US	
	km	kt	km	kt
$V(Y)$	0	0	0	0
$V(p)$	1	3	0	1
$V(dr)$	1	5	4	5
V(criteria)	0	0	0	0

Figure 9: Efficient frontier for the EA with $l_r = 0.5$

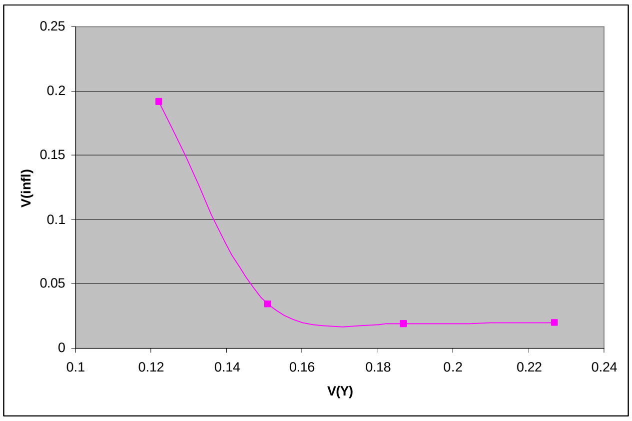
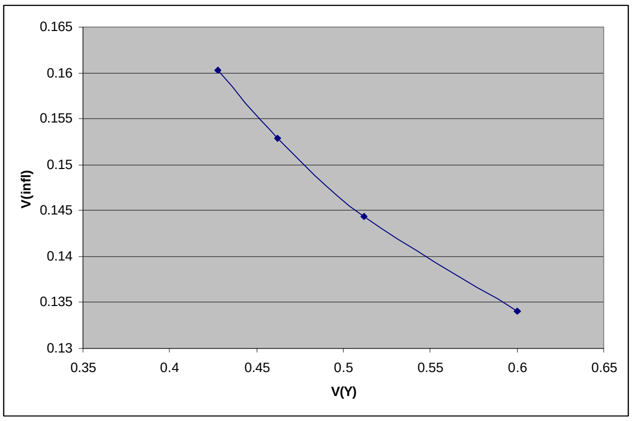


Figure 10: Efficient frontier for the US with $l_r = 0.5$



Concluding Remarks

Three aspects could be improved. So far, we have used a constrained form of the Taylor rule with the changes in the interest rate depending on output-gap and inflation-gap. In other words, we have not examined the degree of interest-rate smoothing in the reaction function. This parameter provides an additional information on the timing of the optimal monetary policy rule. In a same perspective, stochastic simulations could also be extended to minimise the different criteria with respect to parameters and leads of the Taylor rule. However, as noted above, it is an expensive task in CPU time. In a multi-area model perspective, we could also investigate the optimal monetary policy of one area considering the optimality of the reaction function of the other area.

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Appendix 1: The dynamic model

Foreign trade

$$\Delta \ln(X_t) = \Delta \ln\left(DM_t\right) - a_{x,0} \Delta \ln\left(\frac{px_t}{p_t^*}\right) - a_{X,1}\left[\ln\left(\frac{X_{t-1}}{DM_{t-1}}\right) + h_X \ln\left(\frac{px_t}{p_t^*}\right)\right] \quad (1.)$$

$$ABS_t = C_t + I_t + G_t + X_t \quad (2.)$$

$$\Delta \ln(M_t) = \Delta \ln\left(ABS_t\right) + a_{M,0} \Delta \ln\left(\frac{pm_t}{p_t}\right) - a_{M,1}\left[\ln\left(\frac{M_{t-1}}{ABS_{t-1}}\right) - h_M \ln\left(\frac{pm_t}{p_t}\right)\right] + a_{M,2} \quad (3.)$$

$$\mathrm{DEF}_t^{\mathrm{BC}} = \left(M_t\, pm_t - X_t\, px_t\right)\bar{e}_t \quad (4.)$$

$$D_{t+1} = (1 + \bar{r}^*_{t+1} + prime)\, D_{t+1} + DEF_{t+1}^{BC} \quad (5.)$$

