THE ASYMMETRIC EXCHANGE RATE DYNAMICS IN THE EMS: A TIME-VARYING THRESHOLD TEST

Marie BESSEC*

March 31, 2003

Abstract

This paper examines the exchange rate adjustment towards the central parity in the EMS and the role the monetary authorities play in driving the system towards this equilibrium. To this end, we develop a target-zone model where intramarginal interventions occur beyond a band of inaction. We investigate the empirical relevance of this theoretical model by using a time-varying threshold autoregressive model. We implement this analysis on six ERM participants: Belgium, Denmark, France, Ireland, Italy and the Netherlands from March 1979 to December 1998. When we account for the dramatic widening of the official band in August 1993, we obtain results consistent with our theoretical model in five countries.

*bessec@univ-paris1.fr, EUREQua, UMR 8594 du CNRS, Université Paris Panthéon-Sorbonne, 106-112 bd de l'Hôpital 75647 Paris cedex 13, France.

I Introduction

The European Exchange Rate Mechanism (ERM) was founded in March 1979. A major purpose of this system was to reduce real and nominal exchange rate volatility. To this aim, the European currencies were allowed to fluctuate within a band of $\pm 2.25\%$ around an official parity (except for the Italian Lira which initially obtained a margin of $\pm 6\%$ until January 1990). However, the EMS has experienced several crises leading to realignments of the bilateral parities when the central banks intervention was not enough to keep the currencies between the bands and to the broadening of the band to $\pm 15\%$ in August 1993.

The description of the exchange rate behaviour within such a target-zone has been formalized over the ten past years by the target-zone literature. The starting point of this literature is the first-generation model introduced by Krugman in 1991. In the presence of a band, Krugman shows the existence of a non linear S-shaped relationship between the exchange rate and fundamental variables induced by the market expectations. However, an essential drawback of Krugman's specification is the two important assumptions underlying it: the target-zone is perfectly credible and is defended with marginal interventions only that is when the exchange rate hits the edge of the band. Consequently, new models often referred to the second-generation models extend the Krugman model in two alternative directions. Bertola and Caballero (1992), Bertola and Svensson (1993), Tristani (1994) and Torres (2000) allow for a realignment risk. On the other hand, Delgado and Dumas (1991), Klein and Lewis (1993), Lindberg and Söderling (1994a), Tristani (1994) and Lewis (1995) introduce intramarginal interventions occurring when the exchange rate lies inside the band.

Many points give support to this second extension. First, Beetsma and van der Ploeg (1992) point the empirical failure of the Krugman model for the Dutch Guilder, the most credible exchange rate within the EMS from 1987 to 1991, a period during which no realignment occurred and the realignment risk was negligible. Therefore, the introduction of a realignment risk in the Krugman model is not enough. Moreover, from an institutional point of view, the Basle-Nyborg Agreement in 1987 extends credit facilities and allows countries to draw on credit before a currency reaches the limit of its EMS band. At the beginning, these facilities were devoted to the marginal interventions and were to be refunded after 45 days. The Basle-Nyborg Agreement has extended the duration of these loans to 75 days and allowed their use to support intramarginal interventions. At last, many empirical studies (Bini-Smaghi and Micossi, 1989, Dominguez and Kenen, 1992, Lindberg and Söderling, 1994a, Labhard and Wyplosz, 1996) present evidence that central banks defend exchange rate bands by intramarginal interventions, so as to keep the exchange rate well away from the edges.

In this paper, we develop a target-zone model consistent with these observations. We extend the Delgado and Dumas model (1991) to allow for a band of inaction around the central parity in the exchange rate dynamics. To account for intramarginal interventions, Delgado and Dumas introduce mean-reverting fundamentals. This mean-reversion arises whatever the position of the exchange rate in the band. Such a formalisation does not reflect the additional flexibility of the intervention rule with respect to a fixed exchange rate regime. In this model, central banks are assumed to intervene even for small exchange rate deviations from the official parity. Therefore, we develop a representation which combines the Krugman model and the Delgado and Dumas model according to the position of the exchange rate in the band[1]. In the middle of the band, the exchange rate fluctuates freely as suggested in the Krugman model. When the exchange rates are sufficiently far from the central parity, central banks intervene. For these large deviations, the exchange rate dynamics is described by the Delgado and Dumas solution.

To check the empirical relevance of this model, we apply the Self-Exciting Threshold Autoregressive model (SETAR) due to Tong (1978) and Tong and Lim (1980). We extend this specification by allowing a change in the estimated thresholds to capture the potential effect of the changes in the width of the band in January 1990 for the Italian Lira and in August 1993 for all European currencies except for the Dutch Guilder relative to the German Mark. In a three regimes model, if the deviations of the exchange rates from the central parity turn out to be stationary in the outer regimes but exhibit a unit-root behaviour in the corridor regime, it implies the existence of a band of inaction in the exchange rate dynamics, as assumed in the theoretical model. Moreover, the estimated margin gives an indicator of the deviation of the exchange rate from the central parity beyond which central banks interventions drive back the parities towards the target level. These margins can be compared to the official bands. If the official parities encompass the estimated ones, it gives evidence for significant intramarginal interventions. Indeed, it means that the monetary authorities defend an implicit narrower band.

We implement this analysis on six ERM participants: Belgium, Denmark, France, Ireland, Italy and the Netherlands from March 1979 to December 1998. First, we consider a SETAR model with fixed thresholds. In this "fixed" Threshold Autoregressive model, we do not find clear-cut evidence of interventions inside the bands: the estimated margin exceeds the official one in many countries. However, an obvious shortcoming of this fixed SETAR model is the fact that it does not account for the dramatic widening of the band in August 1993. To capture the potential impact of these changes, we estimate the time-varying Threshold Autoregressive model we introduce to allow for the changes in the official band. In this second specification, we find results in line with the existence of a band of inaction and with significant intramarginal interventions inside the target-zone.

The remainder of the paper is organized as follows. In section 2, we present the main theoretical models describing the exchange rate dynamics in a target-zone and introduce on this basis a new representation with intramarginal interventions beyond a band of inaction. Section 3 describes the three regimes threshold autoregressive models we use and develops a strategy of tests to detect a threshold behaviour and a band of inaction in the dynamics of the time series. Section 4 describes the data and presents the results. The last section concludes.

II Target-zone models

II.1 A credible target-zone model with marginal interventions

II.1.1 Model

The basic target-zone (TZ hereafter) model is introduced by Krugman in 1991. He considers a fully credible exchange rate band defended with marginal interventions only.

In this model, the logarithm of the nominal exchange rate at time t, s_t, depends on the fundamental f_t and on the expected change in the exchange rate $E_t(ds_t)/dt$:

$$s_t = f_t + \alpha \frac{E_t(ds_t)}{dt} \tag{1}$$

with $E_t = E[\cdot|\Phi(t)]$ where $\Phi(t)$ is the time-t information set.

To keep the exchange rate in a band $[s_L, s_U]$, the central banks restrict the fundamental to a band $[f_L, f_U]$ through interventions at the boundaries s_L and s_U only. However, the monetary authorities do not intervene when the exchange rate lies inside the band. Thus, the fundamental f_t is assumed to follow a regulated Brownian motion:

$$df_t = \mu dt + \sigma dz_t + dL_t - dU_t \tag{2}$$

where μ and σ are scalar constants and z_t is a standard Wiener process. The regulators dL_t and dU_t are zero except at f_L and f_U:

$$dL_t \begin{cases} = 0 & \text{if } f_t > f_L \\ > 0 & \text{if } f_t = f_L \end{cases} \qquad dU_t \begin{cases} = 0 & \text{if } f_t < f_U \\ > 0 & \text{if } f_t = f_U \end{cases} \tag{3}$$

According to this specification, the fundamental f_t evolves freely, so long as it is located in the interval $[f_L, f_U]$, but the central banks intervene at the boundaries f_L and f_U to prevent the exchange rate from moving outside the band.

II.1.2 Solution

Krugman (1991) and Froot and Obstfeld (1991a,b) show that the general solution to (1) is:

$$s_t = g_K(f_t) = \alpha\mu + f_t + A_1 \exp(\lambda_1 f_t) + A_2 \exp(\lambda_2 f_t) \tag{4}$$

where λ_1 and λ_2 are given by:

$$\lambda_1 = \frac{-\mu - \sqrt{\mu^2 + 2\sigma^2/\alpha}}{\sigma^2} < 0 \qquad \text{and} \qquad \lambda_2 = \frac{-\mu + \sqrt{\mu^2 + 2\sigma^2/\alpha}}{\sigma^2} > 0 \tag{5}$$

Assuming that infinitesimal intervention is expected at the margins, the solution must be tangent to the edges of the band. Therefore, the constants A_1 and A_2 are determined such that the solution (4) is tangent to the limits s_L and s_U of the band. Consequently, they must obey the following conditions (*smooth pasting conditions*):

$$g_K'(f_L) = 0 \qquad \text{and} \qquad g_K'(f_U) = 0 \tag{6}$$

with f_L and f_U the fundamental values at the lower and upper limits of the band, *i.e.* defined by $g_K(f_L) = s_L$ and $g_K(f_U) = s_U$. The constants A_1 and A_2 satisfying (6) are reintroduced in equation (4) to derive the fully credible solution of Krugman.

The solution consists of two components: a linear part $\alpha\mu + f_t$ and a nonlinear one $A_1 \exp(\lambda_1 f_t) + A_2 \exp(\lambda_2 f_t)$. *The linear part* corresponds to the usual solution in the absence of interventions, *i.e.* to the free-float solution. Indeed, under a free-float regime, there is no expected intervention ($A_1 = A_2 = 0$) and the nonlinear part disappears. In particular, in the absence of intervention and if the drift μ is equal to zero, the relation between s_t and f_t is a 45° line, $s_t = f_t$. *The nonlinear part* captures the impact the expected interventions have, when the exchange rate nears the lower or upper limit of the band. These expectations stabilize the exchange rate behaviour within the band. Note that the weight of these terms increases when the fundamentals deviate from the central level, that is[2] when the exchange rate departs from the central parity. On the contrary, the stabilizing expectations are small close to the official parity. For this reason, the first-generation solution will be approximated in the following by the free-float solution, $s_t = f_t$, for small deviations of exchange rate from the central parity.

II.1.3 Limits

However, the model of Krugman is empirically rejected. In particular, the implication of this model on the distribution of the exchange rate within the band is not consistent with the data[3].

The model of Krugman predicts that the exchange rate evolves more often at the edges than in the middle of the band. Indeed, according to this model, the exchange rate reacts strongly to shocks to the fundamentals close to the central parity, whereas it is stabilized at the limits of the band due to the interventions and the expectations of interventions. As a consequence, exchange rate should be more frequently near the limits of the band than in the middle of it. This should imply a U-shaped distribution of the exchange rate observations around the central parity (see Svensson, 1991, for an analytic derivation and a representation of this distribution).

