Modeling Irrigation Agriculture in Bolivia

by
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Abstract

This paper presents a methodology developed for the National Program of Irrigation of Bolivia (PRONAR) in order to provide its technical staff with tools allowing them to formulate, monitor and assess irrigation investment projects. This methodology intends to model farming strategies identified by means of a qualitative inquiry conducted in four different rural areas of Bolivia. We focus our attention on one of the five farming strategies modeled in our project, namely: subsistence farming with production of cash crops. This choice allows us to illustrate the main features of our methodology by avoiding technicalities that can prevent a clear understanding of its scope.

We model, in a dynamic framework, the assignment of irrigation water on the basis of the utility that farmers perceive from changing the composition of their crops. From these behavioral choices, we derive factor demand and production using production functions. Finally, the profitability of farming activity is determined by assuming random sale crop prices, in order to assess the price-risks of irrigation investment projects.

We conclude by presenting and commenting on the results of a simulation experiment performed with a model calibrated using Bolivian data collected at a micro-regional level in order to assess an actual investment project carried out by PRONAR in the Bolivian altiplano of the Department of La Paz.

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1. Introduction

Water is along with land and labor one of the essential factors of agriculture. Without rain, that directly provide fields with water, or without rivers, that carry water to cultivated soils, agriculture would be impossible. When rain is abundant and well spread out over the year, as it is the case for the colonies located in the Bolivian Amazon, north of the city of Santa Cruz, it is possible to farm, with no additional irrigation, cereals as rice, during the wet season, and leguminous plants as soy or beans, during the dry season. When rain is scarce, like in the central valley of Cochabamba, the peasants are prepared to take any effort to provide them with the necessary water for farming. For this purpose, it is instructing to read the testimony of Celestino Rodríguez (Gerbrandy and Hoogendam, 1998, pp. 172-180), a leader of peasants from this valley, reporting the prospect for lagoons in the mountains, then the organization of the community' labors to build the civil works, and, finally, the democratic search for institutional rules necessary to manage the communal irrigation system.

Availability of irrigation water, as much for preparing labored land as for increasing the yield, is a crucial factor of peasants' welfare. Indeed, comparing the incomes of two peasants' families in the valley of Cochabamba, Vega (1996) found that a family that benefit from abundant irrigation water reaches production levels valued at market prices beyond 50'000 bolivianos, whereas a family with limited access to irrigation water hardly reaches 12'000 bolivianos. Irrigation facilitates market gardening and therefore stimulates the transition from subsistence to commercial farming by allowing the peasants' families to substitute the traditional farming of maize and potatoes for self-subsistence with green vegetables that can be sold on the urban markets. In this way, monetary profitability of agriculture becames the driving force of farming activity and the price-ratios of green vegetables on urban markets, the information on which farmers base crops selection.

Fully aware of the stake that a well-developed irrigation system represents for the economic development of the country, the Bolivian Government started, at the end of the last decade, a National Program of Irrigation (PRONAR) aimed at overcoming the productivity limits of an agriculture submitted to the vagaries of weather. This program funded, with the financial support of the Inter-American Bank of Development, the construction and the operation of some 50 irrigation systems, located all-over the country, from the 4'000 meters high altiplano down to the low lands of Chaco. These achievements gave rise to a set of questions that needed to be answered. What is the short, medium and long run impact of incremental water supply over products selection, productivity, profitability as well as on employment? Which role can the incremental water supply play in the transformation of peasants into labor demand farmers? Can the irrigation systems contribute to curb the migration of peasants' population towards the cities? What is the impact of irrigation systems on the carrying capacity of productive lands?

In order to be able to answer such questions, we have been mandated by PRONAR to develop quantitative dynamic models allowing their technical staff to formulate, monitor and assess irrigation investment projects. For this purpose, we have developed. (Carlevaro and Loza, 2002) simulation models describing five different farming strategies, identified by means of a qualitative inquiry conducted in different rural areas of Bolivia, namely: (1) subsistence farming with production of cash crops; (2) subsistence farming combined with cattle farming; (3) subsistence farming combined with commercial fruit production; (4) commercial farming irrigated by gravity; (5) commercial farming irrigated by sprinklers.

We model, in a dynamic framework, the assignment of irrigation water on the basis of the utility that farmers perceive from changing the composition of their crops. From these behavioral choices, we derive factor demand and production using production functions. Finally, the profitability of farming activity is determined by assuming random sale crop prices, in order to assess the price-risks of irrigation investment projects.

This paper presents one of these models that accounts for the farming strategy actually used by the majority of Bolivian peasants. These peasants farm land primarily to provide food to their own families, and subordinately to sell excess production on markets in order to finance the purchase of products that cannot be produced at the farm. The choice of this model also allows us to illustrate the main features of our modeling methodology by avoiding technicalities that can prevent a clear understanding of its scope.

We start our presentation by describing the main characteristics and objectives of the farming strategy we model; then we present and explain the logic of the formal model; finally, we conclude by presenting and commenting on the results of a simulation experiment performed with a model calibrated by Bolivian data collected at a micro-regional level in order to assess an actual investment project carried out by PRONAR in the Bolivian altiplano of the Department of La Paz.