Consumption

$$e = \frac{a^R}{a^W} \quad (6.)$$

$$\Omega = w + (1-v)\, e^{\frac{g}{g-1}} \quad (7.)$$

$$S = \frac{(1-p)(1+g)}{(1+r)(1+n)} S + (1-i)\, E \quad (8.)$$

$$S^W = \frac{v\,(1+g)}{(1+r)(1+n)\,\Omega} S^W + \frac{(1-v)(1+g)}{(1+r)\,\Omega} \hat{S} \quad (9.)$$

$$l = v\,(1-a^R)\frac{1+r}{1+g}\, l + v\,(1-i)\frac{E - a^R S}{FW(1+g)} + 1 - v \quad (10.)$$

$$FW = A + V \quad (11.)$$

$$\left(a^R\right)^{-1} = 1 + \left(1+r\right)^{1/g-1}\left(1+q\right)^{-1/g}(1-p)\left(a^R\right)^{-1} \quad (12.)$$

$$\left(a^W\right)^{-1} = 1 + \left[(1+r)\,\Omega\right]^{1/g-1}\left(1+q\right)^{-1/g}(1-p)\left(a^W\right)^{-1} \quad (13.)$$

$$C^R = a^R\left[(1+r)\; l\; FW + S\right] \quad (14.)$$

$$C^W = a^W\left[(1+r)(1-l)\, FW + H + S^W\right] \quad (15.)$$

$$\hat{S} = \frac{e\,(n+p)}{1-v} S \quad (16.)$$

$$E = prop\_ret \frac{1-v}{n+p}\frac{p}{pc}\, w\, L \quad (17.)$$

$$RS_t^1 = (1-i)\left(1-t_R\right)\left(1-t_{CSG,t}\right)\frac{p_t}{pc_t}\left[\left(1-t^s_{w,t}\right)w_t\, L_t + (\mathrm{TR}_t\, w_t + R_0\, g_{L,t})\left(\overline{L}_t - L_t\right)\right] \quad (18.)$$

$$H_t^1 = \frac{1-p}{(1+r_t)(1+a_1)(1+n)} H_{t+1}^1 + RS_t^1 \quad (19.)$$

$$H_t^2 = \frac{1-p}{(1+r_t)(1+a_2)(1+n)} H_{t+1}^2 + RS_t^1 \quad (20.)$$

$$H_t^3 = \frac{1-p}{(1+r_t)(1+a_3)(1+n)} H_{t+1}^3 + RS_t^1 \quad (21.)$$

$$H_t = \frac{b_H a_1}{(1-p)\left[(1+b_H)(1+a_1)-1\right]} H_t^1 + \frac{b_H a_2}{(1-p)\left[(1+b_H)(1+a_2)-1\right]} H_t^2 + \left[1 - \frac{b_H a_1}{(1-p)\left[(1+b_H)(1+a_1)-1\right]} - \frac{b_H a_2}{(1-p)\left[(1+b_H)(1+a_2)-1\right]}\right] H_t^3 \quad (22.)$$

$$A_t \, pc_t = B_t - \frac{D_t}{\bar{e}_t} \quad (23.)$$

$$W_t = A_t + H_t + V_t \quad (24.)$$

$$C_t^1 = C_t^W + C_t^R \quad (25.)$$

$$R_t^2 = i(1-t_{R,t})(1-t_{CSG,t})\frac{p_t}{pc_t}\left[(1-t^s_{w,t})w_t L_t + (\mathrm{TR}_t w_t + R_0 g_L)(\bar{L}_t - L_t)\right] \quad (26.)$$

$$C_t^2 = R_t^2 \quad (27.)$$

$$C_t = C_t^1 + C_t^2 \quad (28.)$$

$$pc_t \, RPAT_t = (1-t_{R,t})(1-t_{CSG,t})\left[\Pi_t + r_t \, pc_t \, A_t\right] \quad (29.)$$

$$R_t^1 = RS_t^1 + RPAT_t \quad (30.)$$

$$S_t = R_t^1 - C_t^1 \quad (31.)$$

Production

$$\bar{Y}_t = \left[a_k (K_{t-1})^{-r} + a_L (g_{L,t}(1-u^*)\bar{L}_t)^{-r}\right]^{-1/r} \quad (32.)$$

$$F'_{K_t} = \frac{h-1}{h} a_K \left(\frac{Y_t}{K_{t-1}}\right)^{r+1} \quad (33.)$$

$$F'_{L_t} = \frac{h-1}{h} a_L . g_L \left(\frac{Y_t}{L_t g_{L,t}}\right)^{r+1} \quad (34.)$$