This pattern is not consistent with the data. Bertola and Caballero (1992) reports the histogram for the frequency of the exchange rate deviations from the central parity for the French Franc-German Mark from 1979 to 1987. The exchange rate distribution is found hump-shaped, implying that the exchange rate tends to cluster in the middle of the band and is more variable in the neighbourhood of the TZ's edges. We generalize this result to the whole period and to the main members of the ERM. Figures 1 and 2 show the empirical densities[4] for six daily exchange rates relative to the German Mark[5] in the narrow band of $\pm 2.25\%$ (Figure 1) and in the wide bands of $\pm 6\%$ for the Italian Lira and of $\pm 15\%$ for the other currencies except for the Dutch Guilder[6] (Figure 2). As in Bertola and Caballero (1992), we find hump-shaped distributions in the narrow as well as in the wide exchange rate bands.

An obvious shortcoming of the Krugman specification is the two strong assumptions it is based on. Krugman assumes a fully credible exchange rate band, *i.e.* irrevocably fixed and he only allows for marginal interventions. However, many realignments took place during the EMS period (see Table A.2 in Appendix A.2) and many papers present evidence for significant intramarginal interventions, that is when the exchange rate lies inside the band (Bini-Smaghi and Micossi, 1989, Dominguez and Kenen, 1992, Lindberg and Söderlind, 1994a and Labhard and Wyplosz, 1996).

Consequently, new models often referred to the second-generation models extend Krugman specification in two alternative directions. On the one hand, Bertola and Caballero (1992), Bertola and Svensson (1993), Tristani (1994) and Torres (2000) integrate an endogenous realignment risk. Many of them (Bertola and Caballero, 1992 and Torres, 2000) obtain a "reverse-S solution", in line with the stronger concentration of the observations in the centre of the band. On the other hand, Delgado and Dumas (1991), Lindberg and Söderlind (1994a) and Tristani (1994) combine marginal interventions with interventions inside the band, by introducing a mean-reverting mechanism in the fundamental process. Given the relationship between the exchange rate and the fundamentals, the exchange rate also tends to revert to the official parity. This pattern is consistent with the hump-shaped distribution of the European exchange rates.

In the next section, we examine the model developed by Delgado and Dumas as an alternative to the model of Krugman[7].

II.2 A credible target-zone model with intramarginal interventions

II.2.1 Model

The Delgado and Dumas model is similar to Krugman's one. Delgado and Dumas only modify the equation for fundamentals to allow for intramarginal interventions. However, the band is still assumed to be fully credible.

In this model, interventions take place within the band rather than only at the limits of the band. These interventions are continuous and proportional to the exchange rate departure from the central parity s_0, or, equivalently, to the fundamental deviation from the fundamental level f_0 in s_0. These interventions entail mean-reversion in the fundamental process as soon as it deviates from the targeted level. Consequently, the fundamental process can not be represented by a Brownian motion within the band. Instead, the process for f_t can be modelled as regulated Ornstein-Uhlenbeck.

Formally, the equation (2) for fundamentals in the model of Krugman is replaced by:

$$df_t = -\rho(f_t - f_0) + \sigma dz_t + dL_t - dU_t \tag{7}$$

where the fundamental level f_0 corresponds to the targeted exchange rate level within the band $s_0 = g(f_0)$ and ρ is a positive constant parameter controlling the speed of reversion. The regulators dL_t and dU_t positive at the lower and upper limits of the band still represent the infinitesimal interventions at the edges of the TZ. In contrast to (2), equation (7) includes an additional term $-\rho(f_t - f_0)$ negative when the fundamental is greater than f_0 and *vice versa*. This term induces a reversion of the fundamental towards the equilibrium f_0 (that is a convergence of the exchange rate s_t to the official parity s_0). The equations (1) and (3) of the Krugman model are unchanged.

II.2.2 Solution

Delgado and Dumas (1991) and Froot and Obstfeld (1991a) show that the general solution is now given by:

$$s_t = g_{DD}(f_t) = \frac{f_t + \alpha\rho f_0}{1+\alpha\rho} + A_1 M\left(\frac{1}{2\alpha\rho}, \frac{1}{2}, y\right) + A_2 M\left(\frac{1+\alpha\rho}{2\alpha\rho}, \frac{3}{2}, y\right)\sqrt{y(f)} \tag{8}$$

with $\sqrt{y(f)} = \sqrt{\rho}(f_0 - f)/\sigma$ and $M(.,.,.)$ is the hypergeometric function.

According to the boundary conditions, the constants A_1 and A_2 must satisfy:

$$g'_{DD}(f_L) = 0 \qquad \text{and} \qquad g'_{DD}(f_U) = 0 \tag{9}$$

with $g_{DD}(f_L) = s_L$ and $g_{DD}(f_U) = s_U$.

Again, there are two parts in the general solution (8) : the linear part, $(f_t + \alpha\rho f_0)/(1+\alpha\rho)$, corresponds to the solution in a regime with interventions to drive exchange rate towards s_0, but without bilateral limits and marginal interventions to defend them. The two nonlinear terms, $A_1 M\left(\frac{1}{2\alpha\rho}, \frac{1}{2}, y\right) + A_2 M\left(\frac{1+\alpha\rho}{2\alpha\rho}, \frac{3}{2}, y\right)\sqrt{y(f)}$, still measure the effect resulting from marginal interventions and from the expectations of these interventions. Note that equation (8) reduces to the free float solution (without drift), $s_t = f_t$, in the absence of interventions within as well as at the limits of the band ($\rho = A_1 = A_2 = 0$).

II.2.3 Limits

The specification of Delgado and Dumas provides a better description of the data than the model of Krugman (Lindberg and Söderlind, 1994a and Chung and Tauchen, 2001). However, this model also exhibits shortcomings.

In the model of Delgado and Dumas, intramarginal interventions are assumed to be continuous inside the band. In other words, the central banks are supposed to intervene as soon as the exchange rate deviates from the central parity. However, the monetary authorities are likely to allow small departures from the central parity. The Delgado and Dumas representation does not account for the additional freedom they have with respect to a fixed exchange rate regime. Note that the Krugman model has the opposite drawback, since it assumes that only the large departures are corrected.

Consequently, it seems relevant to combine the model of Krugman for small deviations from the central parity and the model of Delgado and Dumas, when the exchange rate departs significantly from the official parity.

II.3 A mixed representation : a model with a band of inaction and intramarginal interventions

We propose here a "mixed" representation with a band of inaction in the neighbourhood of the central parity and intramarginal interventions.

Suppose that the authorities intervene only when the exchange rate exceeds a band $\left[f'_L, f'_U\right]$ and suppose that this band encompasses the central parity and is included in the official margin $[f_L, f_U]$.

These hypotheses imply that the exchange rate fluctuates freely in a band of inaction $\left[f_L^{'}, f_U^{'}\right]$. Given that this band of inaction is included in the official margin, we allow for intramarginal interventions, but these interventions do not occur systematically, when the exchange rate differs from the central parity. Moreover, suppose to simplify that there is no drift μ in the fundamental process.

Under this assumptions, the relationship between the exchange rate and the fundamental depends on the position of the exchange rate inside the band. When the fundamental lies in the band of inaction $\left[f_L^{'}, f_U^{'}\right]$, the monetary authorities do not intervene. Under these conditions, the Krugman solution is still valid. We have shown above that this solution is well approximated by a 45° line in the neighbourhood of the official parity:

$$s_t = g_K(f_t) = f_t$$

with f_t given in equation (2) without regulator ($dL_t = dU_t = 0$), since the exchange rate lies inside the bilateral limits:

$$f_t = \sigma dz_t$$

Hence, the exchange rate follows a Brownian motion in the neighbourhood of the central parity. Such a pattern is consistent with the free behaviour of the exchange rate in the centre of the band.

On the contrary, when the exchange rate moves out the interval $\left[f_L^{'}, f_U^{'}\right]$, the authorities intervene to drive back the exchange rate to the central parity. In this case, the Delagado and Dumas solution should hold:

$$s_t = g_{DD}(f_t) = \frac{f_t + \alpha\rho f_0}{1+\alpha\rho} + A_1 M\left(\frac{1}{2\alpha\rho}, \frac{1}{2}, y\right) + A_2 M\left(\frac{1+\alpha\rho}{2\alpha\rho}, \frac{3}{2}, y\right)\sqrt{y(f)}$$

where f_t follows the regulated Ornstein-Uhlenbeck process (7) and $\sqrt{y(f)} = \sqrt{\rho}(f_0 - f)/\sigma$.

In sum, under the joint hypothesis of a band of inaction and intramarginal interventions, the general solution of the model is given by:

$$s_t = \begin{cases} \frac{f_t+\alpha\rho f_0}{1+\alpha\rho} + A_1 M\left(\frac{1}{2\alpha\rho}, \frac{1}{2}, y\right) + A_2 M\left(\frac{1+\alpha\rho}{2\alpha\rho}, \frac{3}{2}, y\right)\sqrt{y(f)} & \text{if } f_t > f_U^{'} \\ f_t & \text{if } f_L^{'} < f_t \leq f_U^{'} \\ \frac{f_t+\alpha\rho f_0}{1+\alpha\rho} + A_1 M\left(\frac{1}{2\alpha\rho}, \frac{1}{2}, y\right) + A_2 M\left(\frac{1+\alpha\rho}{2\alpha\rho}, \frac{3}{2}, y\right)\sqrt{y(f)} & \text{if } f_t \leq f_L^{'} \end{cases} \tag{10}$$

Now, the relationship between the exchange rate and the fundamentals differs according to the position of the exchange rate in the band.

As before, boundary conditions are used to determine the unique solution. Since infinitesimal interventions still occur at the limits of the band, the solution must be tangent to the edges of the TZ (*smooth pasting conditions*):

$$g_{DD}^{'}(f_L) = 0 \qquad \text{and} \qquad g_{DD}^{'}(f_U) = 0 \tag{11}$$

with $g_{DD}(f_L) = s_L$ and $g_{DD}(f_U) = s_U$. Note that these conditions are defined in function of g_{DD} instead of g_K, since beyond $\left[f_L^{'}, f_U^{'}\right]$ and thus in particular when the exchange rate hits one of the limits of the band s_L or s_U, the Delgado and Dumas solution is valid.

Moreover, the solutions must join at the switching points $f_L^{'}$ and $f_U^{'}$. Therefore, we introduce two additional conditions (*smooth joining conditions*):

$$g_{DD}(f_L^{'}) = g_K(f_L^{'}) \qquad \text{and} \qquad g_{DD}(f_U^{'}) = g_K(f_U^{'}) \tag{12}$$

These two conditions ensure that the functions g_{DD} and g_K join at the edges of the band of inaction $f_L^{'}$ and $f_U^{'}$.

