A Toolkit for Analyzing Alternative Policies in the Chilean Economy[1]

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Abstract

This paper presents a general equilibrium model for a small open economy that can be used to assess the effects of alternative policies. The model is estimated in order to replicate key characteristics of the Chilean economy by using the gradients of an Identified VAR model as matching conditions for the theoretical model. The theoretical model is sufficiently general so that it can be used to analyze issues such as alternative exchange rate regimes, controls to capital inflows, inflation targeting, and other policies used by the monetary authority. A distinguishing feature of this model is that it explicitly models the attitudes of foreign investors.

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JEL Classification: C15, C32, C63, E17, E43, E52, E58.

1 Introduction

As noted by Leeper (1995) "the business pages of leading newspapers give the impression that the effects of alternative monetary policies on the macroeconomy are well understood and predictable." They tend "to write with great certainty that when the monetary authority raises interest rates it slows economic growth, and with it inflation, bidding down stocks and bonds. With equal certainty, press accounts report that the monetary policy responds to economic conditions." Statements like "the recent strength of the economy will prompt the monetary authority to raise interest rates as a preemptive strike against inflation" are not uncommon. With the economy responding to policy and policy responding to the economy, it is hard to tell what causes what. Chile is no exception, statements like the previous are frequently found in local newspapers.

However, there is no consensus regarding the interaction of economic conditions and policy. In fact, while there are several academic papers that directly or indirectly try to identify the effects of alternative policies, most of the results found are, to say the least, inconclusive.[1]

Even though the understanding and measurement of the quantitative effects of monetary policy are essential to evaluate the relative merits of alternative policy arrangements, few papers have addressed the issue in an integrated and consistent way.[2] This paper tries to do so by combining sound statistical representations with theoretical models. In contrast, most of the empirical literature focuses on providing statistical descriptions of the data with no correspondence between the statistical model used to develop the stylized facts intended to explain, and a theoretical model that is consistent with them.

From the statistical standpoint, the effects that different policy arrangements may have over the economy are usually quantified using VARs. While this technique may prove to be valuable for forecasting purposes, it is difficult to obtain a correspondence between the impulse-response functions that are derived from it and economic principles that may come from contesting theories (Hamilton, 1994).

As discussed below, the identifying restrictions imposed on VAR impulse-response functions may not have any meaningful interpretation as they may come from linear combinations of different shocks, thus not providing reliable estimates of the effects of alternative policies. From a theoretical standpoint there are scarce papers that use models derived from first principles to deal in an integrated fashion with their empirical implications, thus being subject to the Lucas critique from the get go.

This paper intends to overcome these shortcomings by integrating statistical models that are able to replicate the intertemporal dynamics of key economic variables with dynamic, stochastic, optimizing models. Thus, in the presence of a rival theoretical model, the statistical description of the data provides an objective metric with which to evaluate their merits.

[1] Rosende and Herrera (1991), Rojas (1993), Eyzaguirre and Rojas (1996), Morandé and Schmidt-Hebbel (1997), Valdés (1998), Parrado (2001), and some specifications of Cabrera and Lagos (2002) find that output and inflation are affected by innovations in monetary policy. Mendoza and Fernández (1994), Morandé, et al (1995), Calvo and Mendoza (1999), and some specifications of Cabrera and Lagos (2002) find complete ineffectiveness of the monetary policy to alter their trajectories.

[2] Schmidt-Hebbel and Servén (2000) develop a deterministic general equilibrium model in which liquidity constraints and wage rigidities are imposed. This model is calibrated and presents exercises regarding the effects of alternative policies.

The paper is organized as follows: Section 2 briefly describes the main problems that traditional statistical models have when they are used to quantify the effects of alternative policies. Section 3 presents a statistical model that is used as the metric with which to compare the empirical implications of alternative theoretical models. Section 4 describes an estimates a simple optimizing model used to replicate the stylized facts reported on section 3. Finally, Section 5 concludes.

2 Identified VARs

VAR models have long been used to describe the dynamic interactions of key macroeconomic variables in an economy. Even though these models have proven successful as forecasting tools, they rarely can be used to test competing theories or to interpret their results with sound economic principles. It is sometimes argued that the main reason for this to happen is that VARs are restricted versions of more "structural" models as VARs usually ignore contemporaneous comovements and no stance regarding the economic principles behind the dynamic interactions encountered is explicitly tested. Furthermore, VAR models impose arbitrary decompositions to the variance-covariance matrix of the innovations (usually a Cholesky decomposition) making the impulse-response functions sensitive to the ordering of the model. Several methodologies have been developed to overcome this shortcoming. However, these functions do not have any direct interpretation in terms of the dynamic consequences of shocks to any of the underlying innovations.[3]

Recently developed models intend to overcome the shortcomings of traditional VARs. They are known as SVARs (for Structural VARs) or IVARs (for Identified VARs). The main characteristic of these models is that they nest traditional VARs and do not impose orthogonality restrictions among the contemporaneous interactions of the variables in the system. They also provide tools that can be used to conduct inference about restrictions of competing statistical models and, in principle, provide estimates of the impulse-response functions that are supposed to recover the underlying structure of the system.[4] Nevertheless, as noted by Cochrane (1998) and more forcefully by Cooley and Dwyer (1998) the robustness of the conclusions drawn from IVAR exercises is questionable.[5]

2.1 The Usual Practice

As discussed above, several attempts to characterize the dynamic consequences of alternative policies in the Chilean economy have been made. Nonetheless, most of them were obtained by using traditional VARs and are subject to the critiques outlined.[6] It is instructive however to discuss

[3] Pesaran and Shin (1998) develop what they call "generalized" impulse-response functions that provide impulse-response functions that are invariant to the ordering of the unconstrained VAR.

[4] Appendix A provides a brief description of the IVAR methodology.

[5] The term Structural VAR is misleading in the sense that it may give the impression that these statistical objects can be understood as representations of behavioral relations grounded on first principles but, as discussed below, this is usually not the case. On this paper we prefer to use the term IVAR that makes explicit that these models provide tests that can help to decompose impulse-response functions in a more formal way, but no structural (behavioral) implications are drawn from them.

[6] Valdés (1998) estimated what he termed a Semi Structural VAR model. However, the identifying assumption imposed there, makes it no different from a specific ordering of an unrestricted VAR model and it is not what we understand as an IVAR. The "restrictions" imposed there correspond to a Cholesky factorization in which the

some of the methodological issues involved in their estimations.

With rare exceptions (e.g. Calvo and Mendoza, 1999 and Valdés, 1998) most of them do not report confidence intervals for the impulse-response functions. Furthermore, the studies that do report them, rely on asymptotic approximations of the confidence intervals of the model but no formal tests for multivariate normality and vector-white noise innovations are performed, nor the impulse-response functions are bias corrected.[7]

When departures from normality are important, confidence intervals based on asymptotic approximations can be deceiving, given that normality imposes symmetry on them. Furthermore, asymmetries may also be present when non-linear structures are important; in which case, positive and negative shocks may imply completely different trajectories. In such cases, confidence intervals for the impulse-response functions may still be constructed relying on bootstrap (Sims and Zha, 1995). However, this practice is itself subject to two problems: First, as discussed below, most of the variables included on the unrestricted VAR are usually statistically nonsignificant, but the bootstrapped model takes their point estimates as given, thus unnecessarily inflating the confidence intervals. Second, and more importantly, the confidence intervals usually considered are constructed using Efron's suggestion; but as is well documented, these intervals do not have the correct coverage if the distribution under consideration is asymmetric. Given that bootstrap is used precisely for these purposes, Hall's confidence intervals are better suited to deal with departures from normality.

Another important consideration that has to be taken into account is the way in which some of the previous studies dealt with non-stationarities. As Sims et al (1990) demonstrated, VAR estimates with some integrated series are super consistent, however they have non-standard asymptotic distributions, thus impulse-response functions from these types of series can be constructed from Monte Carlo or bootstrap approximations (methods that are now readily available and can be routinely performed). However, in case the source of non-stationarity comes from deterministic trends, incorrectly differentiating the series may impose non-trivial dynamics on the model. In particular unit root type of vector-MA processes would now be incorporated to the series thus making OLS estimation not advisable.[8] Thus, care should be given to when and when not to differentiate a

variable used to measure the monetary policy stance comes first, thus making it exactly identified. Obviously, there are no formal tests against alternative orderings or identifying assumptions that can possibly be made in this context. In fact, the impulse-response of that model can be directly computed without estimating the parameters with the methodology described on Appendix A. Other examples of such a practice can be found in García (2001) and Cabrera and Lagos (2002). Parrado (2001) uses the IVAR methodology but, as discussed below, his results are subject to other problems.

[7] When estimating VARs or IVARs, it is often forgotten that they need to be correctly specified prior to conducting impulse-response exercises. As a minimum, vector-white noise innovations are needed. That is, innovations that are not only orthogonal to their own past but also orthogonal to the past of the other innovations of the system. Furthermore, given that the construction of confidence intervals for the impulse-response functions generally rely on asymptotic approximations, formal tests for multivariate normality of the residuals should be conducted. In the former case Ljung and Box type of tests can not be applied as they rely on univariate specifications and Wilks, Portmanteu or LRT tests should be employed (see Lütkepohl, 1991 for details). This fact is independent of the information criteria chosen to select a model, given that it is used only to account for parsimony. In the later case, Jarque-Bera univariate tests for normality are not appropriate and multivariate specification should be used (Doornik and Hansen, 1994).