Investment

$$AJK_t = \frac{m_K}{2}\left[\frac{I_t}{K_{t-1}} - 2(d+g) + (d+g)^2 \frac{K_{t-1}}{I_t}\right] \quad (35.)$$

$$q_t - 1 = m_K \left[\frac{I_t}{K_{t-1}} - (d+g)\right] \quad (36.)$$

$$q_t = \frac{1-t_{IS,t+1}}{(1+r_t)(1-t_{IS,t})}$$

$$\left[\frac{p_t}{pi_t}(1+p_{t+1})F'_{K_t}\ \frac{h-1}{h} + (1+p_{t+1})\left\{\frac{(q_{t+1}-1)^2}{2m_K} + (1+g)(q_{t+1}-1)\right\} + (1-d)\right] \tag{37.}$$

$$V_t = q_t K_{t-1} \tag{38.}$$

$$I_t = K_t - (1-d)K_{t-1} \tag{39.}$$

Employment et wage

$$AJK_t = \frac{m_L}{2}\left[\frac{L_t}{L_{t-1}} - (1+n)\right]^2 L_t \tag{40.}$$

$$w_{t+1}(1+t^{e}_{w,t+1})\frac{p_{t+1}}{p_t} m_L \left(\frac{L_{t+1}}{L_t}\right)^2 \left\{\frac{L_{t+1}}{L_t} - (1+n)\right\} =$$

$$-(1+\bar{r}_t)\frac{1-t_{IS,t}}{1-t_{IS,t+1}}\left[\frac{h-1}{h}F'_{L_t} - w_t(1+t^{e}_{w,t})\left(1+\frac{m_L}{2}\left\{\frac{L_t}{L_{t-1}} - (1+n)\right\}^2 + m_L\left\{\frac{L_t}{L_{t-1}} - (1+n)\right\}\frac{L_t}{L_{t-1}}\right)\right] \tag{41.}$$

$$\Pi_t = (1-t_{IS,t})\left[p_t\left(Y_t - w_t(1+t^{e}_{w,t})(L_t + AJL_t)\right) - pi_t(1+AJK_t)I_t\right] \tag{42.}$$

$$h_{\Pi,w} = (1-h)\left[1 - a_K \left(\frac{Y_t}{K_{t-1}}\right)^r\right] - \frac{r}{1+r} a_K \left(\frac{Y_t}{K_{t-1}}\right)^r \frac{p_t \left(w_t (1+t^e_{w,t}) L_t + (r_t + d) K_t\right)}{\Pi_t} + \frac{r}{1+r} \frac{p_t (r_t + d) K_t}{\Pi_t} \tag{43.}$$

$$h_{L,w} = h\left[a_K \left(\frac{Y_t}{K_{t-1}}\right)^r \left(1 - \frac{1}{h(1+r)}\right) - 1\right] \tag{44.}$$

$$w_t \left(1 - t^s_{w,t}\right)\left(1 - t_{CSG,t}\right) = \left[\frac{1-g_w}{uk} \frac{1}{\left(\frac{1-b}{b} h_{\Pi,w} + h_{L,w}\right)} + 1\right]^{\frac{1}{g_w - 1}} \left[\mathrm{TR}_t \; w_t + R_0 g_{L,t}\right] \tag{45.}$$

$$u_t = 1 - \frac{L_t}{\overline{L}_t} \tag{46.}$$

Public Finance

$$t_{T,t} = \left(1 - a^{t_R}\right) t^{exo}_{T,t} + a^{t_R} . \left[c_{t_R} \left(\frac{B_{t+1}}{PNB_{t+1}} - \overline{TB}_{t+1}\right) + \sum_{i=-2}^{2} \frac{t_{T,t+i}}{5}\right] \tag{47.}$$

$$t_{T,t} PNB_t = t_{R,t} \left(1 - t_{CSG,t}\right) p_t \left[\left(1 - t^s_{w,t}\right) w_t L_t + \left(\mathrm{TR}_t \; w_t + R_0 g_{L,t}\right)\left(\overline{L}_t - L_t\right)\right] + t^C_{TVA,t} pd_t C_t + t^I_{TVA,t} pd_t I_t + t_{IS,t} \frac{\Pi_t}{1 - t_{IS,t}} + TAXE^{exo}_t \tag{48.}$$