In the next section, we propose an empirical approximation of (10) to assess the relevance of this model on European exchange rate data.

III Econometric methodology

III.1 SETAR Models

III.1.1 Fixed threshold autoregressive model

To account for a potential nonlinear adjustment of the European exchange rates towards the central parity, we propose to use the Self-Exciting Threshold Autoregressive (SETAR) model due to Tong and Lim (1980) and Tong (1983)[8].

This class of models can be applied to characterize the dynamics of adjustment of the exchange rate towards the central parity in the ERM. To this end, we study in this framework the stationarity of the deviation of the exchange rate from the central parity:

$$x_t = s_t - c_t \tag{13}$$

where s_t and c_t denote respectively the logarithm of the exchange rate and the logarithm of the central parity. If the process for x_t is stationary around a zero mean in a regime, the exchange rate converges towards the central parity in this regime. On the contrary, if x_t is $I(1)$, there is no adjustment of the exchange rate towards the central parity.

As monetary authorities are expected to intervene for large upwards and downwards deviations from the central parity, we consider a three regimes SETAR model[9] for x_t:

$$\Delta x_t = \begin{cases} \phi_{\sup} x_{t-1} \\ 0 \\ \phi_{\inf} x_{t-1} \end{cases} + \sum_{i=1}^{p} \delta_i \Delta x_{t-i} + \varepsilon_t \qquad \begin{array}{l} \text{if } \lambda_{\sup} < x_{t-d} \\ \text{if } \lambda_{\inf} < x_{t-d} \leq \lambda_{\sup} \\ \text{if } x_{t-d} \leq \lambda_{\inf} \end{array} \qquad t = 1, \ldots, T \tag{14}$$

In the outer regimes ($\lambda_{\text{sup}} < x_{t-d}$ or $x_{t-d} \leq \lambda_{\text{inf}}$), *i.e.* for large deviations from the official parity (negative or positive), the autoregressive coefficients ϕ_{sup} and ϕ_{inf} are nonzero. If ϕ_{sup} and ϕ_{inf} are negative, x_t is stationary in the outer regimes, *i.e.* tends to move towards the zero equilibrium value. In the corridor regime ($\lambda_{\text{inf}} < x_{t-d} \leq \lambda_{\text{sup}}$), x_t follows a random walk. In this regime, there is no tendency for the system to drift back towards the equilibrium relationship. The equilibrium value is equal to zero and common to the three regimes[10]. Thus, we assume a convergence to the central parity exactly[11]. Note also that the coefficients δ_i, $i = 1, \ldots, p$, are constrained to be identical in the three regimes in order to make easier the estimation and testing procedures.

This specification permits to assess the empirical relevance of the theoretical model introduced in the second section. First, finding ϕ_{sup} and ϕ_{inf} significantly negative and rejecting the linear specification in favour of (14) is consistent with the existence of a band of inaction around the central parity. Indeed, it means that x_t is stationary beyond the thresholds λ_{inf} and λ_{sup}, while there is a unit-root in the middle regime. Thus, there is a significant mean-reversion of the series when it departs largely from the equilibrium, whereas small deviations from the equilibrium tend to persist in the centre of the band. Second, the existence of intramarginal interventions can be assessed by comparing the estimated thresholds λ_{sup} and λ_{inf} to the official margin. Indeed, the thresholds λ_{sup} and λ_{inf} provide an estimation of the zone $[\lambda_{\text{inf}}, \lambda_{\text{sup}}]$ where the exchange rate is free to diverge and an estimation of the regions $[-\infty, \lambda_{\text{inf}}]$ and $[\lambda_{\text{sup}}, +\infty]$ where the exchange rate reverts to the central parity. As a consequence, the outer regimes can be interpreted as regimes of interventions by the monetary authorities. Thus, if the official margin encompasses the estimated band $[\lambda_{\text{inf}}, \lambda_{\text{sup}}]$, it means that the exchange rate is pushed back to the central parity before the official limits. Such a result can be interpreted as an evidence of intramarginal interventions[12].

III.1.2 Time-varying threshold autoregressive model

The margins of the ERM have been modified at various times. At the beginning, the exchange rates were allowed to fluctuate within a band of $\pm 2.25\%$ around the official parity. Following the crisis in August 1993, the margins widened to $\pm 15\%$ for all currencies (except for the Dutch Guilder relative to the German Mark). Moreover, Italy initially obtained a margin of $\pm 6\%$, which was reduced to the narrow band in January 1990. These changes are not neutral. The dynamics of adjustment of the exchange rates is likely to differ according to the width of the band. Indeed, the monetary authorities can allow larger deviations from the central parity in the wide band. As a consequence, the width of the band of inaction is also likely to change.

Consequently, we introduce a second model where the narrowing of the Italian band from ±6% to ±2.25% in January 1990 and the widening of the band from ±2.25% to ±15% in August 1993 for the other currencies can lead to a change in the estimated thresholds:

$$\Delta x_t = \begin{cases} \phi_{\sup} x_{t-1} \\ 0 \\ \phi_{\inf} x_{t-1} \end{cases} + \sum_{i=1}^{p} \delta_i \Delta x_{t-i} + \varepsilon_t \qquad \begin{array}{l} \text{si } \lambda^*_{\sup,t} < x_{t-d} \\ \text{si } \lambda^*_{\inf,t} < x_{t-d} \leq \lambda^*_{\sup,t} \\ \text{si } x_{t-d} \leq \lambda^*_{\inf,t} \end{array} \qquad t = 1, \ldots, T \tag{15}$$

with $\lambda^*_{\sup,t} = \lambda_{\sup,1}$ if $t < \tau$, $\lambda^*_{\sup,t} = \lambda_{\sup,2}$ if $t \geq \tau$ and $\lambda^*_{\inf,t} = \lambda_{\inf,1}$ if $t < \tau$ and $\lambda^*_{\inf,t} = \lambda_{\inf,2}$ if $t \geq \tau$, where the change point τ is not estimated but corresponds to the time of the change in the official margins (January 1990 for Italy and August 1993). However, the autoregressive coefficients $\phi_{\sup}$ and $\phi_{\inf}$ are fixed on the whole period.

An alternative approach would consist in estimating (14) on two distinct subsamples, before and after the change in the band. However using equation (15) is more appealing for several reasons. First, the second subsample spans from January 1990 to August 1992 for the Italian Lira and from August 1993 to December 1998 for the other currencies. The number of observations is not enough to generate robust estimates. Moreover, equation (15) is more appropriate to determine the impact of the change in the official band on the exchange rate dynamics. Indeed, the thresholds can vary, but the autoregressive coefficients are fixed on the whole period. Such a specification permits to assess if the same strength of convergence applies at an identical level of the band before and after the change in the margin and thus if the dynamics of adjustment is the same whatever the width of the band[13].

III.2 Estimation procedure

The estimation of the SETAR models (14) and (15) consists of several stages.

First, the linear AR model is estimated by OLS for different orders p and the maximum value of p is chosen on the basis of a *k-max* criterion at the 10% significance level.

The SETAR model (14) is then estimated for the order p selected in the linear model by recursive least squares over the possible threshold values $\lambda_{\inf}$ and $\lambda_{\sup}$ and the possible delay values d. In order to have enough observations in each regime, we search from the 5th to the 30th percentile of the arranged sample of the threshold variable to find $\lambda_{\inf}$ and from the 70th to the 95th percentile to find $\lambda_{\sup}$[14]. For each possible $\lambda = \{\lambda_{\inf}, \lambda_{\sup}\}$ and d values, equation (14) is estimated by OLS. We choose $\hat{\lambda} = \left\{\hat{\lambda}_{\inf}, \hat{\lambda}_{\sup}\right\}$ minimizing the residual variance. The estimates of the other parameters are the OLS estimators, $\hat{\phi}(\hat{\lambda}, d)$

and $\hat{\delta}(\hat{\lambda}, d)$ computed for $\hat{\lambda}$. This procedure is reiterated for different $d \in [1, \bar{d}]$ and the values of the parameters minimizing the residual variance are chosen.

The estimation of the time-varying threshold model (15) is similar. In addition to the coefficients ϕ and δ and to the delay parameter d, we have to estimate the thresholds $\lambda_{\text{sup},1}$ and $\lambda_{\text{inf},1}$ before the change of the band in τ and the thresholds $\lambda_{\text{sup},2}$ and $\lambda_{\text{inf},2}$ after τ. The first ones are searched over the values of the threshold variable before the break τ and the second ones after τ. As above, subsamples are defined in order to keep in the outer regimes a minimum of 5% of the observations from 1 to $\tau - 1$ and 5% of the observations from τ to T, that is 5% of the observations of the whole sample.

III.3 Testing procedure

Once the models (14) and (15) estimated, we have to check that the nonlinearities involved in these models are present in the data. We propose here a strategy of tests to check for the existence of three autoregressive regimes and for the non stationarity of the series in the corridor regime. We describe first the specifications involved in this procedure and then the sequence of tests.

III.3.1 Specifications

Four specifications must be compared.

The two first models (M1) and (M2) are the linear autoregressive models. The first one is *I(1):*

$$\Delta x_t = \sum_{i=1}^{p} \delta_i \Delta x_{t-i} + \varepsilon_t \qquad t = 1, \ldots, T \tag{M1}$$

and the second one is stationary:

$$\Delta x_t = \phi x_{t-1} + \sum_{i=1}^{p} \delta_i \Delta x_{t-i} + \varepsilon_t \qquad t = 1, \ldots, T \tag{M2}$$

with $\phi < 0$. The comparison of these two models constitutes a necessary preliminary step to the linearity test, since the hypotheses and the statistics differ according to the stationarity of the linear process.

The specification (M3) (or equation (14) or (15)) is the empirical approximation of the theoretical model of the previous section:

$$\Delta x_t = \begin{cases} \phi_{\text{sup}}\, x_{t-1} \\ 0 \\ \phi_{\text{inf}}\, x_{t-1} \end{cases} + \sum_{i=1}^{p} \delta_i \Delta x_{t-i} + \varepsilon_t \qquad \begin{array}{l} \text{si } \lambda_{\text{sup}} < x_{t-d} \\ \text{si } \lambda_{\text{inf}} < x_{t-d} \leq \lambda_{\text{sup}} \\ \text{si } x_{t-d} \leq \lambda_{\text{inf}} \end{array} \qquad t = 1, \ldots, T \tag{M3}$$

with $\phi_{\text{sup}} < 0$ and $\phi_{\text{inf}} < 0$.