[8] In this case, exact (unconditional) maximum likelihood estimation of the parameters should be conducted; this

variable prior to estimating the VAR.

2.2 Unit Roots and Impulse-Response Functions

With the exception of Parrado (2001), all of the studies discussed on the first section chose to differentiate the variable that captures the level of activity of the economy (usually the Monthly Activity Index, hereafter IMACEC). Typical examples of the unbelievable dynamics that result from impulse-response functions that use first differences on the scale variable can be found in Valdés (1998) and García (2001). Even if we assume that their models were correctly specified in terms of lag selection, normality, and that there were no biases associated with the parameter estimated; they find a significantly negative effect of what they refer to as the "monetary policy innovation" to the first difference of the scale variable. If the monetary policy innovation has a negative though transitory effect on the growth rate of the scale variable, what are the implications for the level of the series?.

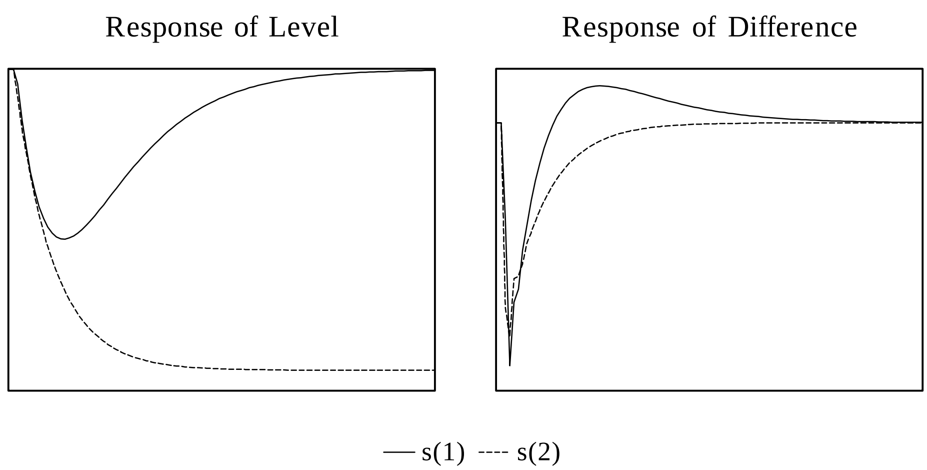

Figure 1: Implications of different impulse-response functions

Figure 1 shows the implications for both levels and growth rates of a unit shock on the innovations estimated in that case.[9] For the sake of comparison, we consider two types of shocks: The first (termed s(1)) corresponds to the effect (in both levels and differences) of a transitory shock when the scale variable is modeled in levels thus making the shocks transitory and the level to revert to its deterministic trend. The second (s(2)) corresponds to the same exercise when the scale variable is modeled in differences. As can be seen, the s(2) shocks on monetary policy have increasing and

practice is rarely (if ever) taken, given that it is computationally demanding. Chumacero (2001b) describes a computationally efficient way to deal with this problem.

[9]The parameters are not chosen to match exactly the impulse-response of the studies in which significant effects are found, but are arbitrarily chosen to demonstrate the effects for the level of the series. The essence of the results would not change if the actual impulse-response functions reported were used.

permanent effects on the level of the series even when the shock is not very persistent in terms of growth rates.

Which type of shock is correct? Chumacero and Quiroz (1996) and Chumacero (2000) show that there is no evidence to support the practice of differentiating series such as IMACEC.[10] Even in that case, it is important to consider what are the implications of the shocks for the levels of scale variables, once we swallow a unit root. As Figure 1 makes clear, such a mighty power of the monetary policy is difficult to rationalize even in the most extreme of Keynesian models.

2.3 Ordering, Causality, and Interpretation

An even more important problem with these results comes from the interpretation that can be given to them. As mentioned previously, Valdés (1998) and Cabrera and Lagos (2002) used a specific ordering in the construction of the impulse-response functions of their VARs, in which the "monetary policy" innovation is not "caused" by any other innovation. Even though it is a common practice to order VARs according to Granger causality results of the variables in levels, the decomposition of the variance-covariance matrix has little to do with that ordering. As a matter of fact, there is no theoretical basis for justifying a specific ordering on the impulse-response functions that come from a Cholesky decomposition based on Granger causality as they may have no relation with the order of precedence of the levels. More fundamentally, it is not difficult to think of a theoretical economy in which there is no effect whatsoever of any monetary variable on the real sector but that presents the dynamics that are claimed to justify the results of Valdés (1998) or Cabrera and Lagos (2002).

Consider for example, the case of a closed endowment economy with a representative agent that is interested in maximizing:

$$\mathcal{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t)$$

subject to:

$$y_t + (1 + r_{t-1}) b_{t-1} \geq c_t + b_t$$

where y is the level of the endowment, c is the level of consumption, b_t is the demand of a risk-free private bond that pays off a net return of r_t the following period (this return is known at t), $u(\cdot)$ is strictly increasing and strictly concave, β is the subjective discount factor, and $\mathcal{E}_t$ denotes the expectation operator conditional on the information available at time t.

Under the conditions stated above, the gross return on this asset is given by:

$$(1 + r_t)^{-1} = \beta \mathcal{E}_t \left[\frac{u'(y_{t+1})}{u'(y_t)} \right] \tag{1}$$

which simply states that the gross return of the asset is a function of the intertemporal marginal rate of substitution (stochastic discount factor).

[10] Furthermore, Chumacero (2001a) shows both at the theoretical and empirical levels, that it is unlikely for a unit root to be present in scale variables such as IMACEC.

Consider now a special case of (1) in which we impose a Constant Relative Risk Aversion (CRRA) utility function with the Arrow-Pratt relative risk aversion coefficient denoted by γ (inverse of the intertemporal elasticity of substitution). Then (1) can be expressed as:

$$(1+r_t)^{-1} = \beta \mathcal{E}_t \left[\frac{y_{t+1}}{y_t} \right]^{-\gamma} \tag{2}$$

To determine the return of the asset, we need to solve (2), thus needing to explicitly introduce a law of motion for the endowment process. Consider two of such cases. The first assumes that the (log of the) endowment is difference-stationary (DS) and the second that it is trend-stationary (TS):

$$\begin{aligned}
&\text{Case 1 (DS)} \quad &&: \Delta \ln y_{t+1} = \alpha + \sum_{i=0}^{k} \delta_i \Delta \ln y_{t-i} + \varepsilon_{t+1}, \text{ where } \varepsilon_t \sim \mathcal{N}\left(0, \sigma_\varepsilon^2\right) \\
&\text{Case 2 (TS)} \quad &&: \ln y_{t+1} = \eta + \alpha t + \sum_{i=0}^{l} \delta_i \ln y_{t-i} + v_{t+1}, \text{ where } v_t \sim \mathcal{N}\left(0, \sigma_v^2\right)
\end{aligned}$$

where ε and v are innovations and k and l denote the number of lags necessary to produce them. Under these assumptions, the return of the asset can be computed as:

$$r_t \cong \ln(1+r_t) = \begin{cases} a_\varepsilon + \gamma \sum\limits_{i=0}^{k} \delta_i \Delta \ln y_{t-i} & \text{(DS)} \\ a_v + \gamma \sum\limits_{i=0}^{l} \delta_i \Delta \ln y_{t-i} - \gamma v_t & \text{(TS)} \end{cases}$$

where $a_i = \alpha\gamma - \ln\beta - 0.5\gamma^2\sigma_i^2$ for $i = \varepsilon, v$.

The purpose of this example is to show that Granger causality and VAR results may be completely misleading when we attempt to identify impulse-response functions as effects of alternative policies, thus we focus on rather simple dynamics that help us build the case. For that purpose, consider an AR(1) process for DS; then the dynamics of the system can be compactly characterized by:

$$\begin{bmatrix} \Delta \ln y_{t+1} \\ \ln(1+r_t) \end{bmatrix} = \begin{bmatrix} \alpha \\ a_\varepsilon \end{bmatrix} + \begin{bmatrix} \delta & 0 \\ \delta\gamma & 0 \end{bmatrix} \begin{bmatrix} \Delta \ln y_t \\ \ln(1+r_{t-1}) \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \varepsilon_{t+1} \\ \varepsilon_t \end{bmatrix} \tag{3}$$

As r_t is known at period t, VAR estimates and Granger causality tests would typically be made in a system that comprises $\Delta \ln y_t$ and $\ln(1+r_t)$. How would Granger causality results from a system like this look like? Not surprisingly, given that r_t is an exact function of the growth rate of the endowment in period t we will find that there should be a strong contemporaneous correlation between variables whose sign will depend exclusively on the value of δ (in fact, as the relationship among these variables is deterministic, the contemporaneous correlation should be -1 or 1). As VAR models and Granger causality tests typically rely on regressions of lagged values of the variables, Granger causality tests will display bidirectional Granger causality between the asset return and the growth rate.

If y is TS, $\Delta \ln y$ has a unit root in its MA component. If we consider a pure trend stationary process, the system can be conveniently expressed as:

$$\begin{bmatrix} \Delta \ln y_{t+1} \\ \ln(1+r_t) \end{bmatrix} = \begin{bmatrix} \alpha \\ a_v \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta \ln y_t \\ \ln(1+r_{t-1}) \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ 0 & -\gamma \end{bmatrix} \begin{bmatrix} v_{t+1} \\ v_t \end{bmatrix} \tag{4}$$

As both variables are functions of innovations, it is not difficult to show that in this case we would find strong evidence in favor of unidirectional causality from the asset returns to the growth rate! As the endowment presents a combination of two independent innovations, the contemporaneous correlation between growth and the asset return should be negative (on average) but possibly not significant and rather small.

