INTRODUCTION OF ACRE: AN AGRO-ECONOMIC PRODUCTION MODEL ON REGIONAL LEVEL

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Abstract

ACRE is an **A**gro-e**C**onomic model for agricultural p**R**oduction on r**E**gional level. Based on an extension of Positive Mathematical Programming (PMP) this model was developed as a decision tool for politics with respect to questions of global change scenarios. The validity of the theoretical approach as well as the algorithms has been tested by calculations with empirical data. Currently, ACRE is applied in large interdisciplinary projects. This paper introduces the development of ACRE, from the theoretical framework to testing its validity and current application.

Keywords:

Positive Mathematical Programming, variant-activity, regional model, agricultural production model, interdisciplinary projects

1. Introduction

This paper introduces ACRE, which is an **A**gro-e**C**onomic model for agricultural p**R**oduction on r**E**gional level. Regional models are applied as instruments for supporting political decisions. For this issue agricultural models require extensive data on farmers' production decision with the possibility to simulate scenarios of global, climatic, political or economical changes.

Several optimisation models, which optimise the regional total gross margin, have been developed. The Linear Programming (LP) method used traditionally was replaced recently by the Positive Mathematical Programming (PMP) method. Extended versions of PMP made it possible to develop regional models with respect to specific questions of politics and production in agriculture. ACRE is based on a new extension of PMP. The statistical data used in ACRE reflect reality well and thus the model is an adequate tool for simulating agricultural production. ACRE is calculated in General Algebraic Modeling System (GAMS) and programmed in such a way that interfaces to other models are possible. The current application in two interdisciplinary projects proves that ACRE is a suitable tool for regional modelling of agricultural production.

The paper is structured as follows: Section 2 gives a short introduction of several agricultural regional models. In Section 3 the development of ACRE and its theoretical framework is outlined. The tests by which ACRE was validated are described in Section 4, followed by Section 5 where we explain the application of ACRE in current projects. In Section 6 we summarize the paper and conclude with some remarks on the potential of further developments. In Section 7 the references are listed.

2. Agro-economic regional models

In the past 20 years the relevance of modelling agricultural production on regional scale has grown. Including ecological and economic aspects, regional models have been developed as decision tools for agro-environmental questions. With respect to cost-benefit aspects, e.g. for evaluation of political instruments, the results of corresponding calculations usually support strategic decisions (Dabbert et al., 1999).

During the past 10 years, several agro-economic regional models have been created, five of which are presented in Table 1 and shortly summarized on the basis of five criteria in the following.

Table 1. Comparison of selected agro-economic regional models by different criteria.

	Spatial resolution	Temporal resolution	Optimization method	Activities	Application in projects
RAUMIS (Weingarten, 1995)	district	static	LP	crops	For all German district-level districts, used by BML since 1993
MODAM (Zander, 2003)	20 ha	comparative-static	LP	crops, livestock	Several applications
ROMEO (Umstätter, 1999)	250 m²	comparative-static	PMP	crops, livestock	Kraichgau-Project
Röhm's-Model (Röhm, 2001)	district	comparative-static	PMP	crops, production intensities	"Market Effects of Countryside Stewardship Policies"
ACRE (Winter, 2005)	district	comparative-static	PMP	crops, livestock, production intensities	"GLOWA-Danube", "Rivertwin"

Source: adopted from Dabbert et al., 1999.

The scale of **spatial resolution** indicates the extension of the modelled region and the availability of agricultural data. Data required for the models MODAM and ROMEO have to be available in such a high resolution that they are usually available only for small model regions. Agricultural statistic data are available and accessible on district-level[1] in several European countries. Therefore, it is possible to calculate agricultural production patterns for larger regions by RAUMIS, Röhm's-Model and ACRE. The resolution on district-level delivers sufficient results, nevertheless there is a need of investigation, how large the aggregation error in results of regional models on district-level is.

As **temporal resolution** a comparative static approach has been established in most of the regional models. The advantages of dynamic modelling (e.g. quantification of economic adaptation processes) are compensated by insufficient accuracy of model result (Umstätter, 1999, p. 96). The traditional optimization method in regional modelling is Linear Programming (LP), used in RAUMIS

[1] Districts-level for German model regions is equivalent to NUTS 3-level

and MODAM. This method was more and more replaced by Positive Method Programming-Method (PMP), which is used by the other three models mentioned in Table 1. The advantages of PMP are sketched later, in Section 3.