At last, (M4) is a SETAR model with three unconstrained regimes:

$$\Delta x_t = \begin{cases} \phi_{\sup} x_{t-1} & \\ \phi_{med} x_{t-1} & + \sum_{i=1}^{p} \delta_i \Delta x_{t-i} + \varepsilon_t \\ \phi_{\inf} x_{t-1} & \end{cases} \quad \begin{array}{l} \text{si } \lambda_{\sup} < x_{t-d} \\ \text{si } \lambda_{\inf} < x_{t-d} \leq \lambda_{\sup} \\ \text{si } x_{t-d} \leq \lambda_{\inf} \end{array} \qquad t = 1, \ldots, T \tag{M4}$$

In contrast to (M3), the autoregressive coefficient of the inner regime can be different from zero. This last specification is necessary to test for a band of inaction when the linear process is stationary.

III.3.2 Tests

Using the previous notations, Figure 3 shows the sequence of tests to assess if (M3) provides a relevant description of the exchange rate dynamics around the central parity.

First, the random walk (M1) is tested against the stationary linear model (M2).

$$H_{01} : \phi = 0 \qquad \text{against} \qquad H_{11} : \phi < 0 \tag{16}$$

The result conditions the linearity and unit-root tests performed in the following. If the linear process is found stationary, that is if (M1) is rejected in favour of (M2), (M3) can not be tested directly against the linear alternative (M1). In this case, testing for the equality of the three autoregressive coefficients, that is testing for the joint nullity of $\phi_{\sup}$ and $\phi_{\inf}$ could lead to validate wrongly the specification (M3), because (M3) may provide a (global) stationary description of the dynamics and not because (M3) induces different speeds of convergence towards the equilibrium. On the contrary, if the random walk is not rejected, (M3) can be compared to (M1). Thus, the testing procedure depends on the result of the unit root test in the linear AR model. As a consequence, we distinguish between the linear stationary case and that of a random walk.

Case of a linear stationary process

If the random walk (M1) is rejected in favour of the stationary process (M2), the SETAR model with a band of inaction (M3) does not encompass the linear alternative. Consequently, the linearity and the existence of a band of inaction must be tested separately. This situation is depicted on the left side in Figure 3. We must proceed in two steps.

First, (M4) is tested against (M2) to determine whether a threshold behaviour is present in the time-series:

$$H_{02} : \phi_{\sup} = \phi_{med} = \phi_{\inf} \qquad \text{against} \qquad H_{12} : \overline{H_{02}} \tag{17}$$

Under H_{02}, (M4) reduces to the linear autoregressive model (M2).

Three statistics are commonly used to test for the linearity against a SETAR alternative. They are the supremum, the average and the exponential of the Wald statistics defined respectively by:

$$W^{\sup} = \sup_{\lambda \in \Gamma} W^i(\lambda_{\inf}, \lambda_{\sup}),\ W^{avg} = \frac{1}{\neq \Gamma} \sum_{i=1}^{\neq \Gamma} W^i(\lambda_{\inf}, \lambda_{\sup}),\ W^{\exp} = \frac{1}{\neq \Gamma} \sum_{i=1}^{\neq \Gamma} \exp\left(\frac{W^i(\lambda_{\inf}, \lambda_{\sup})}{2}\right)$$

where $\neq \Gamma$ represents the number of elements of the threshold parameter grid and $W^i(\lambda_{\inf}, \lambda_{\sup})$ is the Wald statistics computed for the i-th point of the grid. The distribution of these statistics is non standard due to the nuisance parameters λ and d not identified under the null of linearity (Davies, 1987, Andrews and Ploberger, 1994 and Hansen, 1996). We adopt the approach suggested by Hansen (1996) to derive the critical values[15].

The rejection of (M2) in favour of (M4) proves the existence of different speeds of convergence. To validate the theoretical representation, it remains to show that the adjustment is inactive in the neighbourhood of the equilibrium. In a second step, we test for the nullity of the autoregressive coefficient in the inner regime of (M4):

$$H_{03} : \phi_{med} = 0 \qquad \text{against} \qquad H_{13} : \phi_{med} \neq 0 \tag{18}$$

The statistics is the t-statistics computed for ϕ_{med}:

$$t_{\hat{\phi}_{med}} = \frac{\hat{\phi}_{med}}{\hat{\sigma}_{\hat{\phi}_{med}}} \underset{H_{03}}{\sim} N(0,1) \tag{19}$$

This statistics has a $N(0,1)$ distribution under H_{03}. Indeed, if the threshold variable is stationary and ergodic, the estimation of the model by least squares provides consistent and asymptotically normal estimators[16]. Here, the dependent variable and thus the threshold variable is stationary (M1 rejected in favour of M2). Hence, the autoregressive coefficient of the middle regime is said significant at the 5% level, if the t-statistics in absolute value exceeds 1.96. In the same manner, the local stationarity in the outer regimes can be assessed by comparing the t-statistics for $\phi_{\sup}$ and $\phi_{\inf}$ to -1.65 at the 5% significance level.

Case of a linear I(1) process

Examine now the situation where the random walk (M1) is not rejected in favour of the stationary process (M2). This case is illustrated in the right part of Figure 3. Now, the linear process we keep (M1) is a particular case of the SETAR model with a band of inaction (M3). Thus, the linear process can be tested directly against a SETAR model with a $I(1)$ corridor.

The threshold model (M3) reduces to a random walk (M1), if the autoregressive coefficients of the outer regimes ϕ_{sup} and ϕ_{inf} are equal to the autoregressive coefficient of the inner regime equal to zero. Therefore, we test for a unit-root in the three regimes:

$$H_{02}^{'} : \phi_{\text{sup}} = \phi_{\text{inf}} = 0 \qquad \text{against} \qquad H_{12}^{'} : \phi_{\text{sup}} < 0 \text{ and } \phi_{\text{inf}} < 0 \tag{20}$$

This test is problematic for two reasons. The usual statistics (LM, Wald or LR) do not follow the usual chi-squared distribution due to nuisance parameters not identified and due to the non stationarity of the process under the null. Several solutions have been developed recently to test a unit-root versus TAR models (Balke and Fomby, 1997, Gonzalez and Gonzalo, 1998, Caner and Hansen, 2001, Bec, Ben Salem and Carrasco, 2002 and Shin and Kapetanios, 2002). Here, we propose to apply the procedure developed by Shin and Kapetanios (2002) allowing to test directly a linear unit-root process against a three-regimes SETAR process with a unit root in the inner regime (*i.e.* M3).

Shin and Kapetanios examine the three statistics commonly used in the presence of nuisance parameters not identified under the null and noted previously as W^{sup}, W^{avg} and $W^{\exp}$. They derive the asymptotic distribution of the statistics under $H_{02}^{'}$, when the thresholds are known *a priori*, and generalize this result for estimated thresholds. This distribution does not depend on unknown fixed threshold values. Their Monte Carlo evidence shows that the exponential average of the Wald statistics has the better size and power. On the contrary, the statistics W^{sup} shows substantial size distortions and must not be used in this case.

In sum, we can conclude that the three regimes threshold model with a unit-root in the corridor regime is relevant:

(i) when the linear process is stationary, if (M2) is rejected in favour of (M4) and if (M3) is then preferred to (M4) (left side of Figure 3),

(ii) when the linear process is *I(1)*, if (M1) is rejected in favour of (M3) (right side of Figure 3).

IV Data and estimation results

IV.1 Data

We perform the TAR analysis described in section 3 on European exchange rate data.

We consider six European countries taking part in the ERM since its inception : Belgium (BF), Denmark (DK), France (FF), Ireland (IP), Italy (IL) and the Netherlands (NG). The data consists of monthly observations on exchange rates (in units of foreign currency per Deutsche Mark) and central parities for these six members. The exchange rates were obtained from the *Datastream* database[17] and the central parities were provided by the Banque de France[18]. The sample period runs from March 1979 (beginning of the ERM) to December 1998 (end of the ERM) for Belgium, Denmark, France, Ireland and the Netherlands (238 points) and to August 1992 for Italy (162 observations), since the Italian Lira withdrew from the ERM in September 1992 and re-entered the ERM in November 1996 only.

All series have been transformed by taking their logarithm. We then construct the deviation of the exchange rates from the central parities. We prefer to use the process x_t rather than the nominal exchange rate. Indeed, the exchange rates are subject to occasional jumps related to the realignments (see Figure 4 for a plot of the BF/DM, DK/DM, FF/DM, IL/DM, IP/DM and NG/DM exchange rates). These jumps are likely to alter the properties of the tests, in particular the properties of the unit root tests (Perron, 1989). The extraction of the central parity, subject to similar jumps, removes (or at least attenuates) these breaks in the exchange rate series[19].

IV.2 Fixed threshold autoregressive model

IV.2.1 Tests

We apply the previous strategy of tests to the six series of deviations of the exchange rates from the central parities x_t. The statistics and the p-values are reported in Table 1.

First, the stationarity of the linear autoregressive process is assessed. The first line of Table 1 shows the results of the ADF test (without drift and trend) applied to x_t. The unit root is always rejected at the 5% or 1% significance level in the six countries. Thus, the process for x_t is globally stationary. There exists a global mean-reversion of the series towards the central parity. This result illustrates the stabilizing impact a target-zone exerts on the exchange rate dynamics, as suggested in the target-zone models. It remains to be seen whether this process of adjustment is active, whatever the position of the exchange rate in the band. It is the purpose of the following tests. Since all linear processes are found stationary, the linearity and the non stationarity of the middle regime are tested separately (left side of Figure 3).

Second, we test for the existence of three autoregressive regimes in the dynamics of x_t. We use the statistics W^{sup}. We consider six possible values $d = \{1, 2, \ldots, 6\}$ for the delay parameter. The second line of Table 1 reports the results of this linearity test. The p-values are calculated using Monte Carlo simulations of the models estimated under the null (10 000 replications). The linear hypothesis is rejected in five countries: Belgium, Denmark, France and the Netherlands at 5% and Ireland at 13%. The failure of the three regimes SETAR for the Italian Lira is probably due to the non significance of the AR(1) terms in the lower and inner regimes. A two-regimes model could be more relevant to represent the dynamics of this series, but we do not explore this alternative[20]. In the five other countries, there exists different speeds of convergence towards the equilibrium according to the size and the sign of the deviations of the exchange rates from the central parity.

At last, we test for the existence of a band of inaction in the five countries where the linear specification is rejected. We test for the nullity of the autoregressive coefficient in the inner regime and the non stationarity of the series in the outer regimes. The results are provided in the three last lines of Table 1. In the five countries, x_t follows a random walk in the corridor regime (ϕ_{med} not significantly different from zero) and is stationary in the outer regimes (ϕ_{sup} and ϕ_{inf} significantly negative). Hence, the deviations from the central parity are corrected upon and below a band of inaction.