$$TAXE_t = t_{R,t} \left(1 - t_{CSG,t}\right) p_t \left[\left(1 - t^s_{w,t}\right) w_t L_t + \left(\mathrm{TR}_t \; w_t + R_0 g_{L,t}\right)\left(\overline{L}_t - L_t\right)\right] + t^C_{TVA,t} pd_t C_t + t^I_{TVA,t} pd_t I_t + t_{IS,t} \frac{\Pi_t}{1 - t_{IS,t}} + TAXE^{exo}_t \tag{49.}$$

$$G_t = l^G \frac{TAXE_t}{pg_t} + \left(1 - l^G\right) G^{exo}_t \tag{50.}$$

$$DEF_t = pg_t G_t - TAXE_t + p\left[(\mathrm{TR}_t w_t + R_0 g_{L,t})\left(\overline{L}_t - L_t\right) - t^e_{w,t} w_t L_t - t^s_{w,t} w_t\right] - t_{CSG,t} \left[\left(1 - t^s_{w,t}\right) p_t \; w_t L_t + p_t \left(\mathrm{TR}_t w_t + R_0 g_{L,t}\right)\left(\overline{L}_t - L_t\right) + r_t \; pc_t \; A_t + \Pi_t\right] \tag{51.}$$

$$B_t = (1 + \overline{r}^G_t) B_{t-1} + DEF_t \tag{52.}$$

Equilibrium

$$Y_t = C_t + I_t + G_t + X_t - M_t \tag{53.}$$

$$PIB_t = C_t pc_t + I_t pi_t + G_t pg_t + X_t px_t - M_t pm_t \tag{54.}$$

$$t^Y_{TVA,t} \; p_t \; Y_t = pd_t \left(t^C_{TVA,t} C_t + t^I_{TVA,t} I_t\right) \tag{55.}$$

$$PIB_t = p_t \left(1 + t^Y_{TVA,t}\right) Y_t \tag{56.}$$

$$PNB_t = PIB_t - \bar{r}^*_t D_t \qquad (57.)$$

Prices

$$\bar{e}_t \; p^*_t = p^{**}_t \qquad (58.)$$

$$pm_t = p_t^{m_0} \left(p^*_t\right)^{1-m_0} \qquad (59.)$$

$$px_t = p_t^{l_0} \left(p^*_t\right)^{1-l_0} \qquad (60.)$$

$$pc_t = (1+t^{C}_{TVA,t})\, pd_t \qquad (61.)$$

$$pi_t = (1+t^{I}_{TVA,t})\, pd_t \qquad (62.)$$

$$pg_t = pd_t \qquad (63.)$$

$$p_t = \frac{pd_t}{pd_{t-1}} - 1 \qquad (64.)$$

$$1+p^{TTC}_t = \frac{1+t^{C}_{TVA,t}}{1+t^{C}_{TVA,t-1}}(1+p_t) \qquad (65.)$$

$$p^a_t = z_a p_{t+1} + (1-z_a) p_t \qquad (66.)$$

$$p_t = z p_{t-1} + (1-z) p^a_t + y\,(u_t - u^*_t) \qquad (67.)$$

$$1+p_{10,t} = \left[\prod_{i=0}^{9}(1+p_{t+i})\right]^{1/10} \qquad (68.)$$

Interest Rates

$$\bar{r}_t = \bar{r}^*_t + m\,(p_{t+1} - p^*_{t+1}) + t\left(\frac{Y_t}{\bar{Y}_t} - 1\right) \qquad (69.)$$

$$1+r_t = \frac{1+\bar{r}_t}{1+p_{t+1}} \qquad (70.)$$

$$1+\bar{R}_{10,t} = \left[\prod_{i=0}^{9}(1+\bar{r}_{t+i})\right]^{1/10} (1+PT) \qquad (71.)$$

$$1+R_{10,t} = \frac{1+\bar{R}_{10,t}}{1+p_{10;t+1}} \qquad (72.)$$

$$\bar{r}^G_t = l^{r^G}\, \bar{R}_{10,t} + \left(1 - l^{r^G}\right)\bar{r}_t \qquad (73.)$$