2. Subsistence farming in Bolivia

The primary objective of peasant agriculture is to provide peasants’ families with the foodstuff necessary to satisfy their subsistence needs. Therefore, production is organized to achieve this very vital goal. To tackle the many tasks of the farming cycle, the peasant must use products that cannot be produced at the farm, like improved and certified seeds, fertilizers and treatments. Further, he must provide his family with manufactured goods such as sugar, coffee or tea, clothes, school supplies and books for children, and others products of the consumer basket. Thus, he has to raise money income by selling the excess of his subsistence food production as well as products originally intended for local fairs (cash crops).

This farming strategy can be observed everywhere in Bolivias’ countryside. For instance, during our inquiry conducted in different rural areas of the country, we met peasants in the Chaco sowing potatoes in rotation with maize as subsistence crops, and vegetables like tomato, onion and carrot, as cash crops for the nearby markets of Villamontes or Yacuiba. Likewise, in Punata, located in the Department of Cochabamba, we visited farms practicing the same subsistence farming combined with commercial production of flowers or green vegetables. The peasants of Qollana, in the altiplano of La Paz, provide another example of this kind of farming strategy, as they sow potatoes in rotation with beans, quinua, corn and barley, for their subsistence needs, and produce onions specifically for the market. At last, we mention the farmers of Santa Ana, in Vallegrande, that farm for subsistence potatoes in rotation with maize or corn, and growth green vegetables for cash.

3. An explanatory model of irrigated subsistence farming with production of cash crops

According to the above description of the characteristics of peasant agriculture in Bolivia, we model subsistence farming by assuming that irrigation water is first assigned to the production of subsistence crops. Only excess water, not used to satisfy this primary goal, is then used to produce cash crops.

Our model intends to describe the dynamics of the subsistence farming when the size of peasant families is growing, while assuming unchanged farming technology as well as subsistence crops composition. As a consequence of family growth, the production of subsistence crops should increase, and with a fixed quantity of available irrigation water, the supply of cash crops to markets by peasantry will eventually decrease.

We formulate our model as a mixed set of dynamic and static equations of the type suggested by Simon and Iwasaki (1988), in particular. Our formal presentation will use capital letters to denote explained or dependent variables, lower-case letters to indicate non-negative parameters and dotted variables for time derivatives[2].

3.1. Family subsistence needs

The first equation, namely:

$$\dot{N} = r_1(r_2 - N)N \qquad (1)$$

explains the evolution of the size N of the family as determined by an instantaneous growth rate that linearly decreases towards zero when the family size increases. The limit to family growth is reached when the family size approaches the carrying capacity of the natural environment where the peasant family is settled. Such a limit, that also depends on the farming technology used by peasant agriculture and, in particular, on the available volume of irrigation water, is represented in equation (1) by means of parameter r_2. This equation generates the well-known logistic growth curve.

The growth of family size determines the demand for subsistence food. We quantify this demand through the quantities E of calories required to sustain the family size N. Therefore:

$$E = r_3 N \quad . \qquad (2)$$

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[2] As a consequence of the seasonality of agriculture, it would have been more natural to specify time as a discrete variable. We have chosen a continuous representation to simplify the mathematical analysis of the solutions to our models.

This quantification obviously assumes the existence of an equivalence scale allowing to measure the size of any member of the family according to the same unit, based on the calorie needs of individuals.

The demand of subsistence food is met by means of *n* crops, whose composition is assumed to be determined by the traditional tastes of the community and by the nutritional contribution to the peasants diet. Therefore, we specify the family demand for subsistence crops $\overline{Y}_{1i}$, $i = 1,\ldots,n_1$, according to the following fixed share relations:

$$E_i = r_{4i}E\,; \quad i = 1,\ldots, n_1; \qquad \sum_{i=1}^{n_1} r_{4i} = 1 \tag{3}$$

$$\overline{Y}_{1i} = r_{5i}E_i\,; \quad i = 1, \ldots, n_1 \tag{4}$$

where E_i stands for the demand of subsistence crop *i* measured in terms of calorie content, and r_{5i} for the unit energy content of the crop.

3.2. Assigning available irrigation water to crops

As far as farming is concerned, we assume that the production of crops is restrained by the availability of irrigation water rather than by land. Therefore, the quantity of irrigation water assigned to growth subsistence crops, W_1, and the one allocated to the cash crops production, W_2, should add-up to the seasonal quantity of irrigation water, w_0, available for farming:

$$W_1 + W_2 = w_0 \quad . \tag{5}$$

To be consistent with the above stated assumption of an exogenous determination of subsistence needs, we assign the water resource W_1 to the production of each single subsistence crop according to fixed shares complying with community tastes and production technology:

$$W_{1i} = r_{6i}W_1\,; \quad i = 1,2, \ldots, n_1; \qquad \sum_{i=1}^{n_1} r_{6i} = 1\,. \tag{6}$$

The allocation of available water between subsistence and cash crops is modeled through a dynamic decision process that rescales such an allocation at the beginning of each farming season according to the gap between the actual production of subsistence crops, Y_{1i} , $i=1,\ldots,n_1$, and the corresponding demand $\overline{Y}_{1i}$, $i=1,\ldots,n_1$. More precisely, if the actual subsistence crops production exceeds demand, part of the irrigation water assigned to subsistence crops production will be transferred to cash crops production and vice versa. This adjustment process is expressed by the following differential equation:

$$\dot{W}_1 = a_1 \left(W_2 \frac{U_{21}}{U_{12}+U_{21}} - W_1 \frac{U_{12}}{U_{12}+U_{21}} \right) W_1 \tag{7}$$

where:

$$U_{12} = exp\left(b_{12} \sum_{i=1}^{n_1} \frac{1}{r_{5i}} \left(Y_{1i} - \overline{Y}_{1i} \right) \right) \tag{8}$$

$$U_{21} = exp\left(b_{21} \sum_{i=1}^{n_1} \frac{1}{r_{5i}} \left(\overline{Y}_{1i} - Y_{1i} \right) \right) \quad . \tag{9}$$

are farmer's preferences to improve cash crop production or subsistence crops production respectively.