IV.2.2 Estimates

The SETAR model with a corridor *I(1)* is found relevant in five countries: Belgium, Denmark, France, Ireland and the Netherlands. The SETAR model (14) is estimated in these five countries for the order p selected in the linear specification and for threshold and delay values minimizing the residual variance in the unconstrained specification (M4). The results are given in Table 2. We note that the coefficients ϕ_{sup} and ϕ_{inf} still have the expected negative sign in the constrained specification (M3) and are still significant, as found in the unconstrained specification (see Table 1).

Table 3 reports the results of the specification tests in the SETAR model. We compute the Ljung Box statistics on the residuals and the squared residuals to detect autocorrelation of order 1, 5 and 20 and ARCH effects of order 1, 5 and 10 in the models. The results show no evidence of residual autocorrelation and ARCH effects except in the Netherlands.

IV.2.3 Partial validation of the theoretical model

The results of the tests and the estimates are not fully consistent with the theoretical model.

In Belgium, Denmark, France, Ireland and in the Netherlands, the results of estimation and tests support the existence of a band of inaction in the neighbourhood of the central parity, beyond which the exchange rate reverts to the official parity. Indeed, the linear specification is rejected in favour of the threshold model in these five countries and the test for the local non stationarity of each regime shows that a unit root can not be rejected in the middle regime, while the AR(1) coefficients of the outer regimes are significantly negative. This last result still holds in the constrained specification. In terms of exchange rate management, it means that the exchange rates are allowed to fluctuate freely within the estimated margin, but revert to the central parity due to interventions when outside this band.

However, the comparison of the estimated thresholds to the official parities before August 1993 is not always consistent with interventions inside the band. Figure 5 depicts the series of deviations x_t, the estimated margin and the official band. The estimated margin is well inside the official one in Belgium and in the Netherlands. However, before August 1993, the upper threshold is above the official limits in France, the Danish threshold is relatively close to the upper edge of the TZ and the lower threshold in Ireland is close to the lower limit. Therefore, there is no evidence of significant intramarginal interventions in these three countries before August 1993.

However, an obvious shortcoming of this model is the fact that it does not account for the dramatic widening of the band from $\pm 2.25\%$ to $\pm 15\%$ in August 1993. This broadening seems to lead to larger fluctuations of the Danish, French and Irish parities (see Figure 5). In the fixed threshold Autoregressive model, this pattern could entail a broadening of the margin estimated on the whole sample and thus could be responsible for the mixed results in these countries. The estimated band could also widen and lie inside the official band before August 1993. To allow and measure the potential effect of the changes in the width of the band, the estimation of a second specification with variable thresholds is necessary.

IV.3 Time-varying threshold autoregressive model

IV.3.1 Tests

First, we apply the same procedure of tests, but now we allow a change in the estimated margin in January 1990 for the Italian Lira and in August 1993 for the other currencies[21].

Table 4 reports the results of the tests. The first line of the table (M2 vs M1) shows the results of the

ADF test already used in the procedure involving the fixed thresholds model (see Table 1). The Wald statistics of the linearity test given in the third line is optimised with respect to five parameters: the lower and upper thresholds before and after the change in the official margin and the delay parameter. The computation burden is very important. Consequently, we restrict to 1000 the number of replications to derive the p-values of the linearity test.

The results are similar. The nonlinear model with a unit-root in the middle regime is still preferred to the linear specification except for Italy. However, note that the linear model is more strongly rejected in favour of the time-varying SETAR model in France and Ireland. The autoregressive coefficients of the outer regimes are still significantly negative.

IV.3.2 Estimates

The estimates of the time-varying SETAR models (15) are reported in Table 5 for the five countries where the linear model is rejected. Several results are worth commenting on.

First, the autoregressive coefficients of the outer regimes are still significantly negative in the constrained specification. With respect to the fixed SETAR model, these coefficients are relatively unchanged in Belgium and in the Netherlands and diminish strongly in Denmark, France and in Ireland. As these three countries are also the countries where the thresholds vary the most strongly, this decrease results probably from a better definition of the band of inaction in the time-varying SETAR model relative to the fixed SETAR model. We also note that the fit is improved in (15) relative to (14). Indeed, the coefficient of correlation increases in the time-varying SETAR model. The results of the specification tests relatively similar to those of the fixed SETAR model are given in Table 6.

As far as the threshold values are concerned, we observe an important broadening of the estimated margin in Denmark, France and Ireland in August 1993. However, the widening of the official margin does not lead to a widening of the estimated band in Belgium. The Netherlands where the narrow band was maintained show the same pattern. On the contrary, we note a reduction in the estimated margin in these two countries. This is probably due to the pegging of the Dutch Guilder and the Belgian Franc to the German Mark respectively in 1984 and 1990. These observations allow us to refine the findings of Anthony and MacDonald (1999). They apply univariate unit root tests to daily European exchange rates before and after the widening of the band, in order to assess if the narrow band is more stabilizing than the wide band. They show that mean-reversion is as strong in the wide band ERM as in the narrow one. In fact, if the exchange rate remains globally mean-reverting after the change in the width of the band as they show, we find a significant increase of the zone in which it exhibits a unit root behaviour in many countries.

IV.3.3 Validation of the theoretical model

The results are now consistent with the hypotheses of the theoretical model except for Italy.

First, the SETAR with a band of inaction is not rejected except for the Italian Lira. Moreover, the autoregressive coefficients of the outer regimes are significantly negative. Thus there exists a band of inaction where the exchange rates fluctuate freely and beyond which the exchange rates are dragged back to the official parity. It means that the central banks accept small departures from the central parity and intervene when the exchange rate gets too far from the targeted parity.

Second, the results support now the introduction of intramarginal interventions in the theoretical models. Indeed, Figure 6 shows that the official band encompasses the estimated margins in the two subsamples. It means that the monetary authorities intervene when the exchange rate lies inside the TZ. However the results remain mixed in Denmark before August 1993. The estimated margin is very close to the official one in this period. The Danish authorities seem to have made fuller use of the band and have defended the TZ by an active policy at the edges of the band.

V Conclusion

This paper examines the exchange rate adjustment towards the central parity in the EMS and the role the monetary authorities play in driving the system towards this equilibrium.

First, we have introduced an extension of the usual target-zone models. This extension combines the Krugman model and the Delgado and Dumas model. In the first one, the central banks intervene only at the limits of the band. On the contrary, Delgado and Dumas develop a model where the authorities intervene, as soon as the exchange rate deviates from the targeted level. We have developed a representation with a more realistic intervention rule constructed with the Krugman solution when the exchange rate lies in the middle of the band and with the Delgado and Dumas solution when it departs from the central parity.

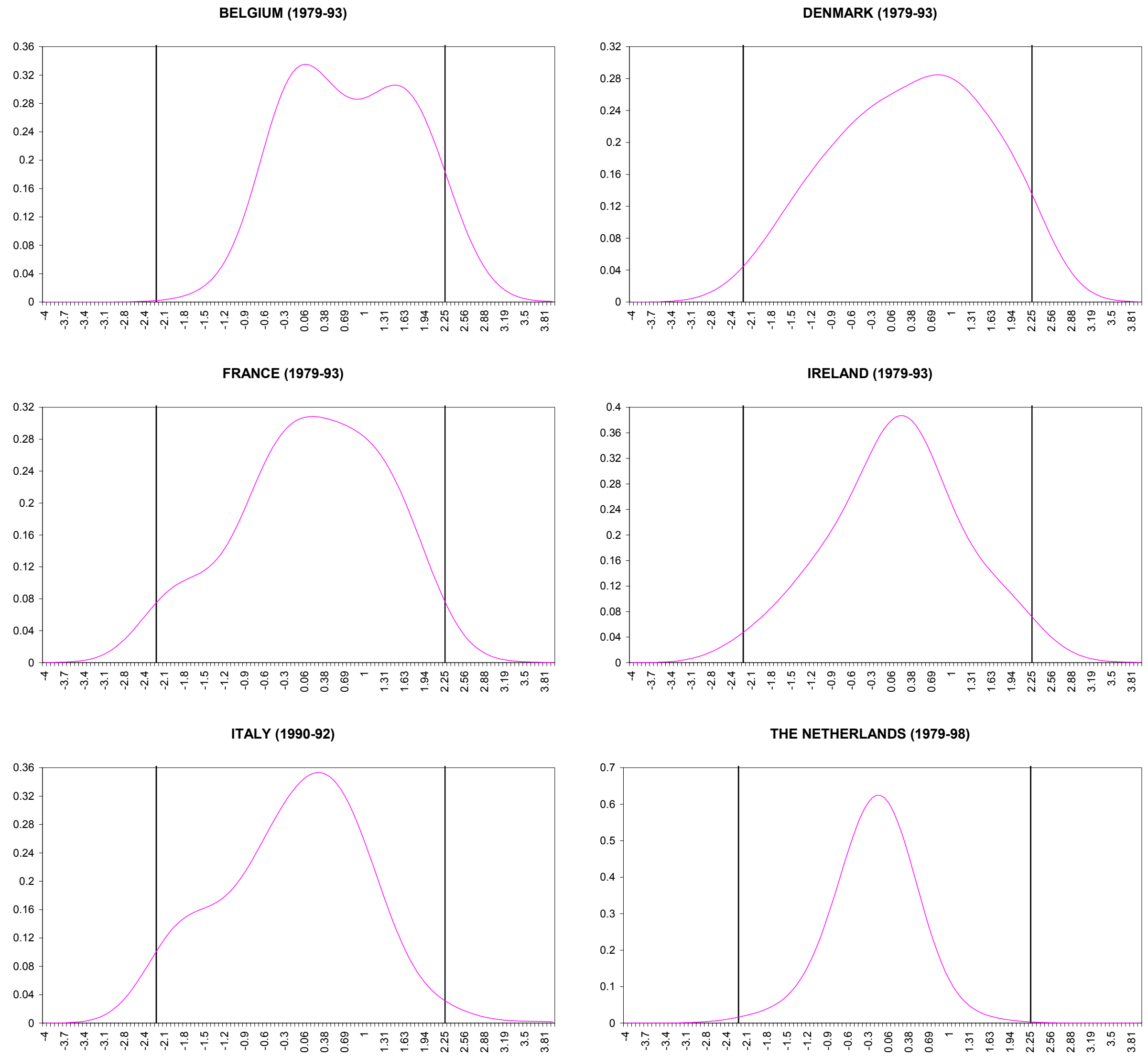
The estimation and the test of a three regimes SETAR model provide results consistent with this representation. We apply the threshold specification to the exchange rates of six European countries relative to the German Mark. The tests do not reject the threshold specification, except for the Italian

Lira. The band of inaction obtained in the five other countries lies well inside the official margin, in particular in Belgium, France, Ireland and the Netherlands. These results are in line with our theoretical model which suggests that the monetary authorities of these countries do not counteract small departures from the central parity, but intervene before the exchange rate hits the edges of the target-zone. Moreover, we show that the widening of the band in August 1993 has affected the exchange rate dynamics of the European currencies not strongly pegged to the German Mark, even if the series remains globally mean-reverting in the wide band, as shown by Anthony and MacDonald (1999).