This exercise intends to show that not all that glitters is gold. In both cases we find statistical evidence in favor of Granger causality from the asset return to the growth rate of the endowment, even though, in this simple set up there is no real (economic) causation whatsoever in that direction. As a matter of fact, if there is a variable that causes (in the real sense) the other, is the endowment. Thus an econometrician that mechanically chooses to interpret Granger causality tests and VAR results may be completely misinterpreting the actual structure of the economy.

It would not be difficult to replicate impulse-response functions such as the ones described in Figure 1 from economies such as (3) or (4) if we take the leap of faith that the policy instrument that the monetary authority uses is the real rate of interest (comparable to our r). As demonstrated below, this can not be the case. Even if it were, and the instrument of the authority accurately reflected the real return of a risk free bond, impulse-response and Granger causality results can not be interpreted as useful tools for identifying the effects of alternative policies. Economics and not pure statistics must be used to do so.

As the examples make clear, VARs or IVARs cannot be used to tackle the task of identifying the effects of alternative policies. However, there is a reason why well-specified time series models could be used, and it is simply to provide a (statistically) objective metric under which alternative theoretical models (that are, at least in principle, robust to the Lucas critique) could be compared. Next, we discuss this issue.

3 The Metric

This section presents the results of a nine variable IVAR model for the Chilean economy. This model is intended to provide a good statistical description of the variables included, but we are careful of not providing any "structural" interpretation to it. Special attention is taken to test the proper order of the model and to test whether or not the innovations are jointly Gaussian.

The variables taken into consideration correspond to monthly time series from 1985:01 to 2001:07 of the (log of the) industrial production index of the US (y^*), the (log of the) first different of the US WPI (p^*), the (log of) real money holdings in the US (m^*), the (log of the) FEDs fund rate (i^*), the (log of the) real exchange rate (e), the (log of the) monthly activity index of Chile (IMACEC and denoted by y), the (log of the) first difference of the Chilean CPI index (p), the (log of) domestic real money holdings (denoted by m), and the (log of the) Chilean monetary policy rate (set by the Central Bank and denominated in UF, d). In all cases a quadratic trend was included to take into account possible smooth changes in trends over time.

3.1 Parsimony

The first step in estimating the IVAR is to compute the unrestricted VAR. This computation is done following the usual OLS regressions for each variable on the system and choosing the optimal lags. Privilege must be given to a representation that is able to obtain innovations prior to reducing it to a parsimonious representation.

As is well known, model selection based on the Akaike Information Criteria (AIC) tends to choose models that are less parsimonious than the Bayesian Information Criteria (BIC), Hannan-Quinn Criteria (HQC), or Final Prediction Error Criteria (FPE). In our case, AIC prefers a model with 13 lags while BIC and HQC choose only 1 lag. Finally, FPE prefers a VAR(2) model. Extensive LRT tests on the residuals show that even a VAR(1) is able to produce residuals that can be characterized as vector-white noise processes but that present important departures from Gaussianity.

Order	Number of Parameters	Saturation Ratio	% of Nonsignificant Variables
1	108	0.061	0.509
2	189	0.107	0.614
3	270	0.154	0.704
4	351	0.201	0.729
5	432	0.249	0.771
6	513	0.300	0.791
7	594	0.346	0.806
8	675	0.395	0.824
9	756	0.444	0.783
10	837	0.495	0.808
11	918	0.545	0.849
12	999	0.597	0.822

Table 1: Implications of the choice of different lags

Table 1 shows the effect of a phenomena that is often over-looked in practice. As all models consider the dynamic interactions of nine variables, increasing the number of lags has non trivial effects on the parsimony and accuracy of the estimation. In particular, even simple unconstrained VAR(1) models include more than 50% of its parameters that are not statistically significant at standard levels. Thus, any unconstrained version of the model may induce to spurious dynamics that are not present in the data if we ignore this fact. Furthermore, even small order VAR models (such as for example a VAR(4)), have a huge saturation ratio (ratio between the number of parameters estimated in each equation and the sample size). In that particular example, more than 20% of the sample is compromised in estimating the parameters of each equation.

The effects of not accounting for parsimony, not only affect inference when obtaining bootstrapped confidence intervals, but also may substantially modify the impulse-response functions themselves. Figure 2 shows that this is indeed the case. Even when using the traditional Cholesky decomposition for the computation of the impulse-response functions, obtaining them from the unconstrained VAR(2) model that ignores that more than 61% of the variables are redundant enhances what appear to be the responses of the variables to the interest rate innovation.[11]

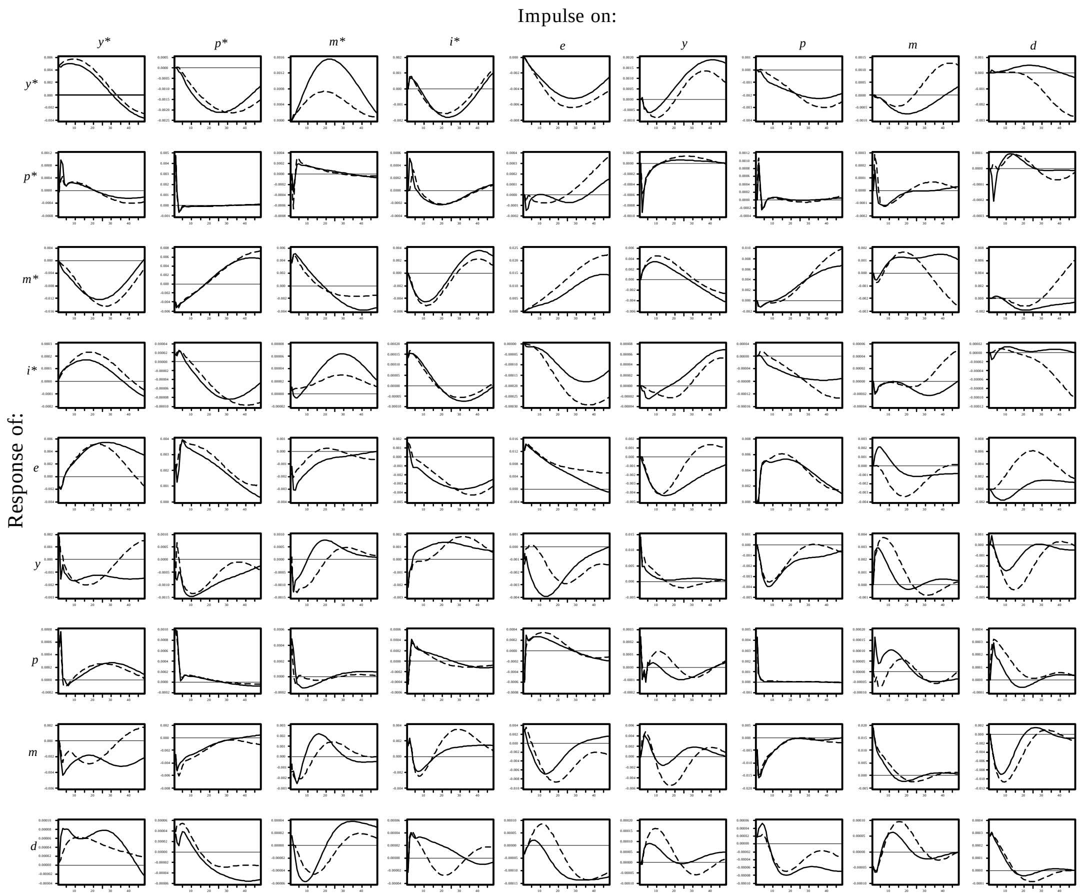

Figure 2: Effects of not considering parsimony in Cholesky impulse-response functions. Continuous line: parsimonious VAR. Dashed line: unconstrained VAR.

3.2 Choice of the Impulse-Response Function

Once the VAR model is estimated, tests of identification can be performed in order to assess the characteristics of the B_0 matrix that best fits the data if IVAR models are to be considered. Recall that different specifications of this matrix will modify non-trivially the impulse-response functions, thus special attention should be given to this point. Likelihood ratio tests, AIC, and BIC in the

[11] As discussed above, the FPE criteria chooses the VAR(2) model, while BIC and HQ prefer the VAR(1) model. Nevertheless, the parsimonious VAR model in the case of the VAR(1) model fails to produce vector-white noise errors. Thus, we conduct all the following exercises using a VAR(2) as the baseline model.

line of Leeper (1995) and Leeper, et.al. (1996) were used while testing this specifications.[12]

The preferred specification for B_0 has a similar structure to the Cholesky decomposition, but contrary to the identifying assumption of Morandé and Schmidt-Hebbel (1997) and Valdés (1998), monetary instruments should be ordered precisely in the opposite direction, tending to react contemporaneously to innovations in the price equation and output equation. One important feature of using IVAR models is that inference regarding the contemporaneous associations of the innovations can be performed once the parameters of B_0 are estimated by maximum likelihood. If this is done, most of the variables considered in B_0 can not be considered as statistically significant. Furthermore, Parrado (2001) imposed a different structure on his IVAR model, but most of the variables he considered are also insignificant.