To assign the residual quantity of water W_2 to the production of n_2 different cash crops, according to the resource constraint:

$$\sum_{i=1}^{n_2} W_{2i} = W_2 \ , \tag{10}$$

we rely on an adjustment process similar to (7), by assuming that allocations W_{2i} of water to the n_2 cash crops are revised at the beginning of each farming season according to the relative profitabilities of the crops. More specifically, if expected profitability of cash crop *i* exceeds that of cash crop *j*, part of the irrigation water previously assigned to cash crop *j* will be transferred to cash crop *i* and vice versa. This assumption leads to the following system of n_2-1 differential equations:

$$\dot{W}_{2i} = a_{2i} \left(\frac{1}{\sum_{j=1}^{n_2} U_{2ij} + \sum_{j=1}^{n_2} U_{2ji}} \left(\sum_{j=1}^{n_2} W_{2j} U_{2ji} - W_{2i} \sum_{j=1}^{n_2} U_{2ij} \right) \right) W_{2i} \, , \qquad i=1,\ldots,n_2-1, \tag{11}$$

where :

$$U_{2ij} = exp\left(b_{2ij} \left(v_j - v_i \right) \right) \ , \quad i \neq j \ , \quad U_{2ii} = 0 \ , \quad i,j=1,\ldots,n_2 \ , \tag{12}$$

are farmer's preferences to improve the production of cash crop *j* at the expense of cash crop *i*, according to the difference between their profitabilities defined as:

$$v_i = \frac{B_{2i}}{W_{2i}} \quad , \tag{13}$$

with B_{2i} standing for the profit of cash crop *i*. As a consequence of the assumed production technology, presented later on, these profitabilities are expressions of structural parameters only. For this reason, we denote them with lower-case letters. Of course, such derived parameters can be of any sign.

3.3. Production technology and the profitability of peasant farming

The production of a crop requires resources other than irrigation water. At the most aggregate description level of the farming process, we distinguish, besides irrigation water W_{hi} required to produce crop *i* of type *h* (*h*=1 for subsistence crop and *h*=2 for cash crop), four other production factors, namely: labored land surface ready to be sown S_{hi}, farming works to maintain the fertility of soil (weeding and treatments) M_{hi}, labor for sowing, irrigating and harvesting T_{hi}, tools and materials to perform cultural works H_{hi}. Clearly such broad categories of factors are all complementaries and contribute, therefore, to actual production Y_{hi} of crop *i* of type *h* according to the Leontief-Walras production function:

$$Y_{hi} = min\left\{\frac{W_{hi}}{d_{1hi}};\frac{S_{hi}}{d_{2hi}};\frac{M_{hi}}{d_{3hi}};\frac{T_{hi}}{d_{4hi}};\frac{H_{hi}}{d_{6hi}}\right\}.$$

Then, with a volume W_{hi} of irrigation water, a non-wasting farm can achieve the production level:

$$Y_{hi} = \frac{W_{hi}}{d_{1hi}} \tag{14}$$

and uses the following quantities of the other four production factors:

$$S_{hi} = d_{2hi}Y_{hi} \tag{15}$$

$$M_{hi} = d_{3hi}Y_{hi} \tag{16}$$

$$T_{hi} = d_{4hi}Y_{hi} \tag{17}$$

$$H_{hi} = d_{5hi}Y_{hi} \qquad . \tag{18}$$

Among these broad categories of factors, the first two are intermediate products, requiring the use of primary factors. The preparation of a surface A_{hi} of land for sowing requires labor L_{hi} and services of tools and machines K_{hi} according to the production function:

$$S_{hi} = min\left\{\frac{L_{hi}}{c_{1hi}};\frac{K_{hi}}{c_{2hi}};A_{hi}\right\}.$$

Therefore, a non-wasting production of this intermediate product requires the following use of primary factors:

$$L_{hi} = c_{1hi}S_{hi} \qquad (19)$$

$$K_{hi} = c_{2hi}S_{hi} \qquad (20)$$

$$A_{hi} = S_{hi} \ . \qquad (21)$$

Farming works to maintain the fertility of soil, can benefit from the substitutability of labor by chemicals, at least within a given range. We analytically specify these substitution possibilities according to a constant return Cobb-Douglas production function:

$$M_{hi} = J_{hi}^{e_{hi}}Q_{hi}^{1-e_{hi}} ,$$

where J_{hi} stands for the quantity of labor, Q_{hi} for the quantity of chemicals for treatments and $0 < e_{hi} < 1$ the constant elasticity of the intermediate product M_{hi} with respect to labor J_{hi}. From this production function, we derive factor demands for J_{hi} and Q_{hi} by minimizing the cost of producing M_{hi}. This cost is part of the cost of producing crop output Y_{hi}:

$$C_{hi}=w_{1hi}L_{hi}+w_{2hi}J_{hi}+w_{3hi}T_{hi}+q_{1hi}K_{hi}+q_{2hi}H_{hi}+q_{3hi}Q_{hi}+q_{4hi}W_{hi} , \qquad (22)$$