Acknowledgements

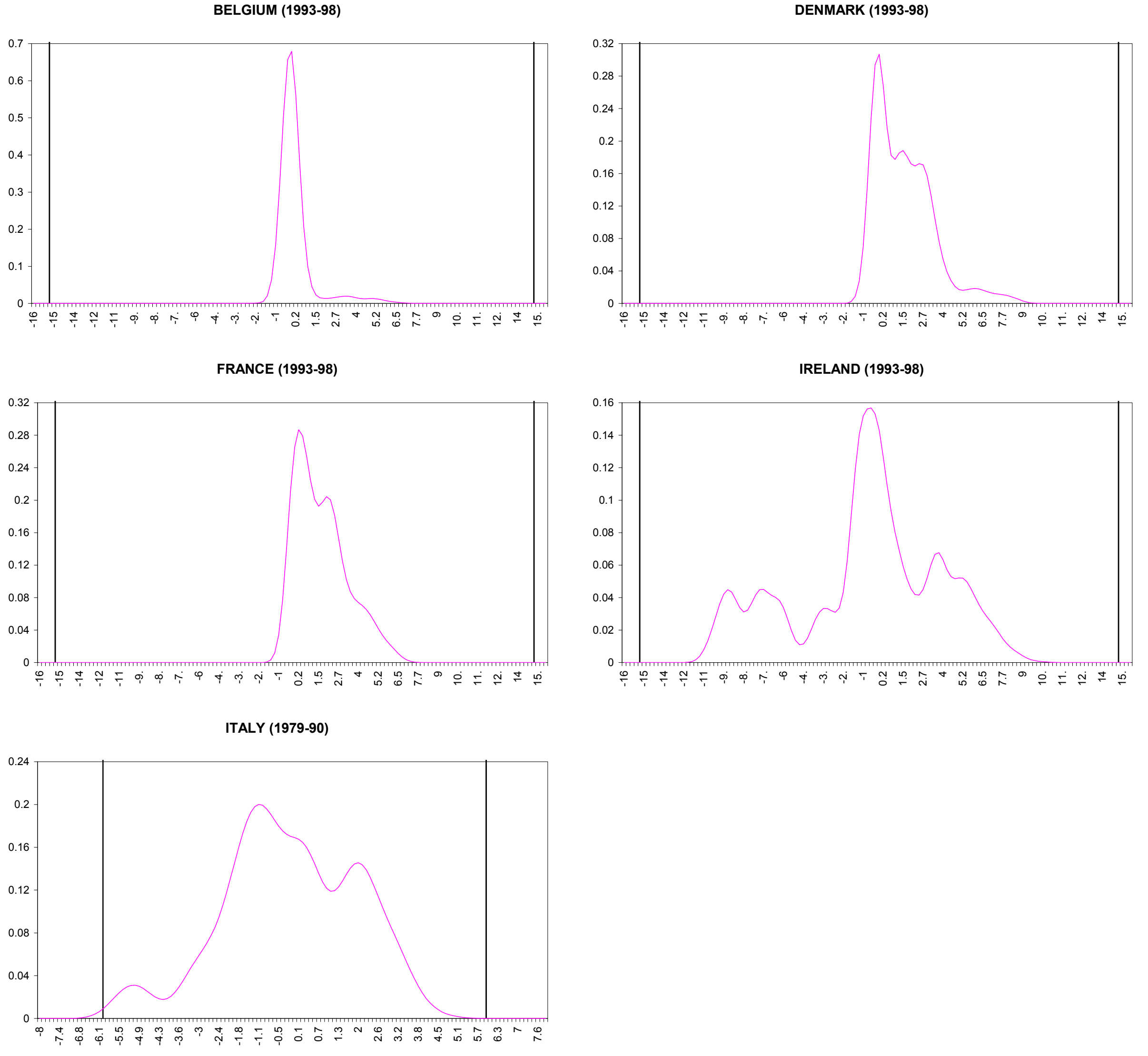
I wish to thank Frédérique Bec, Mélika Ben Salem, Pierre-Yves Hénin, Corinne Perraudin and Christophe Rault for helpful comments. However, all remaining errors are of course mine.

Figure 1. Empirical distributions of exchange rates around the central parity in the narrow band



The distributions are computed from exchange rate data running from 13 March 1979 to 2 August 1993 for the Belgian Franc, the Danish Krone, the French Franc and the Irish Pound, from 8 January 1990 to 17 September 1992 for the Italian Lira and from 13 March 1979 to 31 December 1998 for the Dutch Guilder (exchange rate band of $\pm 2.25\%$). The vertical lines represent the official limits of the zone.

Figure 2. Empirical distributions of exchange rates around the central parity in the wide bands



The distributions are computed from exchange rate data running from 13 March 1979 to 8 January 1990 for the Italian Lira (margins of ±6%) and from 2 August 1993 to 31 December 1998 for the other currencies (margins of ±15%). The vertical lines represent the official limits of the zone.

Figure 3. Detection of threshold effects and a band of inaction

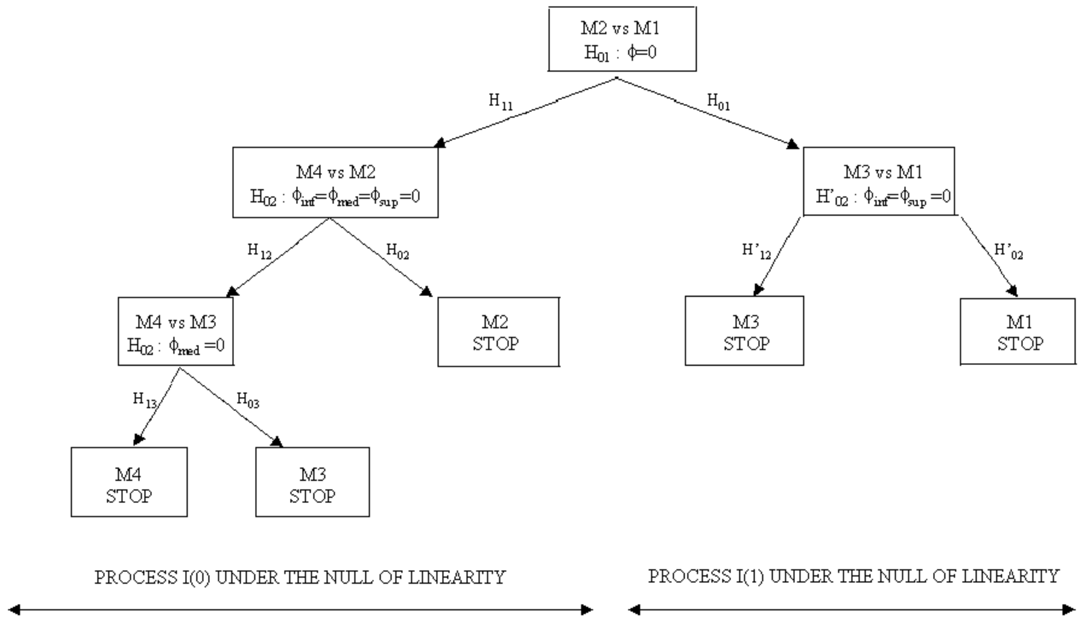
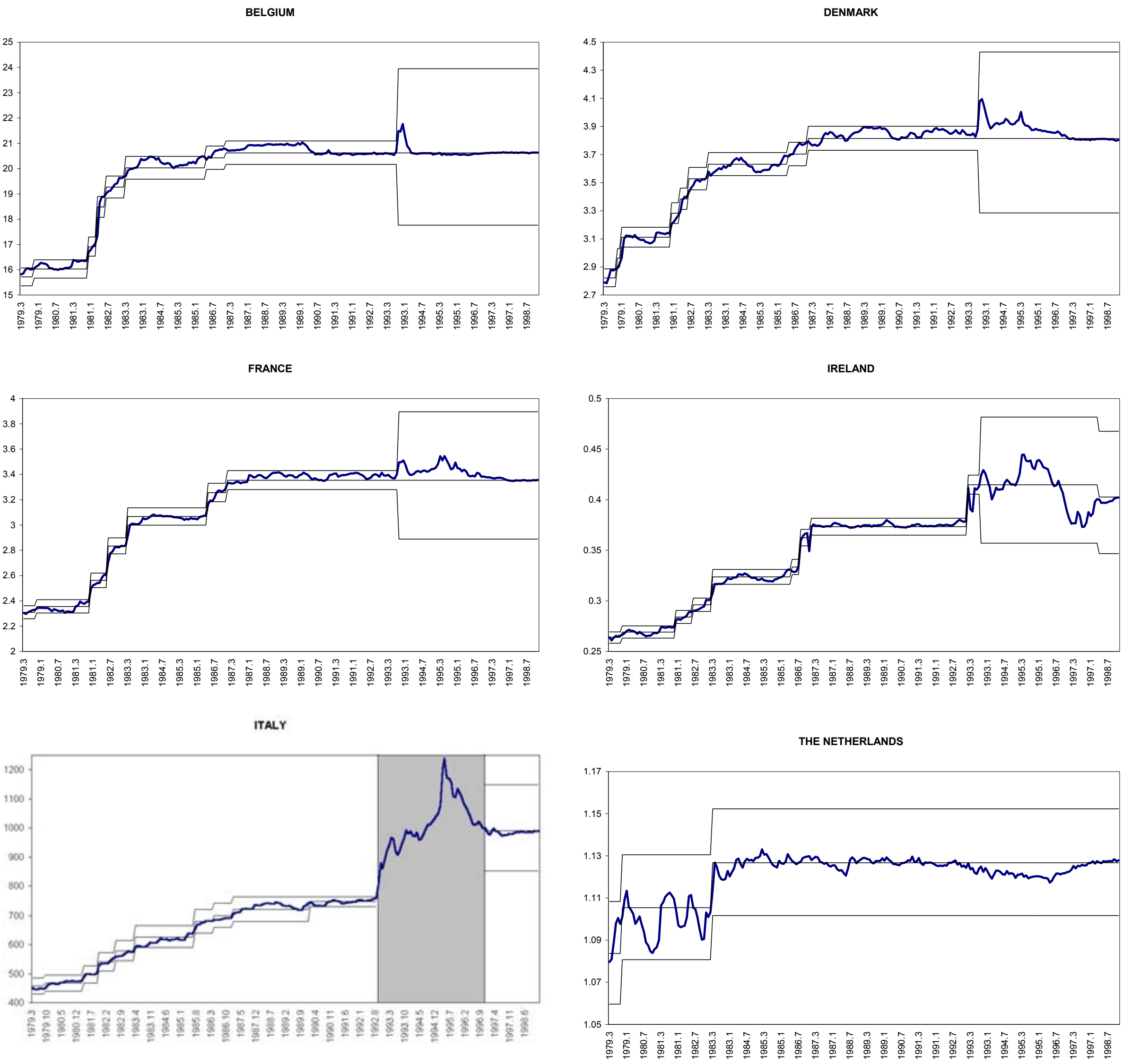
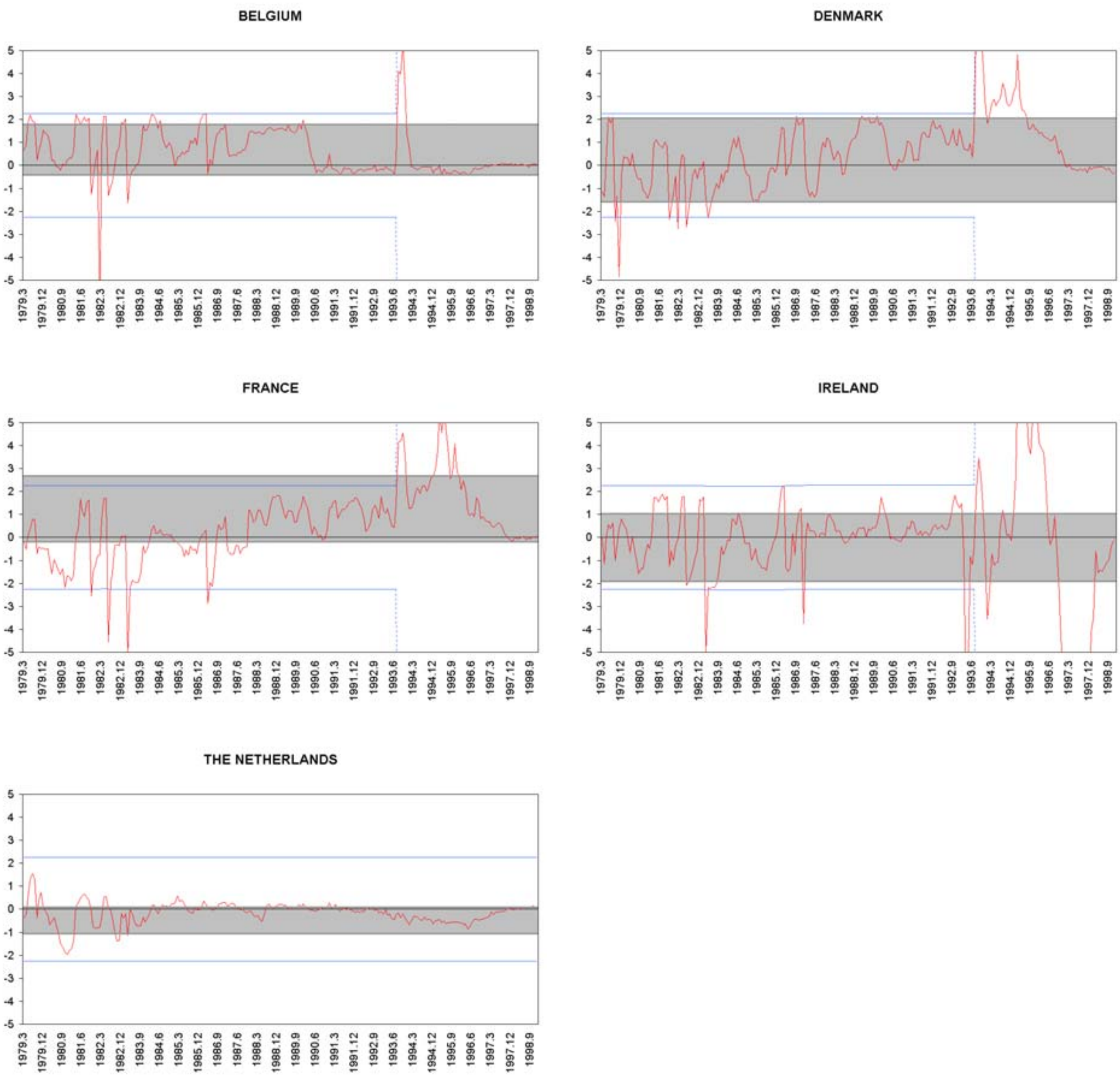