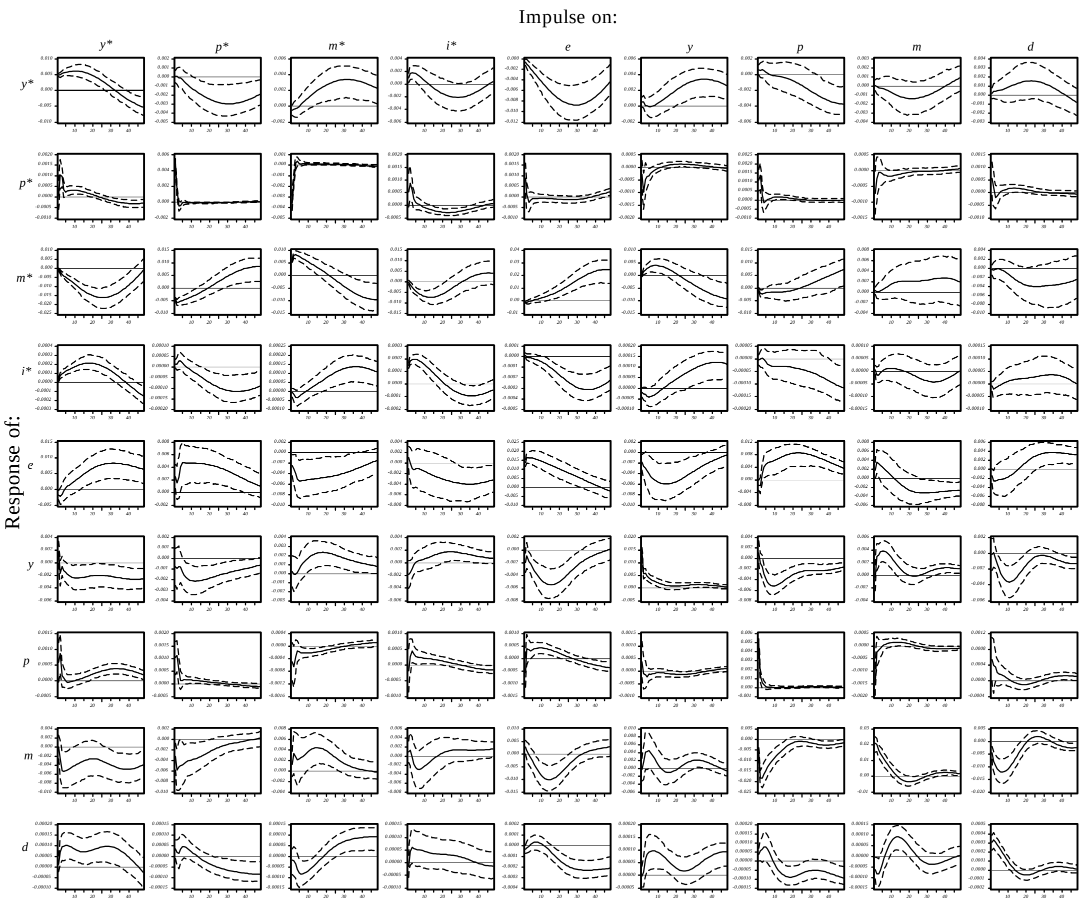

Figure 3: Generalized impulse-response functions. Hall's bootstrapped confidence intervals in parenthesis.

[12] See Appendix A for details.

Thus, IVAR models also impose arbitrary decompositions on the impulse-response functions that results from it if statistically insignificant parameters are considered. Thus, we chose the report the impulse-response functions that are obtained using Pesaran and Sin's (1998) methodology. In contrast with the Cholesky decomposition, generalized impulse-response functions do not depend on the ordering of the equations; however as is the case with the Cholesky decomposition, generalized impulse-responses are exactly identified and tests for reductions can not be performed.

Figure 3 presents the generalized impulse-response functions for four years (48 months). With them, Efron's 95% confidence intervals are also presented. As discussed above, Efron's bootstrapped confidence intervals may not have the correct coverage in the presence of asymmetries. As one of the sources of departures from normality is precisely this, Efron's confidence intervals are not advisable.

If we consider the last two columns of Figure 3 as the effects of potential candidates for a measure of the monetary policy innovation, we observe that if innovations to m are considered as monetary policy we find that surprise changes in the stock of money are persistent and predict subsequent movements in both inflation and output. The later nevertheless is very short lived and dies out almost instantaneously. On the other hand inflation increase only after a few periods (it is not statistically significant initially). However, the response to m innovations in this system show what is sometimes called the "liquidity puzzle": interest rates do not decline when m jumps upward. The liquidity effect, which hypothesizes that the policy-induced increased liquidity of a monetary expansion should lower interest rates, seems not to be present if innovations to m are considered as measures of monetary policy stance.

There is no problem with the liquidity effect if the innovations of the UF interest rates are considered as monetary policy (last column). In this case, the initial shock can be interpreted as a monetary contraction. Here, there is a strong liquidity effect until the fifteenth month and it eventually dies out. In terms of output, the d shocks shock has a short lived effect on contracting y. The problem with this shock arises when analyzing the effect on p. This results that is very common in the literature (Leeper et.al., 1996) has been labeled the "price puzzle". Here, monetary contractions tend to rise prices steadily!

Interpreting either column eight or nine as a monetary contraction therefore requires accepting that monetary contractions produce inflation, which seems as unlikely an idea as the notion that monetary expansions fail to lower interest rates.[13]

However, note that if d is considered as the monetary policy instrument, the ninth row shows very plausible responses. That is, interest rates increase with positive shock on output and inflation.

The results from both IVAR and generalized impulse-response functions show that care should be given in interpreting these type of innovations as monetary policy. Of course, traditional VAR modeling renders even more implausible dynamic responses of prices and output (as we have shown with the example of the $s(2)$ shocks). The following section develops a theoretical model that may help to explain why IVAR modeling is not sufficient for characterizing the effects of alternative policies and that this statistical exercise alone is not sufficient. That is, more structure that the S preceding the VAR has to be considered.

[13] The "price puzzle " is also present in the impulse-response functions of Valdés (1998) when the inflation target is not considered. It is important to mention that several authors have concluded that the price puzzle can be eliminated if terms of trade or the price of oil is included (Parrado, 2001). In our case, this was not possible; including terms of trade did not change our results.

However, VAR estimates provide excellent statistical characterizations of the dynamic interactions of the variables considered. That we have problems trying to translate these statistical objects on structural economic models such not constitute a surprise. Thus, this model should only be considered as an statistical representation and not provide any structural interpretation to it. Once we do that we must explain why supposedly deflationary policies produce inflations. We consider our results simply as a metric with which to obtain estimates of deep parameters of internally consistent dynamic models by using EMM.[14] Once we do that, we can compare the responses of the variables in the theoretical model with those that come from IVARs.

4 The Model

This section develops a simple optimizing model, whose empirical implications will be compared with those of the econometric model described above. To make the model computationally manageable and to gain insights with respect to the characteristics of the data that the model is and is not able to replicate, we introduce stringent assumptions to the stylized economy that we model. Needless to say, many of the assumptions come directly from the observed dynamic interactions of key variables in the IVAR estimated above.

We consider a simple economy in which agents intend to maximize the expected discounted time separable utility of the form:

$$\mathcal{E}_0 \sum_{t=0}^{\infty} \beta^t u\left(c_{h,t}, c_{m,t}, \frac{M_t}{P_{h,t}}\right) \tag{5}$$

where $c_{m,t}$ is the consumption of tradable (importable) goods in period t, $c_{h,t}$ is the consumption of non tradable goods in period t, M_t denotes the nominal money stock that the individual acquires at the beginning of period t and then holds through the end of the period, $P_{h,t}$ is the price level of non tradable goods, β is the subjective discount factor, $\mathcal{E}_t$ denotes the conditional expectation on information available on period t, and $u(\cdot)$ is strictly increasing and strictly concave function in all its arguments.

A few observations are in order. We introduced money in the utility function in order to make money valuable in general equilibrium. Implicit on this assumption is that money may be valuable both as a store of wealth and a medium of exchange. Feenstra (1986) showed that this specification is equivalent to one that derives from the literature of transaction costs. Of course, cash-in-advance constraints are merely special cases of the transaction costs technologies.[15]

The maximization of (5) is done subject to the budget constraint

$$\begin{gathered} q_{h,t} + \frac{(1+T_{m,t})P^*_{m,t}E_t q_{m,t}}{P_{h,t}} + (1+r_{t-1})\, b_{t-1} + \frac{(1+i_{t-1})B_{t-1}}{P_{h,t}} + \frac{(1+d_{t-1})D_{t-1}U_t}{P_{h,t}} + \frac{M_{t-1}}{P_{h,t}} \geq \\ c_{h,t} + \frac{(1+T_{m,t})P^*_{m,t}E_t c_{m,t}}{P_{h,t}} + b_t + \frac{B_t}{P_{h,t}} + \frac{D_t U_t}{P_{h,t}} + \frac{M_t}{P_{h,t}} + \frac{Z_t}{P_{h,t}} \end{gathered} \tag{6}$$

[14] See Appendix D.

[15] The "neutrality" found on the previous section 3 may tempt us to work with specifications such as cash-in-advance. However, that type of specifications impose the assumption of constant velocity which is not supported by the data.

where E is the nominal exchange rate, T_m is the import tariff of a tradable good (q_m) that can be acquired in a competitive international market with price denoted by P_m^*, q_h denotes the output of the non tradable good produced in the economy and sold at price P_h. Agents may also acquire indexed private bonds (in terms of non tradable goods, b) with sure return r that are in zero net supply, nominal government bonds (B) with (net) nominal return i, government bonds indexed to the UF (D) with net return d,[16] and money balances that are carried to the next period. Finally, Z denotes lump sum taxes (or transfers) levied by the government. As a first approximation, the outputs of the different sectors of this economy will be characterized as stochastic endowments, thus making resource allocations irrelevant.