valued at market prices $w_{1hi}, w_{2hi}, w_{3hi}, q_{1hi}, q_{2hi}, q_{3hi}, q_{4hi}$[3]. Solving first order conditions of this constrained optimization problem straightforwardly leads to the following factor demand relations:

$$J_{hi} = \left(\frac{e_{hi}}{1-e_{hi}}\right)^{1-e_{hi}}\left(\frac{q_{3hi}}{w_{2hi}}\right)^{1-e_{hi}} M_{hi} = \bar{j}_{hi}\, M_{hi} \qquad , \qquad (23)$$

$$Q_{hi} = \left(\frac{1-e_{hi}}{e_{hi}}\right)^{e_{hi}}\left(\frac{w_{2hi}}{q_{3hi}}\right)^{e_{hi}} M_{hi} = \bar{q}_{hi}\, M_{hi} \qquad . \qquad (24)$$

Inserting above defined primary factor demands into the cost identity (22) leads to the following total cost function for crop *i* of type *h*, displaying constant unit cost:

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[3] For practical reasons, we have been led to measure tools depreciation, materials and machines rentals in money; as a consequence: $q_{1hi} = q_{2hi} = q_{3hi} = 1$. Furthermore, in our applications, irrigation water is not sold to farmers; therefore $q_{4hi} = 0$.

$$C_{hi} = \bar{c}_{hi} Y_{hi} . \tag{23}$$

with $\bar{c}_{hi} = w_{1hi} c_{1hi} d_{2hi} + w_{2hi} d_{3hi} \bar{j}_{hi} + w_{3hi} d_{4hi} + q_{1hi} c_{2hi} d_{2hi} + q_{2hi} d_{5hi} + q_{3hi} d_{3hi} \bar{q}_{hi} + q_{4hi} d_{1hi}$.

Substracting this cost to market value of production output Y_{hi}, finally leads to farming profit per crop:

$$B_{hi} = p_{hi} Y_{hi} - C_{hi} \quad , \tag{25}$$

as well as to consolidated profit of peasant farming valued at market prices[4]:

$$B = \sum_{h=1}^{2} \sum_{i=1}^{n_h} B_{hi} \quad . \tag{26}$$

The parametric expression of cash crop profitabilities (13) is obtained by dividing farming profit (25) by irrigation water demand (14). This leads to:

$$v_i = \frac{p_{2i} - \bar{c}_{2i}}{d_{12i}} . \tag{27}$$

4. Assessing the improved irrigation system of Qollana

In this section, we apply our model to the assessment of an investment project carried out recently by PRONAR in Qollana, a community of 3,000 inhabitants located in the Bolivian altiplano of the Department of La Paz. The investment were intended to improve the rustic irrigation system benefiting a group of 60 families of the community, by building a dam a few meters high in a narrow arm of a canyon, in order to create a small water reservoir, and by retrofitting the rustic channels with concrete ducts or plastic pipes avoiding seepage water losses in the ground.

We start with the presentation of the demographic, nutritional, hydrous, agricultural and economic data used to calibrate the model; then we display and comment on the results of a simulation performed to assess the economic consequences of the project and the profitability of the investment.

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[4] We value crops production using prices at farm, namely: wholesale market-prices net of transport costs and trade margins.

4.1. Data set and calibration of the parameters

Population dynamics

To estimate the growth of the 60 families (234 individuals) benefiting by the PRONAR irrigation project over the 10 first years of exploitation (reference period for the project assessment), we referred to the average annual rate of growth observed during the last two national censuses for the population size of similar communities in the province of Aroma, where Qollana is located. By assuming a decreasing rate of growth, in accordance with the logistic growth postulated in our model, this procedure generates the estimated evolution of the population size of the above-mentioned group of recipients, shown in *Table 1*.

Table 1
Estimated recipient's population growth

Community	*Rate of growth*	*Year*	*Population*	*Mcal/year*
		0	*234*	*191176*
Umala:	*0.0402*	*2*	*253*	*206855*
Qollana:	*0.0399*	*4*	*274*	*223692*
Calamarca:	*0.0238*	*6*	*287*	*234466*
Ayo-ayo:	*0.0093*	*9*	*295*	*241069*

Using these estimates as observations, we then calibrated parameters r_1 and r_2 of the logistic curve generated by equation (1), by fitting this growth curve to the observations according to the method of least squares. The result of this fit is displayed in *Table 2*.

Table 2
Fitted parameters of model equation (1)

Parameter	*Estimate*	*Unit of measurement*
r_1	*1.2E-03*	*1/capita*
r_2	*295*	*capita*

Demand for subsistence food

The demand for subsistence food in calories is derived from the logistic growth of recipients' population according to equation (2). To perform such a computation, we calibrated parameter r_3 using an average value of the annual per capita demand for subsistence food measured in terms of its calorie content. We used the sex and age composition of Qollana's population observed during the national census of 2001 to derive the total demand for subsistence food calories of the population of recipients shown in *Table 3*. The coefficients of per day demand for calories by sex and age used in this computation are taken from Abela and Pérez (1997). Once divided by the number of recipients of the project, the estimated annual total energy demand of 191 Gcal leads to an estimate of $r_3 = 817\,\text{Mcal}/\text{capita}$.

