Optimisation of strategic investment for market share in a duopoly

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1 Introduction

We outline a simple dynamic model of strategic investment and market share, adapted from the entry deterrence model outlined in [3], which was, in turn, based on the work of Waagstein [5]. Consider a duopoly in which Players I and II have initial market shares m_0 and $1 - m_0$ respectively. Suppose Player I enjoys first-mover advantage and has decided to try to secure an increase in his market share from m_0 to m_1, where $m_1 > m_0$, at time T_1 and thereafter by creating competitive advantage through suitable strategic investments over the period from 0 to T_1. Player II retaliates by investing in the period T_1 to T_2 to increase her market share from $1 - m_1$ to $1 - m_2$, where $m_1 > m_2$, in the period after T_2. Let $\omega > T_2$ denote the planning horizon. This is intended to model a service industry where players seek to maintain or improve their market share by regular enhancements to service quality or attractiveness.

The purpose of the model is the provision to Player I of quantitative support for managerial decision taking about the level and effectiveness of such investments in terms of their impact on present values of anticipated future cash flows.

We include the possibility that $m_0 = 0$ or $m_0 = 1$; in these situations we are studying entry deterrence. If $m_0 = 1$ and Player I cannot increase his market share, then we interpret initial investment by Player I as creating excess capacity. Our results indicate how the possibility of entry varies over time as the value of the market changes and the effect of the potential entrant having multivariable investment opportunities.

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2 Player payoff functions

Let $\mathcal{M}(t)$ be the net present value at time zero of the surplus cash flow from the market in the period (t, ω) after costs, other than those of the strategic investment, have been met. We make no assumptions about the growth of the market or its continuing to be profitable indefinitely into the future [2]. $\mathcal{M}(t)$ plays two roles: first, it determines the total payoff available to the players, in that we assume the players' incentive for seeking increased market share is to obtain a larger share of this surplus. Second, it is the total sum available for the players to commit to strategic investment to gain such an increase.

We assume that each of the players has identified a number of aspects of their business where they believe strategic investment will increase market share. We do not need to assume that these investments are made simultaneously, or in any predetermined order, nor that their effects are independent (though see the footnote on page 2). Let $\boldsymbol{x} = (x_1, x_2, \ldots, x_m)$ be a column vector denoting Player I's strategic investment in areas $1, 2, \ldots, m$ and $X = x_1 + x_2 + \cdots + x_n$ the total investment in the period 0 to T_1 and $\boldsymbol{y} = (y_1, y_2, \ldots, y_n)$ &c. denote Player II's strategic investment in the period T_1 to T_2. In practice the investments would almost certainly be cash flows over time; as with the value of the market, we measure them by their net present value at time 0.

These vectors of investments are non-negative

$$\boldsymbol{x} \geq \mathbf{0}, \qquad \boldsymbol{y} \geq \mathbf{0}$$

in the sense that all their components are non-negative. We will write

$$\boldsymbol{z} > \mathbf{0}$$

for a vector $\boldsymbol{z}$ to mean that all components of $\boldsymbol{z}$ are non-negative with at least one strictly positive. We also give $\leq$ and $<$ the obvious analogous meanings when comparing vectors.

The share m_1 of the market after time T_1 which Player I might hope to gain is clearly a function of $\boldsymbol{x}$ and might also depend on T_1

$$m_1 = m_1(\boldsymbol{x}, T_1)$$

where $m_1(\mathbf{0}, \cdot) = m_0$. We will usually supress the dependence on T_1.

Let us assume that m_1 is increasing and concave along rays in the positive orthant — that is, $(\mathrm{d}/\mathrm{d}\alpha)m_1(\alpha\boldsymbol{x}) > 0$ and $(\mathrm{d}/\mathrm{d}\alpha)^2 m_1(\alpha\boldsymbol{x}) < 0$ for $\alpha > 0$ and $\boldsymbol{x} \geq \mathbf{0}$ — which corresponds to a law of dimishing returns on the investment[1] . Similarly, the share $1 - m_2$ of the market after time T_2 which

[1] In mathematical terms these are equivalent to conditions on directional derivatives of m_1 which can be deduced from weaker conditions on the behaviour of m_1. The simple conditions used here imply, for example, that the areas of investment available to the players are complements rather than substitutes; this could be ameliorated by more subtle mathematical assumptions.

Player II might hope to gain is clearly a function of $\boldsymbol{x}$ and $\boldsymbol{y}$

$$m_2 = m_2(\boldsymbol{x}, \boldsymbol{y})$$

where $m_2(\boldsymbol{x}, \mathbf{0}) = m_1(\boldsymbol{x})$. It could be that m_2 also depends on T_1 and T_2 but we will normally supress this.