The problem of the representative consumer can then be summarized by the value function that satisfies:

$$v(s_t) = \max_{\{c_h, c_m, b, B, D, M\}} \left[u\left(c_h, c_m, \frac{M}{P_h}\right) + \mathcal{E}\beta v(s_{t+1}) \right]$$

subject to (6) and the perceived laws of motion of the states s.[17]

The governments' budget constraint as given by:

$$\begin{aligned} &\tfrac{T_{m,t}P_{m,t}^* E_t(c_{m,t}-q_{m,t})}{P_{h,t}} + \tfrac{B_t}{P_{h,t}} + \tfrac{D_t U_t}{P_{h,t}} + \tfrac{M_t}{P_{h,t}} + \tfrac{Z_t}{P_{h,t}} \\ = \alpha_t g_t + \tfrac{(1-\alpha_t)P_{m,t}^* E_t}{P_{h,t}} g_t + &\tfrac{(1+i_{t-1})B_{t-1}}{P_{h,t}} + \tfrac{(1+d_{t-1})D_{t-1}U_t}{P_{h,t}} + \tfrac{M_{t-1}}{P_{h,t}} + \tfrac{(1+i_{t-1})B_{t-1}^*}{P_{h,t}} \end{aligned}$$

where g is the level of government expenditure (in terms non tradables), α is the fraction of government expenditures destined to the consumption of non tradables, and B^* is the supply of government bonds to foreign investors.

Finally, in order for this problem to be well defined, we consider a representative foreign investor that solves a dynamic portfolio allocation problem, in which he intends to maximize his expected discounted utility:

$$\mathcal{E}_0 \sum_{t=0}^{\infty} {\beta^*}^t w\left(c_{m,t}^*, \frac{M_t^*}{P_{m,t}^*}\right)$$

subject to the constraint:

$$\begin{aligned} q_{m,t}^* + \tfrac{(1+i_{t-1})B_{t-1}^*}{E_t P_{m,t}^*} + \tfrac{(1+i_{t-1}^*)b_{t-1}^*}{P_{m,t}^*} + \tfrac{M_{t-1}^*}{P_{h,t}} \geq \\ c_{m,t}^* + \tfrac{B_t^*}{E_t P_{m,t}^*} + \tfrac{b_t^*}{P_{m,t}^*} + \tfrac{M_t^*}{P_{m,t}^*} + \tfrac{Z_t^*}{P_{m,t}^*} \end{aligned} \tag{7}$$

where q_m^* is an stochastic endowment, c_m^* is the level of consumption of a composite good by the foreign representative agent, B^* is the demand of bonds supplied by the domestic government, and b^* is the demand of international bonds (issued by the foreign authority) that yield a return of i^*. The other variables are analogous to those of the domestic economy.

[16] The UF (Unidad de Fomento) is a unit of account indexed to present and past inflation rates. A further discussion is presented below.

[17] We define $s_t = (q_{h,t}, q_{m,t}, r_{t-1}, i_{t-1}, d_{t-1}, b_{t-1}, B_{t-1}, D_{t-1}, M_{t-1}, P_{h,t}, P_{m,t}^*, E_t)$.

The foreign investor's value function satisfies:

$$v^*(s_t^*) = \max_{\{c_m^*, b^*, B^*, M^*\}} \left[w\left(c_m^*, \frac{M^*}{P_m^*}\right) + \mathcal{E}\beta^* v^*\left(s_{t+1}^*\right)\right]$$

subject to (7) and the perceived laws of motion of the states s^*.[18]

The foreign government satisfies the constraint:

$$\frac{b_t}{P_{m,t}^*} + \frac{M_t^*}{P_{m,t}^*} + \frac{Z_t^*}{P_{m,t}^*} = g_t^* + \frac{\left(1+i_{t-1}^*\right) b_{t-1}^*}{P_{m,t}^*} + \frac{M_{t-1}^*}{P_{m,t}^*}$$

The market-clearing conditions for the tradable and non tradable markets are:

$$q_{h,t} = c_{h,t} + \alpha_t g_t$$

$$CA_t \equiv -\left(B_t^* - B_{t-1}^*\right) = P_{m,t}^* E_t \left(q_{m,t} - c_{m,t} - (1-\alpha_t) g_t\right) - i_{t-1} B_{t-1}^*$$

$$CA_t^* \equiv \left(B_t^* - B_{t-1}^*\right) = P_{m,t}^* E_t \left(q_{m,t}^* - c_{m,t}^* - g_t^*\right) + i_{t-1} B_{t-1}^* = -CA_t$$

where the first equation describes the equilibrium in the non tradable good domestic market, the second presents the equilibrium in the tradable good domestic market that shows that the current account balance must be compensated by the capital account balance. Finally, the third equation shows the equilibrium condition of the foreign economy's good market (expressed in terms of domestic currency) which displays a condition analogous the second. Note that in general equilibrium the current account balance of one economy is the negative of the other; thus the supply and demand of the tradable good equate.

We define a recursive competitive equilibrium for these economies as a set of prices and policy functions such that markets clear. To find the policy functions compatible with the market clearing conditions we must solve the problems faced by each of the agents in these economies.

Appendix B demonstrates that the first order conditions of both optimization problems can be used to obtain the value of the real exchange rate (e),[19] the equilibrium real interest rate for the risk-free indexed private bond, the nominal interest rate of the risk free government bond, the demand for domestic real balances, the foreign nominal interest rate, the no arbitrage condition, and the demand for foreign real money holdings.

4.1 An Example

To gain insight with respect to the characteristics of the economy under consideration, the example that follows shows how prices and real variables are determined. For that purpose we consider that the consumer has a Cobb-Douglas, constant relative risk aversion utility function of the form:

$$u\left(c_h, c_m, \frac{M}{P_h}\right) = \frac{\left[c_h^{\varphi} c_m^{\delta} \left(\frac{M}{P_h}\right)^{1-\varphi-\delta}\right]^{1-\gamma}}{1-\gamma}$$

[18] We define $s_t^* = \left(q_{m,t}^*, i_{t-1}, i_{t-1}^*, b_{t-1}^*, B_{t-1}^*, M_{t-1}^*, P_{m,t}^*, E_t\right)$.

[19] Defined as a relative price between tradables and non tradables.

where γ is the Arrow-Pratt constant relative risk aversion coefficient. In the particular case that $\gamma \to 1$, the last equation can be conveniently expressed as:

$$u\left(c_h, c_m, \frac{M}{P_h}\right) = \varphi \ln c_h + \delta \ln c_m + (1 - \varphi - \delta) \ln \frac{M}{P_h}$$

which is the functional form that we will use for this example.

Suppose further that the domestic endowments follow first order Markov processes that are independent of foreign and domestic nominal variables. Consider that the domestic government expenditures are a constant fraction of the production of non tradables that is partially financed by imposing a fixed import tariff on the tradable good. The monetary authority in the domestic economy sets an UF indexed interest rate by supplying the amount of bonds that the foreign and domestic markets are willing to take at the referred rate. We will introduce more structure to the domestic monetary policy as needed.

Using (B.11)-(B.16), the equilibrium conditions in this case are:

$$e_t \equiv \frac{P^*_{m,t} E_t}{P_{h,t}} = \frac{\delta \left(q_{h,t} - \alpha_t g_t\right)}{\left(1 + T_{m,t}\right) \varphi \left(q_{m,t} - \left(1 - \alpha_t\right) - N_t\right)} \tag{8}$$

$$1 = \beta \left(1 + r_t\right) \mathcal{E}_t \left(\frac{q_{h,t} - \alpha_t g_t}{q_{h,t+1} - \alpha_{t+1} g_{t+1}}\right) \tag{9}$$

$$1 = \beta \left(1 + i_t\right) \mathcal{E}_t \left(\frac{q_{h,t} - \alpha_t g_t}{q_{h,t+1} - \alpha_{t+1} g_{t+1}} \frac{P_{h,t}}{P_{h,t+1}}\right) \tag{10}$$

$$1 = \beta \left(1 + d_t\right) \frac{U_{t+1}}{U_t} \mathcal{E}_t \left(\frac{q_{h,t} - \alpha_t g_t}{q_{h,t+1} - \alpha_{t+1} g_{t+1}} \frac{P_{h,t}}{P_{h,t+1}}\right) \tag{11}$$

$$\frac{M_t}{P_{h,t}} = \frac{1 - \varphi - \delta}{\varphi} \left(q_{h,t} - \alpha_t g_t\right) \frac{1 + i_t}{i_t} \tag{12}$$

where N is defined as the net amount of foreign private capital inflows in terms of the tradable good (current account deficit plus payments of interests).

Even though the results displayed in the previous equations depend on the simple parameterization chosen, their qualitative implications will hold for a wide variety of functional forms for preferences.

Notice that (8) confirms several beliefs in pop culture. The real exchange rate appreciates with a decline on productivity on non tradables, an increase in productive on tradables, increased net capital inflows, and trade protection. As Arrau, et.al. (1992) showed, the effect of an increase in government expenditure has an ambiguous effect on the real exchange depending not only on its propensity to consume non tradable good, but also on the structure of private consumption.