Table 3
Subsistence food demand of recipient's population
(kcal/day)

Age category (year)	*Estimated composition*			*Food demand*		*Total demand of food*
	Total	*Men*	*Women*	*Men*	*Women*	
Less than 1	*6*	*4*	*2*	*783*	*742*	*4738*
1 to 3	*18*	*8*	*10*	*1367*	*1204*	*23384*
4 to 6	*24*	*11*	*12*	*1684*	*1593*	*38694*
7 to 9	*21*	*11*	*10*	*1948*	*1704*	*38198*
10 to 12	*21*	*12*	*9*	*2135*	*1935*	*43893*
13 to 15	*17*	*9*	*7*	*2516*	*2186*	*39740*
16 to 17	*7*	*4*	*3*	*2785*	*2272*	*18968*
18 to 29	*28*	*12*	*16*	*3274*	*2320*	*77014*
30 to 59	*60*	*27*	*33*	*3247*	*2348*	*164824*
60 and more	*31*	*15*	*16*	*2652*	*2098*	*74317*
Total	*234*	*114*	*120*			*523770*

Barley, beans, potato, quinua and corn are the subsistence crops sowed by Qollana's peasants. These foodstuffs provide them with energy, proteins, fibers and vitamins necessary for an equilibrated diet and an appropriate physical and intellectual development. The unit energy content of these subsistence crops necessary for estimating parameters r_{5i} entering in model equations (4) are taken from Guzmán and Villegas (1993). They are displayed in *Table 4* together with unit protein contents and subsistence energy demand shares r_{4i} calibrated by identifying them to the corresponding shares in total subsistence energy production observed in Qollana before PRONAR's investment[5].

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[5] Starting from observed production of subsistence crop i, Y_{1i} , we compute the energy content E_i according to relation (4), using Table 4 estimate of parameter r_{5i} . Calibrated r_{4i} is the share of E_i in total subsistence energy production $\sum E_i$.

Table 4
Nutritional content of subsistence crops
(for 100 gr. of eatable foodstuff)

Crop	*Energy (kcal)*	*Proteins (gr)*	r_{4i}	r_{5i} *(t/Mcal)*
Barley	*373*	*11.8*	*0.201*	*0.000268*
Beans	*93*	*11.38*	*0.288*	*0.001075*
Potato	*125*	*2.75*	*0.194*	*0.000800*
Quinua	*377*	*11.89*	*0.173*	*0.000265*
Corn	*335*	*10.08*	*0.144*	*0.000299*

Supply and demand for irrigation water

During the wet season, from September to April, the retrofitted channels carry, from the dam to the farmed parcels, a water supply of 25 liters per second (l/s). During the dry season, the supply of irrigation water is lower, dropping down to 16 l/s in June, the month of lesser flow. The retrofitting of channels has significantly improved their conduction efficiency, from 20% to 42%, by increasing the monthly supply of water from 12830 to 27216 cubic meters (m^3). This is the volume w_0 of irrigation water, entering in equation (5), that Qollana's recipients assign to the different subsistence and cash crops they grow. This water availability allows them to irrigate only a small part of their parcels, namely 1/3 to 1 hectare per family, during a farming season that lasts no more than six to eight months as a consequence of the extreme climatic conditions of the Bolivian altiplano.

Table 5
Irrigation water supply

Parameter	*Value*	*Unit of measurement*
Conduction capacity	*25*	*(l/s)*
Conduction efficiency (retrofitted duct)	*.42*	
Conduction efficiency (rustic duct)	*.20*	
Monthly water supply (retrofitted duct)	*27216*	*m^3/month*
Monthly water supply (rustic duct)	*12830*	*m^3/month*

To calibrate the unit consumption of water d_{1hi} to grow crops entering in equation (14), we relied on technical information found in different PRONAR projects. More precisely, we chose the highest monthly lamina of irrigation water assigned to crops in order to secure at any stage of the plant development the quantity of water necessary to their growth. The values of these coefficients for the 6 crops farmed in Qollana, measured in cubic meters of monthly consumption of irrigation water per ton of food crop, are presented in *Table 6* together with shares r_{6i} entering equations (6). The latter are estimated according to the same assumption used to calibrate subsistence energy demand shares r_{4i} , namely by identifying them to the corresponding shares of irrigation water required for subsistence crop production observed in Qollana before PRONAR's investment[6]..

Table 6
Irrigation water demand

Crop	*Lamina (mm)*	*Unit consumption of water (m^3/t)*	r_{6i}
Barley	*58*	*232*	*0.116*
Beans	*97*	*162*	*0.468*
Potato	*73*	*122*	*0.176*
Quinua	*53*	*265*	*0.114*
Corn	*63*	*315*	*0.126*
Onion	*87*	*87*	

Demand for other production factors and costs

To calibrate the parameters of the demand equations (15) to (21) and (23)-(24)) for the other production factors necessary to grow crops, we also relied on technical information found in various PRONAR projects together with others sources of information on altiplano's agriculture. A comparative analysis of this data, pinpoints huge unexplainable differences in capital, labor and materials costs of production, casting doubts on the reliability of some of this information. To prevent the influence of extreme values, we avoided the use of average figures and opted for the choice of modal values among selected figures according to plausibility criteria based on common sense. The selected figures are presented in *Table 7* together with the wage rates used to cost crops production. Parameters of equations (19)-(20) are unit consumptions per hectare (ha), whereas those of equations (15) to (18) are unit consumptions per ton (t) of crop yield. The latter are therefore computed by dividing the former by the average productivity of the corresponding crop.