Let us assume that m_2 is decreasing and convex along rays in the positive orthant of its second argument — that is, $(\mathrm{d}/\mathrm{d}\alpha)m_2(\boldsymbol{x}, \alpha\boldsymbol{y}) < 0$ and $(\mathrm{d}/\mathrm{d}\alpha)^2(m_2(\boldsymbol{x}, \alpha\boldsymbol{y}) > 0$ for $\alpha > 0$, $\boldsymbol{x} \geq \mathbf{0}$ and $\boldsymbol{y} \geq \mathbf{0}$ — which again corresponds to a law of diminishing returns. Let us also assume that m_2 is increasing and concave along rays in the positive orthant of its first argument — that is, $(\mathrm{d}/\mathrm{d}\alpha)m_2(\alpha\boldsymbol{x}, \boldsymbol{y}) > 0$ and $(\mathrm{d}/\mathrm{d}\alpha)^2 m_2(\alpha\boldsymbol{x}, \boldsymbol{y}) < 0$ for $\alpha > 0$, $\boldsymbol{x} \geq \mathbf{0}$ and $\boldsymbol{y} \geq \mathbf{0}$.

The value of the market to Player I if he makes a total strategic investment X with profile $\boldsymbol{x}$ and Player II invests Y with profile $\boldsymbol{y}$ is

$$\pi_1(\boldsymbol{x}, \boldsymbol{y}) = \underbrace{m_0[\mathcal{M}(0) - \mathcal{M}(T_1)] - X}_{\text{Proceeds up to } T_1} + \underbrace{m_1(\boldsymbol{x})[\mathcal{M}(T_1) - \mathcal{M}(T_2)]}_{\text{Proceeds } T_1 \text{ to } T_2} + \underbrace{m_2(\boldsymbol{x}, \boldsymbol{y})\mathcal{M}(T_2)}_{\text{Proceeds after } T_2} \quad (1)$$

Similarly, the value of the market to Player II if she makes a total strategic investment Y with profile $\boldsymbol{y}$ and Player I invests X with profile $\boldsymbol{x}$ is

$$\pi_2(\boldsymbol{x}, \boldsymbol{y}) = \underbrace{[1 - m_0][\mathcal{M}(0) - \mathcal{M}(T_1)]}_{\text{Proceeds up to } T_1} + \underbrace{[1 - m_1(\boldsymbol{x})][\mathcal{M}(T_1) - \mathcal{M}(T_2)] - Y}_{\text{Proceeds } T_1 \text{ to } T_2} + \underbrace{[1 - m_2(\boldsymbol{x}, \boldsymbol{y})]\mathcal{M}(T_2)}_{\text{Proceeds after } T_2} \quad (2)$$

3 Feasibility conditions

Given that the improvement in cash flow to Player I resulting from the changes in market share is $[m_1 - m_0][\mathcal{M}(T_1) - \mathcal{M}(T_2)] + [m_2 - m_0]\mathcal{M}(T_2)$, his strategic investment is realistic in business terms only if

$$X \leq [m_1 - m_0][\mathcal{M}(T_1) - \mathcal{M}(T_2)] + [m_2 - m_0]\mathcal{M}(T_2) \quad (3)$$

Player II is the follower so she should only attribute the improvement in her cash flow after time T_1, namely $[m_1 - m_2]\mathcal{M}(T_2)$, to the investment $\boldsymbol{y}$ so this is realistic in business terms only if

$$Y \leq [m_1 - m_2]\mathcal{M}(T_2) \quad (4)$$

Notice that if these feasibility conditions are satisfied then the levels of investment are also smaller than the surpluses available to the players.

4 Static optimality conditions

We assume that strategic investment has no influence on the **size** of the market, but only the split between the players. Then we can easily differentiate π_1 and π_2 to give

$$\begin{aligned}
\frac{\partial \pi_1}{\partial x_i} &= \frac{\partial m_1}{\partial x_i}[\mathcal{M}(T_1) - \mathcal{M}(T_2)] + \frac{\partial m_2}{\partial x_i}\mathcal{M}(T_2) - 1 \qquad (5)\\
&\quad \text{for } i = 1, \ldots, m\\
\frac{\partial \pi_2}{\partial y_j} &= -\frac{\partial m_2}{\partial y_j}\mathcal{M}(T_2) - 1 \qquad (6)\\
&\quad \text{for } j = 1, \ldots, n
\end{aligned}$$