Equation (9) presents the condition that determines the value of the real interest rate in this economy. Contrary to several claims, the monetary authority is not capable of affecting it directly. In this economy, in the long run, the real interest rate displays a positive relation with the growth rate of the non tradable sector. This means, that an economy that is growing at a faster rate than another, will have higher autarkic real interest rates. If there are limitations to the free flow of capital from one economy to the other, the economy that is growing faster, will have a higher interest rate. This rate may have no relation with the "real interest rate" set by the monetary authority and thus, the claim that the Central Bank "sets" the real interest rate is fundamentally incorrect.[20] Then, what did the monetary authority set with instruments indexed to the UF?.

Equations (10) and (11) have the answer. If we combine them, we realize that both instruments must be arbitraged, given that the terms in the expectation operators coincide. Thus, it does not matter if the authority sets a nominal or an UF indexed rate. Nevertheless, the difference between the law of motion of the UF and the actual price level shows that the real interest rate will have fundamental differences with the UF indexed rate. The difference between them, is that the later instrument and truly indexed bonds is contingent on the actual realization of inflation, while the former is (or at least should be) set considering the expected rate of inflation. Thus the difference between these two equations is precisely the same one that prevents the Fischer equation to hold in the presence of uncertainty. The only case in which it would hold (on average) is if the inflation process is independent of the intertemporal marginal rate of substitution. This condition is not likely to hold precisely because the reaction function of the monetary authority (particularly in Chile) is extremely dependent on its perception of the business cycle and the growth rate the economy. In case the monetary authority sets an inflation target, equations (10) and (11) show that it must be consistent with the interest rate chosen. Thus, either of these equations will help to solve for the inflation rate consistent with the perceptions with respect to the evolution of the economy (intertemporal marginal rate of substitution) and the monetary authority policy rule.

Finally, (12) determines the demand for real money holdings. Note that this equation is independent of parameters that may characterize the monetary authority's policy rule. However, it is likely that if it changes, badly specified money demand equations may find evidence of instability even when there is none.

Several monetary policy arrangements can be examined even in this simple case. For example, if the authority sets the nominal exchange rate, no arbitrage conditions with the foreign investor will determine the nominal interest rate consistent with this policy. Likewise, (12) will endogenously determine the money stock consistent with this policy.

4.2 Results

Section 3 showed the problems of using VAR and IVAR impulse-response functions to identify the effects of monetary policy are ill conditioned practices. Irrespective of the method used, it is difficult to rationalize several of the results that are supposed to capture the effects of monetary policy. Furthermore, as the theoretical model discussed previously shows, several dynamic interactions

[20] Prior to the last quarter of 2001, the Central Bank of Chile set its policy rated with UF indexed instruments. This fact, made many specialists and non-specialists to claim that the Central Bank sets the real interest rate. Of course, this claim is fundamentally false.

between variables depend on the particular specification for the policy rule of the government.

Our estimated model closes with a Taylor rule for the monetary authority:

$$\ln(1+i_{t+1}) = a_0 + a_1 \ln\left(\frac{y_t}{y_s}\right) + a_2 \mathcal{E}_t \ln\left(\frac{1+\pi_{t+1}}{1+\pi_s}\right) + a_3 \ln(1+i_t) + \xi_{t+1}$$

where y_s and π_s denote the steady state values for output and inflation.

Our methodological steps are the following: First, use the gradients of the VAR(2) discussed on Section 3 as the matching conditions for the theoretical model. Second, estimate the parameters of the theoretical model using EMM and the gradients of the VAR model as the metric. Third, once the estimates of the model are obtained, simulate a long simulation of the theoretical model, estimate a VAR(2) with it, and derive the generalized impulse-response function. Finally, shock the theoretical model with a transitory innovation to the domestic interest rate and obtain the "true" impulse-response function.[21]

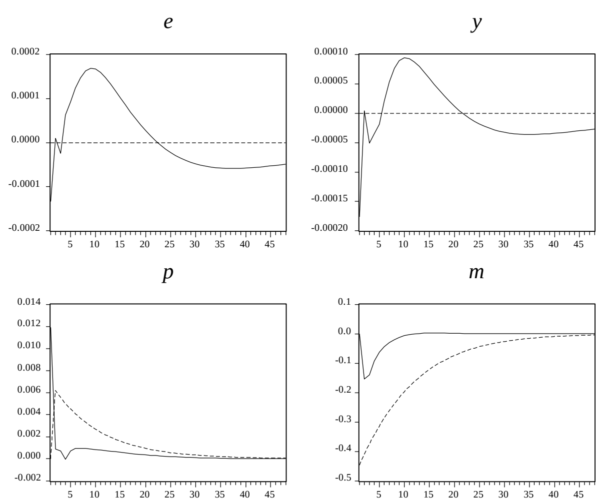

Figure 4: Response to monetary innovations. Continuous line: generalized impulse-response. Dashed line: true impulse-response.

Figure 4 presents the results of comparing the impulse-response functions that are obtained with the VAR(2) estimated with artificial data and the impulse-response functions that come from the theoretical model. Several features are worth mentioning: First, the impulse-response functions estimated with simulated data are broadly consistent with the data; that is the model also produces a price puzzle, a small contraction on the level of activity, a slight appreciation, and strong liquidity effects. Second, the overidentifying restrictions test does not reject the null that the model captures the dynamic interactions of the variables involved. Third, and most importantly, even though the

[21] Here, true means the impulse-response function that is consistent with the theretical model and not the statistical object.

model replicates the impulse-response functions of the data, the true response of the variables with respect to a monetary innovation have little to do with the responses that come from the VAR.

Given that the model has by construction a dichotomy between real and nominal variables, monetary policy has no effect on the real exchange rate and output, even when the impulse response functions of the model show non-neutralities. This is so, because interest rates carry information with respect of the future evolution of the economy; thus a higher interest rate signals lower output today with respect to the long run, as interest rates Granger cause output in this model (as did in the example of Section 2), it is not surprising that the impulse-response functions may show spurious responses of output to interest rate innovations.

More importantly, a case for neutrality (or almost neutrality) can be made precisely because of the presence of the price puzzle. It must be conceded that models that display important non-neutralities (with Phillips curves and such) would have a very difficult time trying to explain this puzzle. In our case, however, the theoretical model predicts that inflation and interest rates should be positively correlated, precisely because of the inability of nominal variables to affect real variables. Thus, if the real interest rate remains basically constant, prices must follow the same direction of the nominal interest rate innovation. This follows simply from the interaction of the feed-back of the Taylor rule and the dichotomy with real variables.

Thus, the model shows that there are only few dimensions of the impulse-response functions that are truly consistent with responses of fundamental models. Thus, the idea that VARs can help to identify the effects of monetary policy is naive.

5 Concluding Remarks

The objectives of this paper were two-folded. First, to show that the traditional practice of quantifying the effects of monetary policy from impulse-response functions of VARs or IVARs is misleading. It can not help as to recover the effects of monetary policies because it is impossible to recover structural shocks (independently of the structure chosen). Second, we constructed a simple metric with which competing theoretical models can be compared. This second objective is important because the theoretical and empirical literature in the field of macroeconomics has not reached a consensus with respect to which metric to use to judge how successful is a model in capturing key features of the data. Our intention is to show is that such a metric can be constructed and that the statistical object that comes from it can help us to understand which features of different theoretical models are important and which are not.

Our statistical model is a VAR(2) model comprised of nine variables and whose impulse-response functions cannot be directly considered as structural. Furthermore, if we chose to do so, we should have to concede that deflationary policies are inflationary or that the money demand depends positively on interest rates.

The theoretical model that is estimated is broadly consistent with the VAR(2) model but has striking implications. First, by construction, it displays neutrality of the monetary policy. Second, precisely because of this feature, it is not difficult to replicate impulse-response functions that appear to be consistent with non-neutralities. Third, the price puzzle is only a puzzle for a model that has as a major driving force important non-neutralities. Finally, even when the model has built-in non neutralities, they must nit have first order implications in order for it to be consistent with the statistical object chosen.

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A Estimation and Inference in an IVAR

This appendix presents a brief summary of the techniques used to estimate IVAR models, their differences with traditional VARs, and the methods developed to test their specification.

Consider a model of the type:

$$\mathsf{B}_0\mathsf{y}_t = \mathsf{k} + \mathsf{B}_1\mathsf{y}_{t-1} + ... + \mathsf{B}_p\mathsf{y}_{t-p} + \mathsf{u}_t \tag{A.1}$$

where y_t is an n-vector, k is an n-vector of constants, B_i is an $n \times n$ matrix $(i = 0, .., p)$, and u_t is an n-vector white noise process with (diagonal) variance-covariance matrix D. Premultiplying (A.1) by the inverse of B_0 we obtain:

$$\mathsf{y}_t = \mathsf{c} + \mathsf{C}_1\mathsf{y}_{t-1} + ... + \mathsf{C}_p\mathsf{y}_{t-p} + \mathsf{e}_t \tag{A.2}$$

where, given that u is a vector white noise process, and $\mathsf{e} = \mathsf{B}_0^{-1}\mathsf{u}$, e is also a vector white noise process with variance-covariance matrix Ω. Equation (A.2) is precisely the representation generally used in VAR models, thus making it interpretable as a reduced form of (A.1). The only case in which the VAR model would be equivalent to (A.1) is when B_0 is an identity matrix. If some of the off-diagonal elements of this matrix are different from zero, the error terms on e will be formed by linear combinations of the "structural" innovation u. Thus, the impulse-response functions estimated with (A.2) can't be interpreted as the dynamic response of the variables in the system to the underlying innovations.