Exchange rate

$$\bar{e}_t = \frac{e_t \; p^*_t}{pd_t} \qquad (74.)$$

$$\frac{e_{t+1}}{e_t} = \frac{1+r^*_t}{1+r_t} \qquad (75.)$$

Appendix 2: the calibration

Table 2.1: EA MARCOS's Steady State: Exogenous Variables During the Calibration

Variable	Simulated value	Comment
Y_t value added	4676057.13	Exogenous during the calibration and set equal to its 1997 observed value
PIB_t GDP in current price	5837583.86	
$\overline{L}_t$ Labour Force	127596.455	
$C_t pc_t \ / \ PIB_t$ household consumption and investment over GDP	62%	Exogenous during the calibration and set equal to its mean value on the sample 1985-1997
$G_t^{exo} pg_t \ / \ PIB_t$ Public spending over GDP	15.2%	
$X_t px_t \ / \ PIB_t$ Exports over GDP	13%	
$M_t pm_t \ / \ PIB_t$ Imports over GDP	16.6%	
B_t / PIB_t Public debt over GNP	60%	
r_t Real short term interest rate	3.8%	
a_t^W Worker marginal propensity to consume the wealth	4.7%	Exogenous during the calibration
a_t^R Retired marginal propensity to consume the wealth	7.5%	
r_t^* Real foreign interest rate	3.8%	Exogenous during the calibration and set equal to its mean value on the sample 1985-1997
t_R Rate of income taxes	14.4%	
u_t^* Equilibrium unemployment rate	8.6%	
l^{r^G} The share bounds in the public debt	60%	

Table 2.2: Endogenous Variables During the Calibration for the EA model

Variable	Simulated value
g_0 Scale factor of the labour saving technical progress	18.9
$TAXE_t^{exo} / PIB_t$ Other taxes on GDP	-3%
DM_t / Y_t Word demand over the value added	8.4%
$a_{M,2}$ Intercept in the imports equation	0.11
q Rate of time preference in utility	1.97%
r Parameter of the CES production function	0.033
m_0 Sensitivity of the import price to the value added price	0.48
$R_{0,t}$ Share of replacement income indexed on the labour saving technical progress	16%
i Share of Keynesian households	34%
$S_t / (R_t^1 + R_t^2)$ Saving rate	-7%
K_t / Y_t Capital coefficient	3.26
$\left(w_t\left(1+t_t^W\right)L_t\right)/Y_t$ Labour share income	56%
H_t / W_t Share of the human wealth in the total wealth	47%
A_t / W_t Share of bonds in the total household wealth	2.4%

Table 2.3: *A-priori* Parameters for the EA model

Parameter	Value	Parameter	Value
g GDP growth rate	2.4%	l_w Indicate if the social contributions are considered as incomes	0
n Employment growth rate	0.5%	k Ratio of the probability to find a job and the unemployment rate	1
p Death probability for retired	5%	y Phillips effect	-0.045
$1-w$ Probability to become retired	2.5%	z Coefficient of the expected inflation in the Phillips curve	0.43
p_t Inflation rate	3.6%	h_X Price elasticity of exports	0.55
d Depreciation rate	4.5%	h_M Price elasticity of imports	0.76
h Price elasticity of the goods demand	11	c_{t_R} coefficient of the budget reaction function	0.2
a_L Technical coefficient of the CES production function	0.6	$t^C_{TVA,t}$ VAT rate for consumption goods	11%
a_K Technical coefficient of the CES production function	0.4	$t^I_{TVA,t}$ VAT rate for equipment goods	10%
m_K Capital adjustment cost	6	$t_{IS,t}$ profit tax rate	15%
$g_{L,t}$ Labour saving technical progress	$(1+0.018905)^t$	m Sensitivity of the reaction function to the inflation	1.5
g Consumer risk aversion	1.7	t Sensitivity of the reaction function to the output gap	0.5
g_w Employee risk aversion	1.5	PT Term premium	0.5pt
b Union bargaining power	0.5	l_0 Sensitivity of the export price to the value added price	0.75
m_L Employment adjustment cost	6		
TR_t ratio of replacement income to wage	25%	$a_{X,1}$ Error correction coefficient in export equation	0.2
l_G Public expenditures exogenous (l_G =0) or endogenous (l_G =1)	0	$a_{M,1}$ Error correction coefficient in import equation	0.21
a_1 Coefficient of the humane wealth	40	a_2 Coefficient of the human wealth	-30

Table 2.3 (next): *A-priori* Parameters

a_3 Coefficient of the human wealth	-13	a_1 Coefficient of the human wealth	0.5%
a_2 Coefficient of the human wealth	1.4%	a_3 Coefficient of the human wealth	0.02%