Besides, being predetermined by the scope of the model, the number of agricultural production activities depends on data availability and on the relevance of the products in the model region. Moreover, the choice depends on the possibility to represent the production activities in the model. An extension of PMP as it was realized in Röhm's-Model and in ACRE adopts the method to program production intensities.

The criteria **"application in projects"** reports which projects the models were applied to.

ACRE was developed within the framework of the interdisciplinary project GLOWA-Danube. The issue was the development of an agro-economic regional model for the upper Danube-River basin that should simulate global change scenarios. The model should be used as an instrument for socio-economic analysis of the farming sector under changing climate and political conditions, as well as for economic evaluation of agricultural landuse. Therefore the model has to simulate agricultural production with respect to economical, political and climate scenarios.

ACRE is a comparative static partial-equilibrium model, which optimizes the total gross margin on regional level by computing optimal production. Dynamic developments can be modelled using a series of modelling runs. Agricultural production in each of the 57 sub regions (on district-level) is represented by one single farm. The simulated time is one year. Agricultural production includes 23 food and non food crops as well as 12 production processes for livestock. Consisting of a process analytical approach, feeding of animals and fertilization of crops are optimized by using feed and manure produced model-endogenously. Trade between the districts is not possible. By variation of parameters included in the model scenarios can be simulated. Changes of production situation can be calculated by varying agricultural input parameters; for example by increasing input prices or reduction of arable and grass land. Market situations can be represented by varying product prices or production quotas. To simulate political instruments parameters for subsidies (CAP-premia and environmental programs) are used. Changes in climate can be simulated by parameters for water supply and crop yields.

3. Theoretical framework

ACRE is based on the method of Positive Mathematical Programming (PMP). PMP is an optimisation approach that maximises an objective value of a total gross margin function. Compared to Linear Programming (LP), PMP has the following advantages:

a) PMP models are exactly calibrated by the reference situation and avoid overspecialization

b) PMP models react continuously to parameter variations and allow a flexible result calculation

c) PMP models tend to require less data than LP models

(adapted from Dabbert et al., 1999; Umstätter, 1999)

These features characterise PMP as a method that promises a fruitful application for regional modelling of agricultural production.

PMP, was developed originally by Howitt (Howitt and Mean, 1983). In several revisions the method was improved: Paris (1988) developed a version that requires a reduced set of information, Howitt (1995) complemented the approach of decreasing marginal cost by the approach of increasing marginal yields and Paris and Howitt (1998), combined PMP with the method of maximum entropy (all cited in Röhm, 2001). Recently, Röhm and Dabbert (2003) published a version of PMP, which extended PMP by a further sub-dimension, the variant-activity. This variant-activity extension differentiates between two levels of activities with different degrees of substitution characteristics.

A PMP-model is built in two steps: A LP-model calculates dual values, which are used to calibrate the second part, the PMP-model. Thus, a PMP-model is not calibrated by means of flexibility constraints. Instead it is calibrated by the shadow prices of restrictions, making sure that the observed production pattern is reproduced by the model. Therefore, PMP simulations are not limited in simulation by fixed restrictions.

According to Röhm and Dabbert (2003) a variant-activity version of PMP is described by the following Equation (1) to Equation (11).

Equation (1) is the total gross margin function (*TGM*), and the objective function of the LP-model. *TGM* is maximised by LP subject to Equation (2) to (6). $GM_{i,v}$ is the gross margin of variant-activity *i*, *v*, with $X_{i,v}$ as the optimised extension of the variant-activities and $\hat{X}_{i,v}$ as the extension of the activity-variant observed in the baseline situation.

$$\max f(X) \text{ where}$$

$$f(X) = TGM = \sum_i \sum_v (X_{i,v} * GM_{i,v}) \tag{1}$$

subject to

$$\sum_i \sum_v (X_{i,v}) \le \sum_i \sum_v (\hat{X}_{i,v}) \tag{2}$$

Equation (2) limits the resource land and produces the dual value λ_{land} which is, as the other dual values, used in the second step by the PMP-model.

$$\sum_v (X_{i,v}) \le \sum_v (\hat{X}_{i,v}) * (1 + \varepsilon_1) \tag{3}$$

Equation (3) constraints the amount of the total-activities. The total-activities (X_i) (which can be e.g. crop activities) are the sums of the corresponding variant-activities ($X_{i,v}$). This total-activity constraint produces the dual value λ_i.

$$X_{i,v} \leq \hat{X}_{i,v} * (1+\varepsilon_2) \tag{4}$$

Analogously, Equation (4) restricts the amount of the variant-activities ($X_{i,v}$) and produces the dual value of the variant-activities $\lambda_{i,v}$.