To recover the "structural" parameters of (A.1) we can use the parameters estimated from (A.2) and obtain an estimate of Ω; with it, we can solve the nonlinear system:

$$\Omega = \mathsf{B}_0^{-1}\mathsf{D}\left(\mathsf{B}_0^{-1}\right)' \tag{A.3}$$

or the log-likelihood function that relates both variance-covariance matrices:[22]

$$\ell\left(\mathsf{B}_0, \mathsf{D}, \widehat{\Omega}\right) \propto \frac{T}{2}\left(\ln|\mathsf{B}_0|^2 - \ln|\mathsf{D}| - \operatorname{tr}\left[\mathsf{B}_0'\mathsf{D}^{-1}\mathsf{B}_0\widehat{\Omega}\right]\right) \tag{A.4}$$

One advantage of this approach is that the variance-covariance matrix of the parameters that solve (A.3) are readily available. Once the "structural" parameters are recovered, inference based on likelihood ratio (LRT) or Wald tests can be conducted as usual.

One important issue as that of identification; recall that as Ω is symmetric, there are $n(n+1)/2$ distinct parameters in the variance-covariance matrix of the residuals of (A.2). Given that D is diagonal, there are at most $n(n-1)/2$ parameters that can be estimated in B_0 if the order condition of identification is to be satisfied. Thus, some restrictions (hopefully testable) have to be imposed. If we denote by z the number of parameters estimated in B_0, the number of overidentifying restrictions is $r = (n(n-1)/2) - z$. In that case the LRT test for overidentifying restrictions is simply:

$$\text{LRT} = 2\left(-\frac{T}{2}\ln\left|\widehat{\Omega}\right| - \frac{T}{2}n - \ell^*\right) \xrightarrow{D} \chi_r^2 \tag{A.5}$$

[22] See Hamilton (1994) for details.

where ℓ^* is the value of the log-likelihood function that maximizes (A.4).

This methodology provides not only a robust way of estimating the effects of orthogonal innovations to the system, but may also be a useful tool for discriminating among models. Recall that VARs impose a somewhat arbitrary ordering of the variables in the system that will affect the resulting impulse-response functions, while IVARs provide formal tests under which to contrast alternative orderings and contemporaneous relations among variables. This feature may constitute a valuable intermediate stage that provides insights with respect to the theoretical models that can and cannot be used in order to replicate the dynamic interactions between variables. Nevertheless, as IVARs heavily rely on the identifying assumptions imposed on them, they are useful only as intermediate devices between data and theory.

A healthy practice, whether using VARs or IVARs is to compute standard errors (and thus confidence intervals) associated with the impulse-response functions. Traditional econometric packages rely on the asymptotic distribution of the impulse-response functions or on Monte Carlo experiments (based on the maintained hypothesis of Gaussian innovations) to construct them. These methods may however provide poor approximations to the confidence intervals in small samples even with the assumption of normality (due to small sample bias of the OLS estimators). Another important problem that this type of confidence intervals has is that they are symmetric (due to the assumption of normality). In finite samples, the innovations may have important departures from normality (typically leptokurtosis) and may not be symmetric (if there is skewness), thus the Monte Carlo approximation may not be advisable. In this case bootstrap, methods may be used to replicate the empirical distribution of the innovations. Sims and Zha (1995) also advise to construct confidence intervals that may help to correct the pervasive nature of the biases implicit in the VAR estimation. This can be done again with bootstrapping.[23]

[23] Christiano, et al (1996) provide a detailed description of the algorithm used to construct both the impulse-response functions and confidence intervals with bootstrapping. Sims and Zha (1995) describe the algorithm used for bias reduction.

B Equilibrium Conditions for the Theoretical Model

This Appendix derives the first order conditions for the optimization problems of the domestic representative consumer and the representative foreign investor. These conditions are latter used to describe the characteristics of the equilibrium conditions of the economies presented in the theoretical model.

The first order conditions with respect to $c_{h,t}$, $c_{m,t}$, b_t, B_t, D_t and M_t for the representative domestic consumer are:

$$u'_{c_{h,t}} - \lambda_t = 0 \tag{B.1}$$

$$u'_{c_{m,t}} - \lambda_t \left(1 + T_{m,t}\right) \frac{P^*_{m,t} E_t}{P_{h,t}} = 0 \tag{B.2}$$

$$\lambda_t - \beta \mathcal{E}_t v'_{b_t} = 0 \tag{B.3}$$

$$\lambda_t - \beta P_{h,t} \mathcal{E}_t v'_{B_t} \tag{B.4}$$

$$\lambda_t - \beta \frac{P_{h,t}}{U_t} \mathcal{E}_t v'_{D_t} \tag{B.5}$$

$$u'_{\frac{M_t}{P_{h,t}}} \frac{1}{P_{h,t}} - \frac{\lambda_t}{P_{h,t}} + \beta \mathcal{E}_t v'_{M_t} = 0 \tag{B.6}$$

where λ is the dynamic multiplier associated with the constraint (6). The corresponding envelope conditions are:

$$v'_{b_{t-1}} = \lambda_t \left(1 + r_{t-1}\right) \tag{B.7}$$

$$v'_{B_{t-1}} = \frac{\lambda_t}{P_{h,t}} \left(1 + i_{t-1}\right) \tag{B.8}$$

$$v'_{D_{t-1}} = \frac{\lambda_t U_t}{P_{h,t}} \left(1 + d_{t-1}\right) \tag{B.9}$$

$$v'_{M_{t-1}} = \frac{\lambda_t}{P_{h,t}} \tag{B.10}$$

Combining (B.1) and (B.2) we find that the real exchange rate (e), defined as the relative price between tradables and non tradables is given by the ratio of marginal utilities between the consumption of both goods, corrected by the import tariff. That is:

$$e_t \equiv \frac{P^*_{m,t}E_t}{P_{h,t}} = \frac{u'_{c_{m,t}}}{(1+T_{m,t})\,u'_{c_{h,t}}} \tag{B.11}$$

Combining (B.3) and (B.7) we find the condition that determines the equilibrium real interest rate for the risk-free indexed private bond, while combining (B.4) and (B.8) we encounter the equilibrium condition that determines the nominal interest rate of the risk free government bond. Finally, combining (B.5) and (B.9) we find the equilibrium interest rate for the UF indexed bond. That is:

$$1 = \beta\,(1+r_t)\,\mathcal{E}_t \frac{u'_{c_{h,t+1}}}{u'_{c_{h,t}}} \tag{B.12}$$

$$1 = \beta\,(1+i_t)\,\mathcal{E}_t \left(\frac{u'_{c_{h,t+1}}}{u'_{c_{h,t}}} \frac{P_{h,t}}{P_{h,t+1}} \right) \tag{B.13}$$

$$1 = \beta\,(1+d_t)\,\frac{U_{t+1}}{U_t}\mathcal{E}_t \left(\frac{u'_{c_{h,t+1}}}{u'_{c_{h,t}}} \frac{P_{h,t}}{P_{h,t+1}} \right) \tag{B.14}$$

Note that in (B.14) U_{t+1}/U_t is known at period t, given that

$$\frac{U_{t+1}}{U_t} = \left[\frac{P_t}{P_{t-1}}\right]^a \left[\frac{P_{t-1}}{P_{t-2}}\right]^{1-a} \tag{B.15}$$

where can be approximated by 21/30.[24]

Finally, combining (B.6) and (B.10), and using (B.13) we can derive the condition that determines the demand for real balances:

$$\frac{u'_{\frac{M_t}{P_{h,t}}}}{u'_{c_{h,t}}} = \frac{i_t}{1+i_t} \tag{B.16}$$

These equations combined with the market clearing conditions and the policy rules followed by the public sector (and the functional form of preferences) will determine the temporal trajectory of these variables.

On the other hand, the first order conditions with respect to $c^*_{m,t}$, B^*_t, b^*_t and M^*_t for the representative foreign investor are:

$$w'_{c^*_{m,t}} - \lambda^*_t = 0 \tag{B.17}$$

[24] The UF has a deterministic law of motion that depends on a weighted average of present and past inflations. Assuming that a typical month has 30 days, the variation of the UF from the last day of month t to the last day of month $t+1$ is given by (B.15), where P is the consumption-based price level derived in Appendix C.