According to Qollana farmers' practices, soil fertility is maintained without use of chemicals, implying: $Q_{hi} = 0$. In such a polar case $M_{hi} = J_{hi}$ and as a consequence: $e_{hi} = 1$.

Table 7
Demand for production factors and costs

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[6] Here again, starting from observed production of subsistence crop *i*, Y_{1i} , we compute the irrigation water requirement W_{1i} according to relation (14), using Table 6 estimate of parameter d_{11i} .

	Unit of measurement	Barley	Beans	Potato	Quinua	Corn	Onion
<u>Soil preparation:</u>							
Labor (L/S)	*Worked day/ha*	*3*	*8*	*12*	*3*	*2*	*12*
Tools (K/S)	*US$/ha*	*11*	*28*	*110*	*11*	*11*	*116*
<u>Crop production:</u>							
Productivity (Y/S)	*t/ha*	*2.5*	*6*	*6*	*2*	*2*	*10*
Labor for fertility maintenance (M/S)	*Worked day/ha*	*4*	*10*	*7*	*4*	*0*	*16*
Labor for sowing, irrigating and harvesting (T/S)	*Worked day/ha*	*15*	*31*	*43*	*15*	*18*	*61*
Tools and materials for farming (H/S)	*US$/ha*	*27*	*77*	*147*	*27*	*12*	*280*
Wage rate	*US$/Worked day*	*3*	*3*	*3*	*3*	*3.88*	*3*

Crop prices

To value crops yield, we estimated market prices of crops paid at the farm by removing trade margins from monthly wholesale prices of La Paz markets. From data published in the "*Boletín quincenal de precios al por mayor de productos agropecuarios*" we estimated trade margins to be 49% for beans, 40% for potato and 37% for onion. As this information was missing for the other crops (barley, quinua and corn), we guessed a trade margin on the basis of these known data. Using the series of monthly wholesale prices for the period 1996-1999 published in the above mentioned bulletin, we ended up with estimates of average prices and price volatilities, measured by standard deviations, presented in *Table 8*.

Table 8
Crop prices at the farm
(US$/t)

Crop	*Average price*	*Standard deviation*
Barley	*137*	*22*
Onion without tail	*142*	*47*
Fresh beans	*113*	*24*
Potato 1st quality	*182*	*48*
Royal quinua	*550*	*47*
Corn in peeled grain	*356*	*31*

4.2. Assessment methodology and simulation results

To assess the investment project carried out by PRONAR in Qollana from an economic and financial point of view, we simulated the trajectories of the variables of our calibrated model over a period of 10 years from the achievement of the improved irrigation system. This simulation period corresponds to the standard employed by PRONAR for assessing its projects. Simulations were performed using a discrete time version of the model defined by replacing time derivatives with first differences, namely $\dot{X}$ by $X(t+1)-X(t)$, where t stands for number of years[7]. As a consequence of a changing hydrological balance (difference between supply and demand of irrigation water) during the farming year, we split up the farming year into hydrological homogeneous farming seasons and run a model for each one of them. Annual production figures are therefore the result of the consolidation of such seasonal simulations.

Subsistence crop production

Subsistence food demand evolves as a proportion of population size whose growth path is exogenously determined by differential equation (1). In turn, this growth determines the irrigation water requirements that provide, through farming, the quantity of food necessary to satisfy such needs. Simulated evolution of these variables over the assessment time span of 10 years are presented in *Table 9*. They show a growth of ¼ of the size of the population of the 60 recipients families in 10 years, asking for a 6% increase of water needs for irrigating subsistence crops and, therefore, a corresponding decrease of water available to grow cash crops.

Table 9
Population growth, subsistence food demand and water requirements

Time period *Year*	*Population size* *Capita*	*Subsistence food demand* *Mcal/year*	*Water requirements* *m³/month*
0	*234*	*191176*	*20489*
1	*249*	*203789*	*20607*
2	*262*	*213672*	*20742*
3	*271*	*221190*	*20876*
4	*278*	*226780*	*21007*
5	*283*	*230867*	*21136*
6	*286*	*233818*	*21264*
7	*289*	*235930*	*21390*
8	*291*	*237432*	*21513*
9	*292*	*238495*	*21635*
10	*293*	*239244*	*21755*

Once we determine the total volume of water required for farming subsistence crops, our model assigns this resource to each subsistence crop according to the fixed shares r_{6i} presented in *Table 6*. As a consequence of these assignments, we can compute, for each subsistence crop, its demand and that of the other factors of production necessary to grow this foodstuff. In order to illustrate these computations, we present in *Table 10* the evolution of these variables for just one

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[7] A formal analysis of the model solutions in a deterministic environnement is presented in Carlevaro and Loza (2002), chapters 3 and 4.

crop, namely beans. In this table we also present a money measure of the production surplus computed as the difference among a money value of the crop yield based on average market price of *Table 8* and the total labor and capital costs of production. This average market profitability of a crop can be used as an efficiency indicator for the technology of production of subsistence crops.