The perturbation coefficients ε_1 and ε_2 in Equation (3) and (4) are small positive numbers. Therefore, the right hand side factor enlarges the restriction of the observed amounts of the activities $(\hat{X}_i \text{ and } \hat{X}_{i,v})$ by a small value. This enforces the LP-model to produce dual value for each activity. Nevertheless, because the number of constraints exceeds the number of variables by one constraint, one total-activity constraint produces the dual value zero. This problem has to be solved by the calibration of the least attractive total-activity (for details cf. Röhm and Dabbert, 2003, Röhm, 2001 and Umstätter, 1999). Equation (5) constraints a stricter restriction given by Equation (3) for the total-activities than the restriction given by Equation (4) for the variant-activities. This effects that the model substitutes the variant-activities in total-activities rather than substitutes total-activities by other total-activities. (For example a production variant in a crop activity is replaced rather than a crop activity by another crop activity).

$$\varepsilon_1 < \varepsilon_2 \tag{5}$$

$$X_{i,v} \geq 0 \tag{6}$$

Equation (10) describes the objective function of PMP version without the variant-activity extension. Only a crop activity (X_i and λ_i) appears and *TGM* is a sum of index *i*. The formulation of Equation (11) shows the hierarchical relationship between variant-activities and total-activities. Inside the curly brackets the summands where summed up by index *v*, whereas the *TGM* is summed up by index *i*.

$$TGM = \sum_i X_i \left[X_i * \left(GM_i + \lambda_i * \left(1 - \frac{X_i}{\hat{X}_i} \right) \right) \right] \tag{10}$$

$$TGM = \sum_i \left\{ \sum_v \left[GM_{i,v} * X_{i,v} + \lambda_{i,v} * X_{i,v} * \left(1 - \frac{X_{i,v}}{\hat{X}_{i,v}} \right) \right] + \lambda_{i,v} * \sum_v X_{i,v} * \left(1 - \frac{\sum_v X_{i,v}}{\sum_v \hat{X}_{i,v}} \right) \right\} \tag{11}$$

Roughly described, this new version extends PMP in two notable aspects:

1) From the methodological point of view it is possible to differentiate sub-activities (called variant-activities) within main-activities (called total-activities). The flexibility of the substitution between variant-activities is larger within the total-activity than the flexibility to substitute between

the total activities. This approach allows modelling of production systems where the hierarchy of substitution flexibilities determines the decision trees of alternative activities.

2) In application, it is possible to model production systems with different products and different alternatives of production. Agricultural production systems are characterised by products, which can be produced by alternative production processes. One aspect of production processes is the level of production intensity. Among other factors the choice of intensity level is influenced by local characteristics (e.g. limits of soil fertility for intensity levels) and/or by political instruments (e.g. subsidies for less intensive production). An important role can be attributed to both factors for agricultural regional and agricultural sector models.

4. Testing ACRE by empirical data

A promising validation of the variant-activity extension was the successful application in ACRE, which was first developed as a regional model for districts of Bavaria and Baden-Württemberg. Winter (2005) programmed ACRE using regional data of high resolution. For representing agricultural production as close to reality as possible, statistical data on district-level were drawn by information from empirical data surveys on community level. Data of production intensity were concluded from the results of a Delphi survey and the regional distributions of production intensities were estimated. Due to the PMP extension of variant-activities and on a data base which represents a good approximation of the regional situation, ACRE provided a realistic simulation of the model region.

Testing the validity of ACRE by ex-post analysis Winter (2005) investigated noticeable facts:

1) The investigated forecasting error is insignificant, so that the model can be regarded as a good forecasting tool.

2) In contrast to literature, the differences of results between the two PMP approaches of decreasing-marginal-yield and increasing-marginal-cost are insignificant.

ACRE was calibrated for the reference year 1995. For ex-post analysis Winter calculated agricultural production for the year 1999 and compared the results with the statistical data from 1999. Thus, ACRE simulated regional scenarios with a time difference of four years. Winter quantified the validity of forecasting by defining the average forecasting error of the crops (*AFE*). This index expresses weighted absolute difference between the forecasted crop production and the real crop production in 1999 for the complete model region. *AFE* is calculated according to Equation (12).

$$AFE = \sum_r \left[\frac{\hat{X}_{i,r}}{\sum_i \hat{X}_{i,r}} * \frac{\left| \hat{X}_{i,r} - \breve{x}_{i,r} \right|}{\hat{X}_{i,r}} * 100 \right] * R^{-1} \qquad (12)$$

with

i = crop

r = region, district

R = number of districts in the total model region

$\hat{X}_{i,r}$ = statistical landuse for crop *i* in the district *r* in 1999 [ha]

$\breve{x}_{i,r}$ = calculated land use for crop *i* in the district *r* in scenario year 1999 [ha].