$$\lambda_t^* - \beta^* P_{m,t}^* E_t \mathcal{E}_t v_{B_t^*}^{*\prime} \tag{B.18}$$

$$\lambda_t^* - \beta^* P_{m,t}^* \mathcal{E}_t v_{b_t^*}^{*\prime} \tag{B.19}$$

$$w'_{\frac{M_t^*}{P_{m,t}^*}} \frac{1}{P_{m,t}^*} - \frac{\lambda_t^*}{P_{m,t}^*} + \beta^* \mathcal{E}_t v_{M_t^*}^{*\prime} = 0 \tag{B.20}$$

where in this case λ^* is the dynamic multiplier associated with (7). The envelope conditions for this problem are given by:

$$v_{B_{t-1}^*}^{*\prime} = \frac{\lambda_t^*}{P_{m,t}^* E_t} (1 + i_{t-1}) \tag{B.21}$$

$$v_{b_{t-1}^*}^{*\prime} = \frac{\lambda_t^*}{P_{m,t}^*} \left(1 + i_{t-1}^*\right) \tag{B.22}$$

$$v_{M_{t-1}^*}^{*\prime} = \frac{\lambda_t^*}{P_{m,t}^*} \tag{B.23}$$

As in the previous problem, we can find the equilibrium conditions for the foreign nominal interest rate, the no arbitrage condition, and the demand for foreign real money balances by combining the envelope and the first order conditions which yield:

$$1 = \beta^* (1 + i_t) \mathcal{E}_t \left(\frac{w'_{c_{m,t+1}^*}}{w'_{c_{m,t}^*}} \frac{P_{m,t}^*}{P_{m,t+1}^*} \frac{E_t}{E_{t+1}} \right) \tag{B.24}$$

$$1 = \beta (1 + i_t^*) \mathcal{E}_t \left(\frac{w'_{c_{m,t+1}^*}}{w'_{c_{m,t}^*}} \frac{P_{m,t}^*}{P_{m,t+1}^*} \right) \tag{B.25}$$

$$\frac{w'_{\frac{M_t^*}{P_{m,t}^*}}}{w'_{c_{m,t}^*}} = \frac{i_t^*}{1 + i_t^*} \tag{B.26}$$

The Euler equations of both problems were solved considering binding constraints (because of the assumption that both utility functions are strictly increasing). The values of these variables will be determined in general equilibrium by their interaction with the market clearing conditions and the laws of motion of the states (including government policies).

With relatively mild conditions, existence and uniqueness for the value functions of both problems can be demonstrated by using Blackwell's conditions for contraction and contraction mapping theorem arguments (see Altug and Labadie, 1994 or Stokey et al, 1989).

C The Aggregate Consumption-Based Price Index

This Appendix derives the aggregate consumption-based price index for the CES utility function, extending the derivations of Obstfeld and Rogoff (1996) for an economy with a tradable good, a non tradable good, and money.

Let $c = f(c_h, c_m, n)$ be a composite consumption good that is a linear-homogeneous function of c_h, c_m, and n, where $n = M/P_h$. We are interested in finding a price index associated with c that will tell us how much of it can a consumer obtain from a given level of expenditure Z (denominated in domestic currency).

We define the aggregate consumption-based price index P as the minimum expenditure $Z = P_h c_h + (1+T_m) P_m^* E c_m + Wn$ such that $c = f(c_h, c_m, n) = 1$, given P_h, P_m^*, E, T_m and W. Where in this case W denotes the user cost of holding currency whose closed form expression will be derived below.

Consider the CES consumption index of the form:

$$c = \left[\varphi^{\frac{1}{\theta}} c_h^{\frac{\theta-1}{\theta}} + \delta^{\frac{1}{\theta}} c_m^{\frac{\theta-1}{\theta}} + (1-\varphi-\delta)^{\frac{1}{\theta}} n^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}} ; \quad \varphi, \delta \in (0,1), \quad \theta > 0 \tag{C.1}$$

The highest value of the index c for a given value of Z (found by substituting the demands for each good on (C.1)) is given by:

$$\left\{\varphi^{\frac{1}{\theta}} \left[\frac{\varphi Z}{P_h D}\right]^{\frac{\theta-1}{\theta}} + \delta^{\frac{1}{\theta}} \left[\frac{\delta Z}{P_h \left(\frac{P_h}{P_m^* E (1+T_m)}\right)^{\theta} D}\right]^{\frac{\theta-1}{\theta}} + A\right\}^{\frac{\theta}{\theta-1}} \tag{C.2}$$

where

$$A = (1-\varphi-\delta)^{\frac{1}{\theta}} \left[\frac{(1-\varphi-\delta) Z}{P_h \left(\frac{P_h}{W}\right)^{\theta} D}\right]^{\frac{\theta-1}{\theta}}, \quad D = \varphi + \delta \left(\frac{P_m^* E}{P_h}\right)^{1-\theta} + (1-\varphi-\delta)\left(\frac{W}{P_h}\right)^{1-\theta}$$

Defining P to be the minimum expenditure needed to obtain $c = 1$, we can solve for P by imposing $P = Z$ and equating (C.2) to 1. After trivial manipulations and using (B.11), the price index is given by:

$$P_t = P_{h,t} \left[\varphi + \delta (e_t (1 + T_{m,t}))^{1-\theta} + (1-\varphi-\delta) W^{1-\theta}\right]^{\frac{1}{1-\theta}}$$

As W results from the ratio of the marginal utility of real money holdings and the marginal utility of consumption of non tradables, we correctly infer that W_t is simply the right hand side of (B.16). Thus the consumption based price index adopts the form:

$$P_t = P_{h,t} \left[\varphi + \delta (e_t (1 + T_{m,t}))^{1-\theta} + (1-\varphi-\delta) \left(\frac{i_t}{1+i_t}\right)^{1-\theta}\right]^{\frac{1}{1-\theta}} \tag{C.3}$$

Expression (C.3) can be used to compute the evolution of the "general" price level, once the other prices are determined.

If (C.1) were Cobb-Douglas (i.e. θ=1), (C.4) converges to:

$$P_t = P_{h,t}\left[1 + \left(e_t\left(1+T_{m,t}\right)\right)^{\delta} + \left(\frac{i_t}{1+i_t}\right)^{1-\varphi-\delta}\right] \tag{C.4}$$

A trivial extension to (C.3) for the case of the foreign economy is analyzed by Obstfeld and Rogoff (1996). In that case, we define the consumption-based price index as:

$$P_t^* = P_{m,t}^*\left[\varphi^* + \left(1-\varphi^*\right)\left(\frac{i_t^*}{1+i_t^*}\right)^{1-\theta^*}\right]^{\frac{1}{1-\theta^*}} \tag{C.5}$$

where the values of the parameters with superscripts correspond to those of the foreign consumers.

D The Efficient Method of Moments

This Appendix is based on Chumacero (1997) and present a brief introduction to the type of Efficient Method of Moments (EMM) estimators that are used in the paper.[25]

Consider a stationary stochastic process $p(y_t | x_t, \rho)$ that describes y_t in terms of exogenous variables x_t and structural parameters ρ which the researcher is interested in estimating. Consider an auxiliary model $f(y_t | x_t, \theta)$ that has an analytical expression whereas the $p(\cdot)$ density may not. Gallant and Tauchen (1996) propose to use the scores

$$\left.\frac{\partial \ln f(y_t | x_t, \theta)}{\partial \theta}\right|_{\widehat{\theta}_T}$$

where

$$\widehat{\theta}_T = \arg\max_{\theta \in \Theta} \sum_{t=1}^{T} \ln f(y_t | x_t, \theta)$$

is the quasi-maximum likelihood estimator of $f(\cdot)$ for a sample of size T, to generate the GMM moment conditions

$$m_T(\rho) = \iint (\partial/\partial\theta) \ln f\left(y \middle| x, \widehat{\theta}_T\right) p(y | x, \rho)\, dy\, p(x | \rho)\, dx$$

In cases in which analytical expressions for these integrals are not available, simulation may be required to compute them; in that case we define the moments as:

$$m_N(\rho) = \sum_{n=1}^{N} (\partial/\partial\theta) \ln f\left(\widetilde{y}_n \middle| \widetilde{x}_n, \widehat{\theta}_T\right)$$

where N is the sample size of the Monte Carlo integral approximation drawn from one sample of y, x generated for a given value of ρ in the structural model.

The GMM estimator of ρ with an efficient weighting matrix is given by:

$$\widehat{\rho}_N = \arg\min_{\rho \in \mathcal{R}} m'_N(\rho)\, \widehat{W}_T^{-1} m_N(\rho) \tag{D.1}$$

where, if the auxiliary model constitutes a good statistical description of the data generating process of y, the outer-product of the gradients (OPG) can be used in the weighting matrix; that is:

$$\widehat{W}_T = \frac{1}{T} \sum_{t=1}^{T} \left[(\partial/\partial\theta) \ln f\left(y_t \middle| x_t, \widehat{\theta}_T\right)\right] \left[(\partial/\partial\theta) \ln f\left(y_t \middle| x_t, \widehat{\theta}_T\right)\right]'$$

[25] The interested reader is referred to Gallant and Tauchen (1996) for a complete and formal treatment of this an other setups in which EMM can be applied. Chumacero (1997) presents Monte Carlo evidence that shows that this technique is superior to GMM in several dimensions.

Gallant and Tauchen (1996) demonstrate the strong convergence and asymptotic normality of the estimator presented in (D.1) as well as the asymptotic distribution of the objective function that $\widehat{\rho}_N$ minimizes. That is, let k by the dimension of ρ and q the dimension of θ, then:

$$TJ_T = Tm_N'\left(\widehat{\rho}\right)\widehat{W}_T^{-1}m_N\left(\widehat{\rho}\right) \xrightarrow{\mathcal{D}} \chi_{q-k}^2$$

which corresponds to the familiar overidentifying restrictions test described by Hansen (1982). As in GMM, identification requires that $q > k$. Thus, statistical inference may be carried out in identical fashion as in GMM. However, depending on the complexity of the auxiliary model, Wald type tests based on the variance-covariance matrix obtained by differentiating the moments may be difficult to construct.

One major advantage of EMM is that the econometrician does not need to observe directly all the variables in the structural model to compute ρ. This feature is extremely attractive, because in many cases the poor small sample performance of GMM estimators is due to the limited amount of observations that the econometrician has in order to estimate the structural model.