so, at a Nash equilibrium, the values of $\boldsymbol{x}$ and $\boldsymbol{y}$ satisfy

$$\begin{aligned}
\frac{\partial m_1}{\partial x_i}[\mathcal{M}(T_1) - \mathcal{M}(T_2)] + \frac{\partial m_2}{\partial x_i}\mathcal{M}(T_2) &= 1 \qquad (7)\\
&\quad \text{for } i = 1, \ldots, m\\
\frac{\partial m_2}{\partial y_j}\mathcal{M}(T_2) &= -1 \qquad (8)\\
&\quad \text{for } j = 1, \ldots, n
\end{aligned}$$

The second equation here is a disguised form of the relationship between the optimal investment decision and the interest rate in Fisher's theory of interest [1]. As in his theory we relate improvements in market share to investment rather than capital.

4.1 Existence of optimal investment levels

Consider first Player II's strategic investment decision. Assume $\mathcal{M}$ is discounted, so that $1/\mathcal{M}(T) \to \infty$ as $T \to \omega$; then either there exist $\boldsymbol{y}$ and T_2 satisfying equation (8) or for some j with $1 \leq j \leq m$

$$\frac{\partial m_2}{\partial y_j} > -\frac{1}{\mathcal{M}(T_2)} \qquad \text{for all } y_j > 0 \text{ and } T_2 > T_1 \qquad (9)$$

In this case $\partial \pi_2 / \partial y_j < 0$ so there is no worthwhile level of investment in area j. The first-mover advantage in this area is so great that Player II cannot profitably respond to the investment made by Player I and Player II should give no further consideration to investing in this area herself; in notational terms, j should be removed from the set $\{1, \ldots, n\}$ and n reduced to $n - 1$.

We can also deduce the strategy which Player I should adopt in order to induce Player II not to invest. The ray convexity conditions on m_2 imply that it is a convex function of each of $y_1, \ldots, y_n$; moreover $\mathcal{M}$ is a decreasing function so inequality (9) is satisfied for every $j = 1, \ldots, n$ if and only if

$$\left.\frac{\partial m_2(\boldsymbol{x}, \boldsymbol{y})}{\partial y_j}\right|_{\boldsymbol{y}=\boldsymbol{0}} > -\frac{1}{\mathcal{M}(T_1)} \qquad \text{for every } j = 1, \ldots, n \qquad (10)$$

If Player I's purpose is to prevent Player II's investment then he should choose $\boldsymbol{x}$ so that these inequalities are satisfied. Of course, such an $\boldsymbol{x}$ is unlikely to be optimal and corresponds to the familiar strategy of a monopolist investing in excess capacity in order to deter entry. Furthermore, if Player I can choose T_1 then he can choose it so that (10) is satisfied for a predetermined value of $\boldsymbol{x}$ since $M(T_2) \to 0$ as $T_2 \to \omega$ and $T_2 > T_1$.

Regarding Player I's optimal investment decision, let us assume for the moment that Player II does not retaliate. In this case $m_2 = m_1$ and so

$$\pi_1(\boldsymbol{x}) = \underbrace{m_0[\mathcal{M}(0) - \mathcal{M}(T_1)] - X}_{\text{Proceeds up to } T_1} + \underbrace{m_1(\boldsymbol{x})\mathcal{M}(T_1)}_{\text{Proceeds after } T_1}$$

so the optimality conditions become

$$\frac{\partial \pi_1}{\partial x_i} = \frac{\partial m_1}{\partial x_i}\mathcal{M}(T_1) - 1 = 0 \quad \text{for } i = 1, \ldots, m \tag{11}$$

Now, either there exist $\boldsymbol{x}$ and T_1 satisfying equation (11) or for some i with $1 \leq i \leq m$

$$\frac{\partial m_1}{\partial x_i} < \frac{1}{\mathcal{M}(T_1)} \qquad \text{for all } \boldsymbol{x} > \mathbf{0} \text{ and } T_1 > 0 \tag{12}$$

In this case $\partial \pi_1 / \partial x_i < 0$ so there is no worthwhile level of investment in area i. Competition in the market is so intense that strategic investment based on exploiting first-mover advantage in this area is unprofitable and Player I should give no further consideration to investing in this area; in notational terms, i should be removed from the set $\{1, \ldots, m\}$ and m reduced to $m-1$. We shall show later that Player II cannot make a credible threat to cause an optimal investment made by Player I to become unprofitable.