Figure 1 illustrates that the *AFE* for different crops is less than 3.5 % and therefore a good forecasting quality can be assumed for the extension of crops simulated by ACRE for 1999. Furthermore the *AFE* resulting from the calculation by the PMP approach of increasing-marginal-cost differs insignificantly from the *AFE* calculated by the approach of decreasing-marginal-yields. Both results suggest two assumptions: 1) the implication of variant-activities leads to an approximation of the results from the two basic PMP approaches provided by Howitt (1995). 2) Following from 1) it seems possible to choose from both approaches the one which runs more stable within GAMS, which seems to be the increasing-marginal-cost approach.

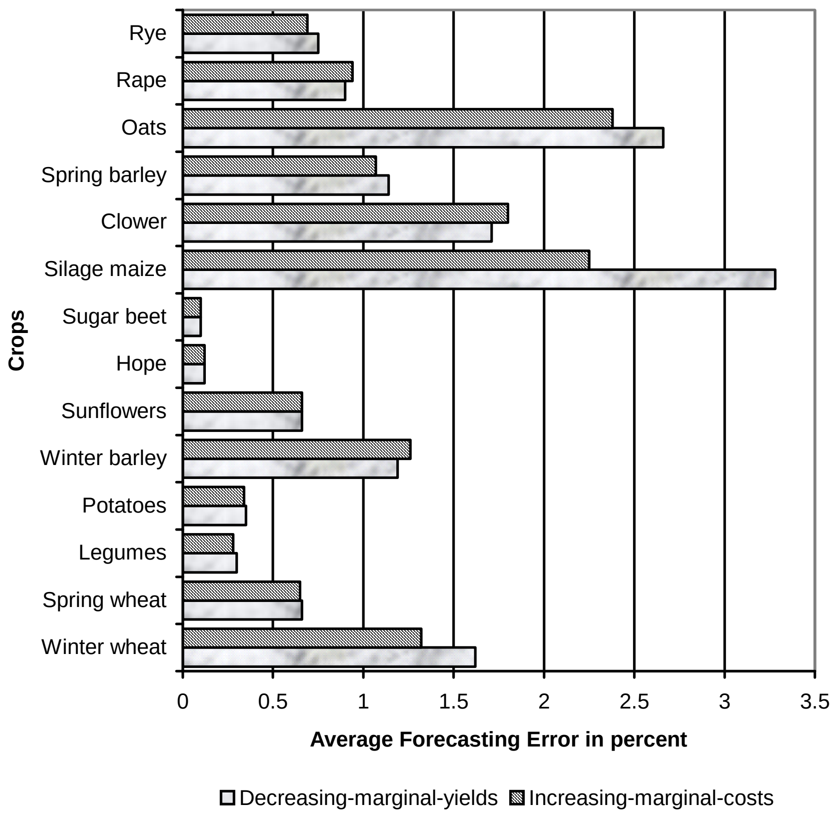

Figure 1. Average Forecasting Error for selected crop production. The forecasted production for the year 1999 was compared to production statistics of the year 1999. (Source: adapted from Winter, 2005.)

Winter calculates scenarios only by the approach of increasing-marginal-cost. Due to its linear constraints this approach provokes a reduction of computational resources as it is necessary for the approach of decreasing marginal yields consisting on non-linear equations.

We try to give some impressions for possible agro-political analysis with ACRE by sketching two results of Winter's calculations. Cause-effect-relationships are described here only shortly and by examples.

1) With respect to a variation of wheat price ACRE reacts by changing the rate of wheat production. As Table 2 shows, changes of landuse differs in extensive production and intensive production. A symmetric reaction can be observed. The absolute change of extensive landuse resulting from an increasing of price by 20 % is approximately equal (+19.9 %) to the absolute decrease of intensive landuse (+19.6 %) resulting from a change of price by –20 %. This symmetry is found also for other price modifications. These results show that ACRE reacts to price variation with specific elasticity of supply for intensive and extensive wheat production. This observation can be explained by the mathematical reaction of the model, however it should be verified by market theory and empirical data to validate ACRE's results with respect to reality.