Table 2.4: US MARCOS's Steady State: Exogenous Variables During the Calibration

Variable	Simulated value	Comment
Y_t value added	79212980	Exogenous during the calibration and set equal to its 1997 observed value
PIB_t GDP in current price	8091339	
$\overline{L}_t$ Labour Force	136381	
$C_t\, pc_t\ /\ PIB_t$ household consumption and investment over GDP	66.2%	Exogenous during the calibration and set equal to its mean value on the sample 1985-1997
$G_t^{exo}\, pg_t\ /\ PIB_t$ Public spending over GDP	15%	
$X_t\, px_t\ /\ PIB_t$ Exports over GDP	9.8%	
$M_t\, pm_t\ /\ PIB_t$ Imports over GDP	11.2%	
$B_t\ /\ PIB_t$ Public debt over GNP	60%	
r_t Real short term interest rate	3.8%	
a_t^W Worker marginal propensity to consume the wealth	4.7%	Exogenous during the calibration
a_t^R Retired marginal propensity to consume the wealth	7.5%	
r_t^* Real foreign interest rate	3.8%	Exogenous during the calibration and set equal to its mean value on the sample 1985-1997
t_R Rate of income taxes	8%	
u_t^* Equilibrium unemployment rate	6.13%	
l^{r^G} The share bounds in the public debt	60%	

Table 2.5: Endogenous Variables During the Calibration for the US model

Variable	Simulated value
g_0 Scale factor of the labour saving technical progress	33.57
$TAXE_t^{exo} / PIB_t$ Other taxes on GDP	0%
DM_t / Y_t World demand over the value added	4.4%
$a_{M,2}$ Intercept in the imports equation	0.187
q Rate of time preference in utility	1.96%
m_0 Sensitivity of the import price to the value added price	0.52
r Parameter of the CES production function	0.15
$R_{0,t}$ Share of replacement income indexed on the labour saving technical progress	10.7%
i Share of Keynesian households	32%
$S_t / (R_t^1 + R_t^2)$ Saving rate	-7.5%
K_t / Y_t Capital coefficient	2.89
$\left(w_t\left(1+t_t^W\right)L_t\right)/Y_t$ Labour share income	60%
H_t / W_t Share of the human wealth in the total wealth	60%
A_t / W_t Share of bonds in the total household wealth	8%

Table 2.6: *A-priori* Parameters for the US model

Parameter	Value	Parameter	Value
g GDP growth rate	2.4%	l_w Indicate if the social contributions are considered as incomes	0
n Employment growth rate	1.4%	k Ratio of the probability to find a job and the unemployment rate	1
p Death probability for retired	5%	y Phillips effect	-0.12
$1-w$ Probability to become retired	2.5%	z Coefficient of the expected inflation in the Phillips curve	0.56
p_t Inflation rate	3.6%	h_X Price elasticity of exports	0.72
d Depreciation rate	4.5%	h_M Price elasticity of imports	1.92
h Price elasticity of the goods demand	11	c_{t_R} coefficient of the budget reaction function	0.2
a_L Technical coefficient of the CES production function	0.6	$t^C_{TVA,t}$ VAT rate for consumption goods	11%
a_K Technical coefficient of the CES production function	0.4	$t^I_{TVA,t}$ VAT rate for equipment goods	10%
m_K Capital adjustment cost	6	$t_{IS,t}$ profit tax rate	15%
$g_{L,t}$ Labour saving technical progress	$(1+0.09862)^t$	m Sensitivity of the reaction function to the inflation	1.5
g Consumer risk aversion	1.7	t Sensitivity of the reaction function to the output gap	0.5
g_w Employee risk aversion	1.5	PT Term premium	0.6pt
b Union bargaining power	0.5	l_0 Sensitivity of the export price to the value added price	0.73
m_L Employment adjustment cost	6		
TR_t ratio of replacement income to wage	25%	$a_{X,1}$ Error correction coefficient in export equation	0.55
l_G Public expenditures exogenous (l_G =0) or endogenous (l_G =1)	0	$a_{M,1}$ Error correction coefficient in import equation	0.26
a_1 Coefficient of the human wealth	40	a_2 Coefficient of the human wealth	-30

Table 2.6 (next): *A-priori* Parameters

a_3 Coefficient of the human wealth	-13.11	a_1 Coefficient of the human wealth	0.5%
a_2 Coefficient of the human wealth	1.4%	a_3 Coefficient of the human wealth	0.0018