Notice that the right-hand side of (12) is an increasing function of T_1. For investment by Player I to be worthwhile over the period 0 to $T_1^* > T_1$ but not over the period 0 to T_1 it is necessary that m_1 be an increasing function of T_1. This is not wholly unrealistic as Player I may be able to make the same levels of investment more effective given a longer period to fully marshal them. On the other hand, this model suggests that market conditions alone cannot cause strategic investment in market share to become viable over the long term if it is not already viable in a shorter term.

5 Some deductions

To progress further, let us assume that the areas under consideration for investment have been reduced to those in which competition is not so intense that there exist $\boldsymbol{x}$, T_1, $\boldsymbol{y}$ and T_2 satisfying (7) and (8); we show that (3) and (4) are also satisfied.

To show that (4) holds, we note that, by the Mean Value Theorem applied to the function

$$f(\lambda) = m_2(\boldsymbol{x}, \lambda\boldsymbol{y})\mathcal{M}(T_2)$$

for any $\boldsymbol{y} > \mathbf{0}$, there exists $\alpha \in (0,1)$ such that

$$\begin{aligned}
& \boldsymbol{y}^\top \left.\frac{\partial m_2}{\partial \boldsymbol{y}}\right|_{(\boldsymbol{x},\alpha\boldsymbol{y})} \mathcal{M}(T_2) \\
&= [m_2(\boldsymbol{x},\boldsymbol{y}) - m_2(\boldsymbol{x},\mathbf{0})]\,\mathcal{M}(T_2) \\
&= [m_2(\boldsymbol{x},\boldsymbol{y}) - m_1(\boldsymbol{x})]\,\mathcal{M}(T_2)
\end{aligned}$$

By the assumed ray convexity of the function m_2 and equation (8)

$$\begin{aligned}
\boldsymbol{y}^\top \left.\frac{\partial m_2}{\partial \boldsymbol{y}}\right|_{(\boldsymbol{x},\alpha\boldsymbol{y})} \mathcal{M}(T_2) &< \boldsymbol{y}^\top \left.\frac{\partial m_2}{\partial \boldsymbol{y}}\right|_{(\boldsymbol{x},\boldsymbol{y})} \mathcal{M}(T_2) \\
&= \boldsymbol{y}^\top(-1,-1,\ldots,-1)^\top \\
&= -Y
\end{aligned}$$

and so

$$Y < (m_1 - m_2)\mathcal{M}(T_2)$$

— that is, the business feasibility condition (4) is satisfied. Indeed, not all the available surplus is invested at the optimal investment level.

To show that (3) holds, we note that, by the Mean Value Theorem applied to the function

$$f(\lambda) = m_1(\lambda\boldsymbol{x})\,[\mathcal{M}(T_1) - \mathcal{M}(T_2)] + m_2(\lambda\boldsymbol{x},\boldsymbol{y})\mathcal{M}(T_2)$$

for any $\boldsymbol{y} > \mathbf{0}$, there exists $\alpha \in (0,1)$ such that

$$\begin{aligned}
& \boldsymbol{x}^\top \left.\frac{\partial m_1}{\partial \boldsymbol{x}}\right|_{\alpha\boldsymbol{x}} [\mathcal{M}(T_1) - \mathcal{M}(T_2)] + \boldsymbol{x}^\top \left.\frac{\partial m_2}{\partial \boldsymbol{x}}\right|_{(\alpha\boldsymbol{x},\boldsymbol{y})} \mathcal{M}(T_2) \\
&= m_1(x)\,[\mathcal{M}(T_1) - \mathcal{M}(T_2)] - m_1(\mathbf{0})\,[\mathcal{M}(T_1) - \mathcal{M}(T_2)] + \\
&\quad [m_2(\boldsymbol{x},\boldsymbol{y}) - m_2(\mathbf{0},\boldsymbol{y})]\,\mathcal{M}(T_2) \\
&= [m_1(x) - m_0]\,[\mathcal{M}(T_1) - \mathcal{M}(T_2)] + \\
&\quad [m_2(\boldsymbol{x},\boldsymbol{y}) - m_0]\,\mathcal{M}(T_2) + \\
&\quad [m_0 - m_2(\mathbf{0},\boldsymbol{y})]\,\mathcal{M}(T_2)
\end{aligned}$$