2) A scenario context to the Agenda2000 mid-term-review was calculated for the year 2010. The scenario includes producer prices in 2010, calculated by an external trade model, a decoupling of product premia as well as decrease of premia by 10 %. For all district-level districts Winter compared the percentage of extensive cereal production in the reference year with that of the scenario year.

Table 2. Modification of product prices and resulting change in regional wheat production.

Modification of wheat prices in percent	Change of land used for wheat production in percent	
	Production variant	
	Extensive	Intensive
+ 20	+ 19.9	+ 29.4
+ 10	+ 9.9	+ 13.2
- 10	- 13.1	- 10.1
- 20	- 26.8	- 19.6

Source: adapted from Winter, 2005.

Figure 2 demonstrates the observed tendency. The dotted diagonal represents the situation in which the rate of extensive production remains constant. Districts with more than 65 % in the reference situation tend to increase extensive cereal production. (Nearly all diamonds lies slightly above the dotted line.) Districts which less than 65 % of extensive cereal production tend to reduce extensive production in the scenario year. (Most of the diamonds are located below the diagonal.) However, districts with less than 65 % of extensive production are distributed more heterogeneous.

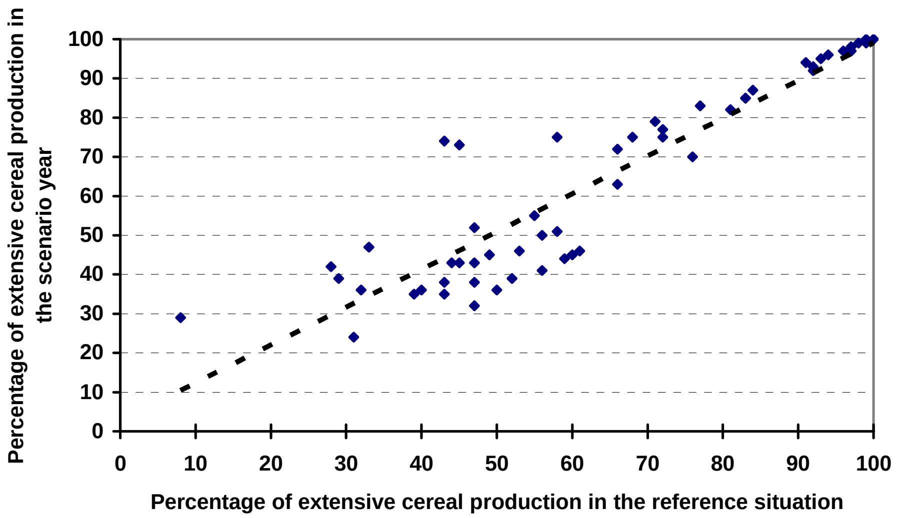

Figure 2. Comparison of the percentage of extensive cereal production in the districts in the reference situation 1995 vs scenario year 2010. Districts are represented by diamonds. (Source: adapted from Winter, 2005.)

Winter explains the observation made for the “less-65percent-farms” by the high level of extensive cattle fattening in these districts. Decoupling of subsidies as well as price- and subsidy-reduction results in a decrease of cattle fattening in the scenario year. Therefore, mineral fertilizer replaces manure produced by cattle used previously to fertilize cereal fields. Mineral fertilizer has to be bought and consequently, the costs of fertilization increase. Furthermore, due to the reduced number of animals, less feed cereals are cultivated. Both, the higher costs for fertilization and the lower need of feed, result in an increase of extensive farming of cereal production.

5. Current application in interdisciplinary projects

5.1 Application in GLOWA-Danube

The GLOWA-Danube research project in the framework of GLOWA (Global Change in the Hydrological Cycle) funded by the Federal Ministry of Education and Research (BMBF) which started in 2001 aims at developing scenarios, strategies and integrative techniques to deal with regional effects of Global Change on the water cycle and its utilization by man in the Upper Danube catchment (Mauser and Ludwig, 2002).

The Upper Danube basin is covering an area of 77,000 km² enclosing 8 Mio. residents in two German States and three major countries. Approximately 55 % of the catchment area is agriculturally used.