Table 10
Evolution of beans production

Period	*year*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
Irrigation water	*m³*	*9634*	*9698*	*9760*	*9821*	*9882*	*9942*	*10000*	*10058*	*10115*	*10171*
Crop yield	*t*	*60*	*60*	*60*	*61*	*61*	*61*	*62*	*62*	*63*	*63*
Farmed surface	*ha*	*10*	*10*	*10*	*10*	*10*	*10*	*10*	*10*	*10*	*10*
Soil preparation:											
Labor	*Worked day*	*79*	*80*	*80*	*81*	*82*	*82*	*82*	*83*	*83*	*84*
Tools	*US$*	*278*	*280*	*282*	*284*	*285*	*287*	*289*	*290*	*292*	*294*
Crop production:											
Labor	*Worked day*	*308*	*310*	*312*	*314*	*316*	*318*	*320*	*321*	*323*	*325*
Tools and materials	*US$*	*760*	*765*	*770*	*775*	*779*	*784*	*789*	*793*	*798*	*802*
Total production cost	*US$*	*3163*	*3184*	*3205*	*3225*	*3245*	*3264*	*3284*	*3303*	*3321*	*3340*
Surplus	*US$*	*3571*	*3594*	*3617*	*3640*	*3662*	*3685*	*3706*	*3728*	*3749*	*3770*

Cash crop production

Cash crops farmed in Qollana include, besides the same crops grown for subsistence, a single “pure” cash crop, namely onion. The residual volume of available water resource, once the needs for subsistence crops farming are deducted, is used to produce these cash crops. As explained in section 3.2, the assignment of this volume of water to cash crops is based on their relative profitabilities, namely the differentials of crops unit profits valued at market prices. As a consequence of observed randomness of market prices, profitabilities are unknown to farmer at the beginning of the production period when he has to decide how to assign the available water to the production of cash crops. This implies that water assignment to cash crops is a risky decision, because to each different constellation of anticipated profitabilities corresponds a different water assignment to cash crops, implying changing crop yields, factors demand, production costs and eventually profits.

We deal with this problem by assuming that farmers assign water to cash crops according to the expected value of the distribution of possible water assignments at equilibrium. These are the water assignments computed by replacing dynamic equations (11) with their static counterparts obtained by setting the right end side of these equations equal to zero and solving them jointly with the resource constraint equation (10). From the set of multiple solutions to this equation system, we selected the stable equilibrium with strictly positive water assignments, obtained by solving the following system of linear equations with respect to variables W_{2i}, $i=1,...,n_2$:

$$\sum_{j=1}^{n_2} W_{2j} U_{2ji} - W_{2i} \sum_{j=1}^{n_2} U_{2ij} = 0, \qquad i=1,\ldots,n_2-1,$$

$$\sum_{i=1}^{n_2} W_{2i} = W_2 .$$

As a consequence of the analytical complexity in computing the expected values of this solution, even in the simplified case of cash crop prices independently and identically distributed according to normal distributions with historical means and standard errors displayed in *Table 8*, we simulated them using a Monte Carlo technique.

As an illustrative example of the consequences of this behavioral assumption on a cash crop farming, we present in *Table 11* the evolution of production, factors demand, cost of production and profit for onion. The series of this table displayed in *Figure 1* reflect important random fluctuations of onion production and profitability due to the sampling variability of Monte Carlo estimates of the expected water assignments at equilibrium and of the high market price volatility of this crop.

Table 11
Evolution of onion production

Period	*year*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
Irrigation water	*m³*	*826*	*679*	*1039*	*684*	*582*	*1033*	*870*	*724*	*650*	*595*
Crop yield	*t*	*9*	*8*	*12*	*8*	*7*	*12*	*10*	*8*	*7*	*7*
Farmed surface	*ha*	*1*	*1*	*1*	*1*	*1*	*1*	*1*	*1*	*1*	*1*
Soil preparation:											
Labor	*Worked day*	*11*	*9*	*14*	*9*	*8*	*14*	*12*	*10*	*9*	*8*
Tools	*US$*	*110*	*90*	*138*	*91*	*77*	*137*	*115*	*96*	*86*	*79*
Crop production:											
Labor	*Worked day*	*58*	*48*	*73*	*48*	*41*	*72*	*61*	*51*	*46*	*42*
Tools and materials	*US$*	*266*	*219*	*335*	*220*	*187*	*332*	*280*	*233*	*209*	*191*
Total production cost	*US$*	*666*	*548*	*838*	*551*	*469*	*833*	*701*	*584*	*524*	*480*
Profit	*US$*	*1210*	*995*	*1522*	*1001*	*853*	*1513*	*1274*	*1061*	*951*	*871*

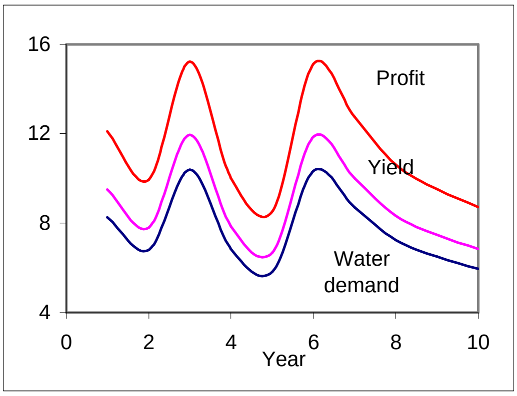

Figure 1. Onion crop: yield, profit (100 US$/year) and water demand (100 m^3/month)

Economic and financial assessment of the project

To assess the economic impact of PRONAR investment in Qollana as well as its financial return, we perform model simulations according to two scenarios: one with no implementation of PRONAR's project (*rustic duct scenario*) and one assuming the implementation of the project (*retrofitted duct scenario*). Using these two simulations, we measure the economic consequences of PRONAR investment by computing the incremental (algebraic) effect of the retrofitted duct scenario on the rustic duct scenario. This incremental effect for the economic activity of irrigated farming in Qollana is presented in *Table 12*. Over the 10 years assessment period, the PRONAR investment increases foodstuffs production to provide food to 15 additional people. This crop yield increase is achieved by extending the irrigated surface devoted to subsistence as well as cash crops by 21 ha (65%) and by creating an additional demand of labor of 628 worked days (112%). The expenses in farming tools and materials also increase by 1943 US$ (115%). Finally, we can globally assess the impact of PRONAR's project on the welfare of Qollana recipients through the increase in the monetary surplus, which amounts to 14624 US$ (163%) with respect to the rustic duct scenario.