By the assumed ray concavity of the functions m_1 and m_2

$$\boldsymbol{x}^\top \left.\frac{\partial m_1}{\partial \boldsymbol{x}}\right|_{(\alpha\boldsymbol{x},\boldsymbol{y})} [\mathcal{M}(T_1) - \mathcal{M}(T_2)] > \boldsymbol{x}^\top \left.\frac{\partial m_1}{\partial \boldsymbol{x}}\right|_{(\boldsymbol{x},\boldsymbol{y})} [\mathcal{M}(T_1) - \mathcal{M}(T_2)] \quad (13)$$

and

$$\boldsymbol{x}^\top \left.\frac{\partial m_2}{\partial \boldsymbol{x}}\right|_{(\alpha\boldsymbol{x},\boldsymbol{y})} \mathcal{M}(T_2) > \boldsymbol{x}^\top \left.\frac{\partial m_2}{\partial \boldsymbol{x}}\right|_{(\boldsymbol{x},\boldsymbol{y})} \mathcal{M}(T_2) \quad (14)$$

According to equation (7), the sum of the right-hand sides of (13) and (14) is X and so

$$X < [m_1 - m_0][\mathcal{M}(T_1) - \mathcal{M}(T_2)] + [m_2 - m_0]\mathcal{M}(T_2) + [m_0 - m_2(\mathbf{0}, \boldsymbol{y})]\,\mathcal{M}(T_2)$$

The term $m_0 - m_2(\mathbf{0}, \boldsymbol{y}) = m_2(\mathbf{0}, \mathbf{0}) - m_2(\mathbf{0}, \boldsymbol{y})$ is negative by the ray convexity of m_2 and so the business feasibility condition (3) is satisfied. Indeed, not all the available surplus is invested at the optimal investment level.

This completes the proof that (3) holds.

6 Correspondence to the Chain-Store Game

In reality, the interactions modelled above take place repeatedly, in different geographical locations for example, so there might be the opportunity for one of the players to develop a reputation for, say, aggressive investment behaviour in an attempt to dissuade other players from making certain responses. However, the argument above shows that the single interaction is strategically equivalent to the one-shot Chain Store Game, whether or not Player I has some market share at the outset. This generalises the analysis in [4], showing that a rational player cannot create such a reputation if the number of interactions is bounded.

The basic logic of a single interaction is shown in Figure 6. Here π_1^{Th} and π_2^{Th} are the payoffs to the players if Player II adopts a "threatening" strategy so as to make $\pi_1^{\mathrm{Th}} < 0$, in the hope of deterring Player I from investing. However, he knows that $\pi_2^{\mathrm{Th}} < \pi_2^{\mathrm{Op}}$ — the payoff to Player II when she makes her optimal investment decision — so will not find the threat credible. Hence rational play follows the double lines in Figure 6.

7 Futher work

The single episode model has been tested on data which aims to capture key features of realistic scenarios. The results indicate some of the issues which decision makers in these environments need to address. Details will be included in the full paper.

References

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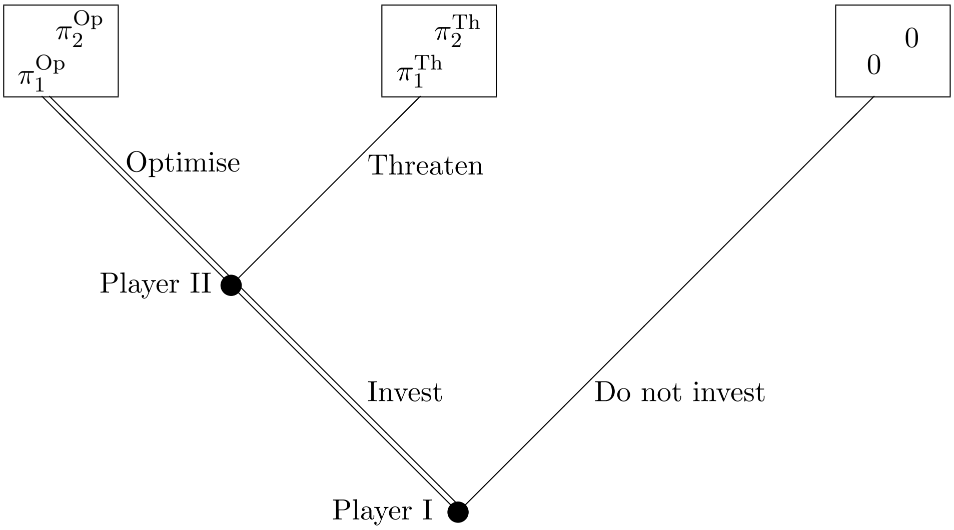
Figure 1: Payoffs in Correspondence to the Chain-Store Game



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