Figure 4. European exchange rates in the ERM



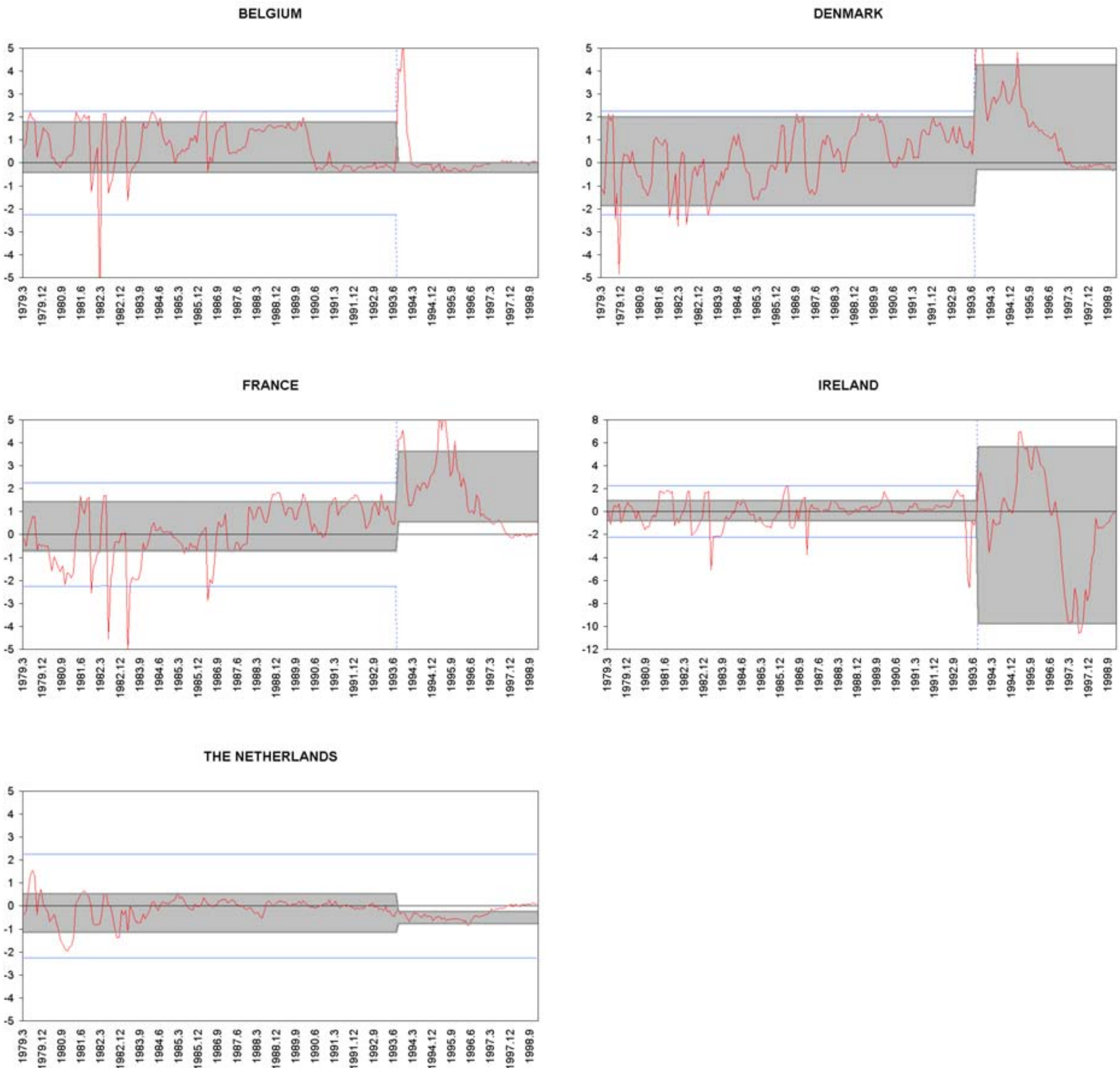
In the Italian case, the dark band corresponds to the phase of withdrawal of the Italian Lira from the ERM.

Figure 5. Estimated and official margins in the fixed SETAR model



The dark band represents the estimated margin, the dotted lines correspond to the official limits and the solid line to the deviation of the exchange rate from the central parity x_t.

Figure 6. Estimated and official margins in the time-varying SETAR model



The dark band represents the estimated margin, the dotted lines correspond to the official limits and the solid line to the deviation of the exchange rate from the central parity x_t.

Table 1. Unit root and linearity tests (fixed thresholds)

Test	BF/DM	DK/DM	FF/DM	IL/DM	IP/DM	NG/DM
M2 vs M1	-2.30	-2.34	-2.06	-2.35	-4.29	-3.17
$\phi = 0$ vs $\phi < 0$	[0.02]	[0.02]	[0.04]	[0.02]	[0.00]	[0.00]
M4 vs M2	98.67	31.00	16.91	10.37	13.68	26.04
$\phi_{\text{sup}} = \phi_{med} = \phi_{\text{sup}}$	[0.00]	[0.00]	[0.05]	[0.37]	[0.13]	[0.00]
M4 vs M3	-0.70	0.45	0.35	-	1.25	-1.47
$\phi_{med} = 0$ vs $\phi_{med} \neq 0$	[0.48]	[0.65]	[0.73]		[0.21]	[0.14]
$\phi_{\text{sup}} = 0$ vs $\phi_{\text{sup}} < 0$	-4.59	-2.57	-2.45	-	-4.72	-1.94
	[0.00]	[0.01]	[0.01]		[0.00]	[0.03]
$\phi_{\text{inf}} = 0$ vs $\phi_{\text{inf}} < 0$	-9.10	-5.72	-3.49	-	-3.56	-6.00
	[0.00]	[0.00]	[0.00]		[0.00]	[0.00]

The p-values are given in brackets.