Table 12

Economic consequences of PRONAR investment in Qollana

Year		1	2	3	4	5	6	7	8	9	10
Population	Capita	5	8	11	13	14	15	15	15	15	15
Farmed surface	ha	22	21	21	21	21	21	21	21	21	21
Labor for soil preparation	Worked day	114	115	118	117	117	117	117	117	117	116
Labor for crop production	Worked day	511	511	527	517	513	524	518	516	515	512
Total labor demand	Worked day	625	626	645	634	630	641	635	633	631	628
Tools for soil preparation	US$	652	663	707	685	677	690	679	678	677	668
Tools and materials for crop production	US$	1282	1275	1378	1304	1282	1362	1326	1305	1293	1275
Tools and materials total expense	US$	1933	1937	2085	1989	1959	2053	2005	1984	1970	1943
Surplus	US$	15856	15576	15348	15283	15270	15218	15076	14894	14710	14624

Our financial assessment of PRONAR investment is based on the computation of its internal rate of return (IRR). It is the annual discount rate that equalizes the present value of the flow of annual increments in surpluses to the amount of the investment, as illustrated in *Table 13* for the particular random simulation experiment also used to compute the above mentioned economic consequences of the project[8].

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[8] Analytically, the internal rate of return on an investment of amount *I* producing a flow of increments in surpluses *DB(t)* during the assessment time span of length *T*, is the solution with respect to ρ of the following equation:

$$\int_0^T \Delta B(t) e^{-\rho t} dt = I \quad .$$

Another criterion to assess an investment from a financial point of view is represented by the pay-off period on investment, namely the shortest period of time that profits generated by an investment will return the initial investment outlay. This criterion is also a solution of the above equation but with respect to time period *T* rather than ρ, which denotes here the instantaneous interest rate paid to finance the investment.

Table 13
Financial assessment of PRONAR investment in Qollana

Year	*Flow of funds*	*Discount rate*	*Net present value*	*Internal rate of return*
0	*-76866*	*1*	*0*	*0.12*
1	*14023*	*.89*	*12482*	
2	*13757*	*.79*	*10901*	
3	*14079*	*.71*	*9931*	
4	*13957*	*.63*	*8763*	
5	*13931*	*.56*	*7786*	
6	*13609*	*.50*	*6771*	
7	*13488*	*.44*	*5973*	
8	*13450*	*.39*	*5302*	
9	*13459*	*.35*	*4723*	
10	*13553*	*.31*	*4234*	

As we consider random market prices for cash crops, the internal rate of return on the investment is actually a random variable whose distribution can be derived from the one assumed for market crop prices through the mathematical formula used to compute IRR. Due to the complexity of an analytical derivation of the IRR distribution, we estimated a Monte Carlo frequency distribution by first computing the IRR for 30 independent random simulations of the model in both rural duct and retrofitted duct scenarios and then compiling the histogram presented in *Table 14*. To visualize the shape of this frequency distribution we display it in *Figure 2* and compare it with a normal distribution calibrated with the same mean value and standard error of 0.13 and 0.02 respectively. We observe a fairly good agreement of the two distributions and a strong concentration of the computed values of the annual IRR around its mean value. Indeed, 90% of IRR values are located between 15% and 10% and 2/3 between 14% and 11%, suggesting a high no risky return of PRONAR investment. Clearly, this result is due to the high share of incremental water provided by the irrigation project that is assigned, in Qollana, to subsistence crops farming.

Table 14
Monte Carlo frequency distribution of IRR

Upper bound	*Absolute frequency*	*Relative frequency*	*Normal distribution*
0.10	*1*	*0.03*	*0.06*
0.11	*5*	*0.17*	*0.12*
0.12	*6*	*0.20*	*0.21*
0.13	*7*	*0.23*	*0.25*
0.14	*7*	*0.23*	*0.20*
0.15	*2*	*0.07*	*0.11*
0.16	*2*	*0.07*	*0.04*
Total	*30*	*1*	*0.99*

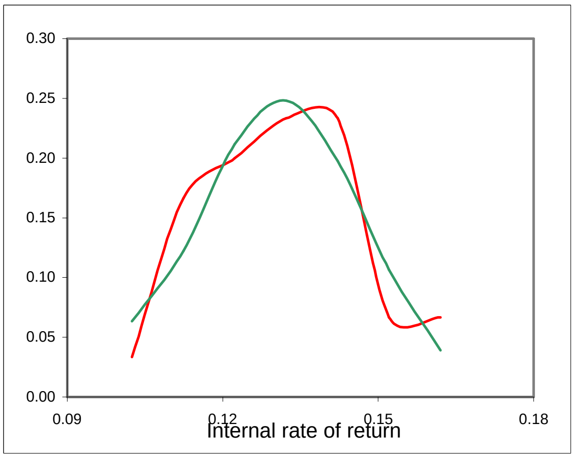

Figure 2. Frequency distribution of internal rate of return

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