Table 2. TAR estimates (fixed thresholds)

Country	BF/DM	DK/DM	FF/DM	IP/DM	NG/DM
λ_{sup}	1.77	2.06	2.69	1.02	0.08
λ_{inf}	-0.43	-1.58	-0.21	-1.89	-1.04
d	4	1	6	2	6
ϕ_{sup}	-0.454	-0.108	-0.192	-0.290	-0.139
	(-4.55)	(-2.64)	(-2.43)	(-4.90)	(-1.64)
ϕ_{inf}	-1.158	-0.598	-0.323	-0.154	-0.483
	(-9.50)	(-5.83)	(-3.74)	(-3.62)	(-5.83)
$\overline{R}^2$	0.480	0.242	0.207	0.155	0.173
p_{sup}	12%	12%	6%	20%	25%
p_{mid}	83%	83%	66%	67%	70%
p_{inf}	5%	5%	28%	13%	5%

The t-statistics are given in parentheses. p_{sup}, p_{mid} and p_{inf} denote the percentage of observations which lies respectively in the upper, middle and lower regimes.

Table 3. Specification tests (fixed thresholds)

Country	BF/DM	DK/DM	FF/DM	IP/DM	NG/DM
AR(1)	0.16	0.70	0.05	0.12	2.63
	[0.69]	[0.40]	[0.83]	[0.73]	[0.11]
AR(5)	1.99	4.50	0.38	0.18	9.83
	[0.85]	[0.48]	[1.00]	[1.00]	[0.08]
AR(20)	16.76	12.19	6.84	5.82	33.48
	[0.67]	[0.91]	[1.00]	[1.00]	[0.03]
ARCH(1)	0.00	0.21	0.18	3.17	5.31
	[0.98]	[0.65]	[0.67]	[0.08]	[0.02]
ARCH(5)	4.94	1.25	0.38	8.82	65.02
	[0.42]	[0.94]	[1.00]	[0.12]	[0.00]
ARCH(10)	5.52	1.95	52.22	12.25	104.72
	[0.85]	[1.00]	[0.00]	[0.27]	[0.00]

The table reports the statistics and the p-values in brackets.

Table 4. Unit root and linearity tests (variable thresholds)

Test	BF/DM	DK/DM	FF/DM	IL/DM	IP/DM	NG/DM
M2 vs M1	-2.30	-2.34	-2.06	-2.35	-4.29	-3.17
$\phi = 0$ vs $\phi < 0$	[0.02]	[0.02]	[0.04]	[0.02]	[0.00]	[0.00]
M4 vs M2	98.83	46.89	25.02	12.28	38.99	26.93
$\phi_{\text{sup}} = \phi_{med} = \phi_{\text{sup}}$	[0.00]	[0.00]	[0.03]	[0.48]	[0.00]	[0.00]
M4 vs M3	-0.74	0.10	1.24	-	-1.63	-1.47
$\phi_{med} = 0$ vs $\phi_{med} \neq 0$	[0.46]	[0.92]	[0.21]		[0.10]	[0.14]
$\phi_{\text{sup}} = 0$ vs $\phi_{\text{sup}} < 0$	-4.57	-3.89	-3.69	-	-6.15	-2.09
	[0.00]	[0.00]	[0.00]		[0.00]	[0.02]
$\phi_{\text{inf}} = 0$ vs $\phi_{\text{inf}} < 0$	-9.14	-6.92	-3.63	-	-5.58	-5.96
	[0.00]	[0.00]	[0.00]		[0.00]	[0.00]

The p-values are given in brackets.

Table 5. TAR estimates (variable thresholds)

Country	BF/DM	DK/DM	FF/DM	IP/DM	NG/DM
$\lambda_{\text{sup},1}$	1.77	1.99	1.41	0.96	0.09
$\lambda_{\text{sup},2}$	0.01	4.28	3.64	5.64	-0.11
$\lambda_{\text{inf},1}$	-0.43	-1.86	-0.70	-0.81	-1.04
$\lambda_{\text{inf},2}$	-0.43	-0.31	0.56	-9.73	-0.66
d	4	1	5	2	6
ϕ_{sup}	-0.451	-0.226	-0.267	-0.484	-0.192
	(-4.52)	(-3.93)	(-3.70)	(-5.93)	(-1.86)
ϕ_{inf}	-1.164	-0.774	-0.409	-0.382	-0.483
	(-9.52)	(-6.96)	(-3.93)	(-5.40)	(-5.78)
$\overline{R}^2$	0.480	0.287	0.231	0.236	0.176
p_{sup}	19%	5%	14%	14%	12%
p_{mid}	76%	90%	64%	71%	83%
p_{inf}	5%	5%	22%	15%	5%

The t-statistics are given in parentheses. p_{sup}, p_{mid} and p_{inf} denote the percentage of observations which lies respectively in the upper, middle and lower regimes.

Table 6. Specification tests (variable thresholds)

Country	BF/DM	DK/DM	FF/DM	IP/DM	IL/DM	NG/DM
AR(1)	0.15	0.77	0.03	0.05	0.20	1.83
	[0.70]	[0.38]	[0.86]	[0.82]	[0.65]	[0.18]
AR(5)	2.10	3.98	1.58	1.48	0.65	10.26
	[0.83]	[0.55]	[0.90]	[0.91]	[0.99]	[0.07]
AR(20)	17.31	14.07	11.27	12.01	9.83	34.36
	[0.63]	[0.83]	[0.94]	[0.92]	[0.97]	[0.02]
ARCH(1)	0.00	0.28	0.21	0.89	0.05	6.27
	[0.97]	[0.59]	[0.65]	[0.34]	[0.82]	[0.01]
ARCH(5)	4.96	1.14	0.59	4.92	0.68	63.97
	[0.42]	[0.95]	[0.99]	[0.43]	[0.98]	[0.00]
ARCH(10)	5.58	2.31	43.09	7.83	12.31	103.47
	[0.85]	[0.99]	[0.00]	[0.65]	[0.26]	[0.00]

The table reports the statistics and the p-values in brackets.

APPENDIX

A.1 Sources of the data

Table A1. Datastream codes

Country	Exchange rate
Germany	BDI..RF.
Belgium	BGI..RF.
Denmark	DKI..RF.
France	FRI..RF.
Ireland	IRI..RF.
Italy	ITI..RF.
Netherlands	NLI..RF.

A.2 Realignment dates and central parities

Table A2. Realignment dates and central DM parities in the ERM

Realignment dates	BF/DM	DK/DM	FF/DM	IP/DM	IL/DM	NG/DM
13/03/79	15.7164	2.82237	2.30950	0.263932	457.314	1.08370
24/09/79	16.0307	2.96348	2.35568	0.26921	466.460	1.10537
30/11/79	...	3.11165	...	...	...	...
23/03/81	...	...	...	...	496.232	...
05/10/81	16.9125	3.28279	2.56212	0.284018	539.722	...
22/02/82	18.4837	3.38433	...	...	...	...
14/06/82	19.2693	3.52817	2.83396	0.296090	578.574	...
21/03/83	20.0285	3.63141	3.06648	0.323703	626.043	1.12673
22/07/85	...	...	...	...	679.325	...
07/04/86	20.4252	3.70332	3.25617	0.333416	699.706	...
04/08/86	...	...	...	0.362405	...	...
12/01/87	20.6255	3.81443	3.35386	0.373281	720.699	...
08/01/90	...	...	...	...	748.217	...
15/09/92	...	...	...	...	802.488	...
02/02/93	...	...	...	0.414757	...	...
25/11/96	...	...	...	...	990.004	...
16/03/98	...	...	...	0.402676	...	...

These parities have been provided by the Banque de France.

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Footnotes:

(1) Klein and Lewis (1993) present a model with similar implications. They introduce in the Krugman model a stochastic process for intramarginal interventions which probability increases when the exchange rate deviates from the target level.

(2) According to (4), the exchange rate s_t is an increasing function of the fundamental f_t.

(3) Other predictions of the Krugman model are also rejected: the nonlinearity of the solution (Flood *et al.*, 1991, Lindberg and Söderling, 1994b) or the form of the relationship between the interest rate differentials and the position of the exchange rate within the band (Bertola and Caballero, 1992, Lindberg and Söderling, 1994b).

(4) The density functions are estimated using a fixed-window-width kernel estimator with a Gaussian kernel.

(5) The exchange rate data are provided by the Federal Reserve of New York.

(6) The Italian Lira initially obtained a margin of $\pm 6\%$ from March 1979 to January 1990 and the EMS band was broadened to $\pm 15\%$ in August 1993 except for the Dutch Guilder/German Mark exchange rate.

(7) Note that the models due to Lindberg and Söderlind and Tristani are similar as far as the intervention rule is concerned. They consider the model of Delgado and Dumas and include expectations of devaluation in the specification.

(8) See Tong (1983, 90) for detailed reviews.

(9) This specification is called "Equilibrium-TAR" by Balke and Fomby (1997).

(10) Other attractors could be used: a constant μ or a linear trend $\mu + \alpha t$ (see Enders and Granger, 1998, for a more detailed discussion on this point).

(11) Many authors (Avouyi-Dovi and Laffargue, 1994 or Pentecôte and Roncalli, 1996) suggest that the European exchange rate could be stabilized around implicit central parities different from the official parity. We do not take this possibility into account.

(12) There is a potential problem with this interpretation. This mean-reversion may be due to the market forces rather than to a change in the intervention policy. To check the validity of this interpretation, we should confront our estimates to EMS intervention data and check that the periods when the

exchange rate lies in the outer regimes correspond to a higher frequency of intervention. Unfortunately, this information is confidential. However, data on reserve changes are publicly available, but as shown in Mastropasqua *et al.* (1988), changes in reserves are poor proxies for amounts of interventions.

(13) On the contrary, two separate estimations would lead to different thresholds λ_{sup} and λ_{inf} as well as to different autoregressive coefficients ϕ_{sup} and ϕ_{inf} for the two subsamples. Therefore, the comparison of the two periods would be less easy.

(14) In the empirical studies, a minimum of 10% rather than 5% of the observations are often let in the outer regimes. This partition is not chosen here, because it would impose thresholds closer to zero and thus could alter the conclusions in favour of intramarginal interventions.

(15) Observations are simulated from the model estimated under the null and the test statistics is computed from these observations. From a number of replications of this procedure, the distribution is estimated, so that p-values can be obtained.

(16) See, for example, Gonzalez and Gonzalo (1998) for a discussion on this point.

(17) The codes of the series are reported in Appendix 1.

(18) The central parities are given in Appendix 2.

(19) An alternative approach consists in excluding the observations corresponding to the realignments and considering separately the subsamples of observations between two devaluations (see, for instance, Anthony and McDonald, 1998 and 1999). However, we can not perform this approach on monthly data.

(20) A two thresholds model does not correspond to the theoretical representation introduced in the second section.

(21) The time-varying SETAR model (15) is also applied to the Netherlands, although this country maintained a margin of $\pm 2.25\%$ relative to the German Mark in August 1993. Indeed, we use this country as a benchmark. If we obtain a broadening of the band of inaction in the Netherlands in August 1993, the widening of the estimated margin in the other countries could not be attributed to the extension of the official limits.