GLOWA-Danube involves various disciplines such as hydrology, ecology, glaciology, geography, water resources management, agricultural economy, tourism, environmental economy, environmental psychology and computer science. In order to build up a mutual transdisciplinary communication platform and to bridge the gap between the participating disciplines DANUBIA an integrated environmental decision support system is being set up. Beyond water related Global Change management issues under economic, cultural, social and ecological aspects such as conflicts in water utilization, water quality and environmental protection, tourism, vulnerability of mountain environments, flood risk protection and transboundary water management GLOWA-Danube focuses on socio-economic analysis and evaluation of drinking water demand for agricultural production, by using the regional economic optimization model ACRE described above (Krimly et al. 2003).

The Upper Danube watershed is represented by a rectangular mesh of 1 x 1 km proxel-(process-pixel)-objects. The initial agricultural land use situation is based on CORINE land cover data (Coordination of Information on the Environment) which was calibrated by means of statistical information on district-level. An aggregation tool allocates the proxel-based information on district level which is the mutual spatial basis for ACRE. Seventeen classes of agricultural land use (hop, sugar beet, potato, maize, crop silage, summer barley, oleaginous, winter barley, oats, summer wheat, rye, legumes, forage, set aside, winter wheat, intensive, and extensive grassland) are computed by ACRE for a total of 73 districts by means of the maximization of agricultural income as objective function. Figure 3 shows the gross margin per hectare calculated for the reference year 1995.

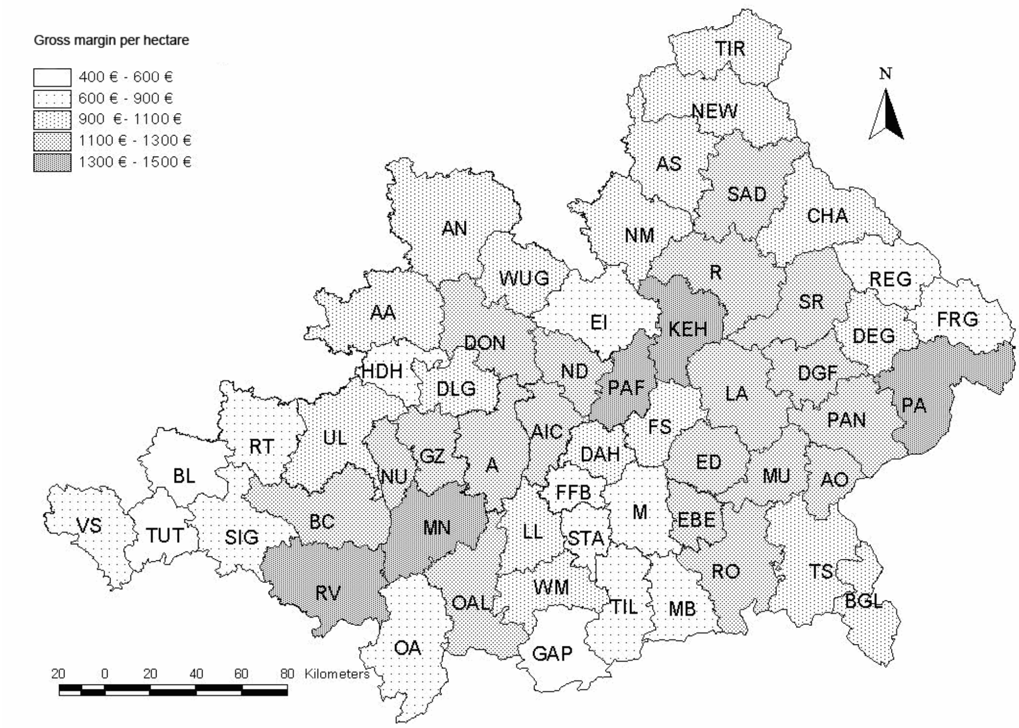

Figure 3. Gross margin per hectare of arable land on district level in the German part of the Upper Danube Basin. (Source: ESRI Geoinformatik GmbH (2000), calculations by Winter, 2005.)

A disaggregation tool distributes the district based modelling results to each proxel. Its internal multi-criteria-decision-matrix is based on a rule frame-work derived from expert knowledge and information on site conditions as well as on historical development. Furthermore, a ‘farming tool’ decides on a daily basis about agricultural management activities like sowing, fertilization, harvesting depending on precipitation and soil saturation. The distribution of main agricultural land use types (grassland and arable) and the arable crop ratio affect ground-water recharge and other hydrological factors, whereas the fertilization level has an impact on water quality. The concept of conjunction with the central modelling system DANUBIA is realized with the unified modelling language (UML) and implemented with JAVA™.

Figure 4 illustrates the methodological concept of the farming model. DANUBIA shows great promise for developing a tool for integrative research, which allows the representation and of sustainable water use.

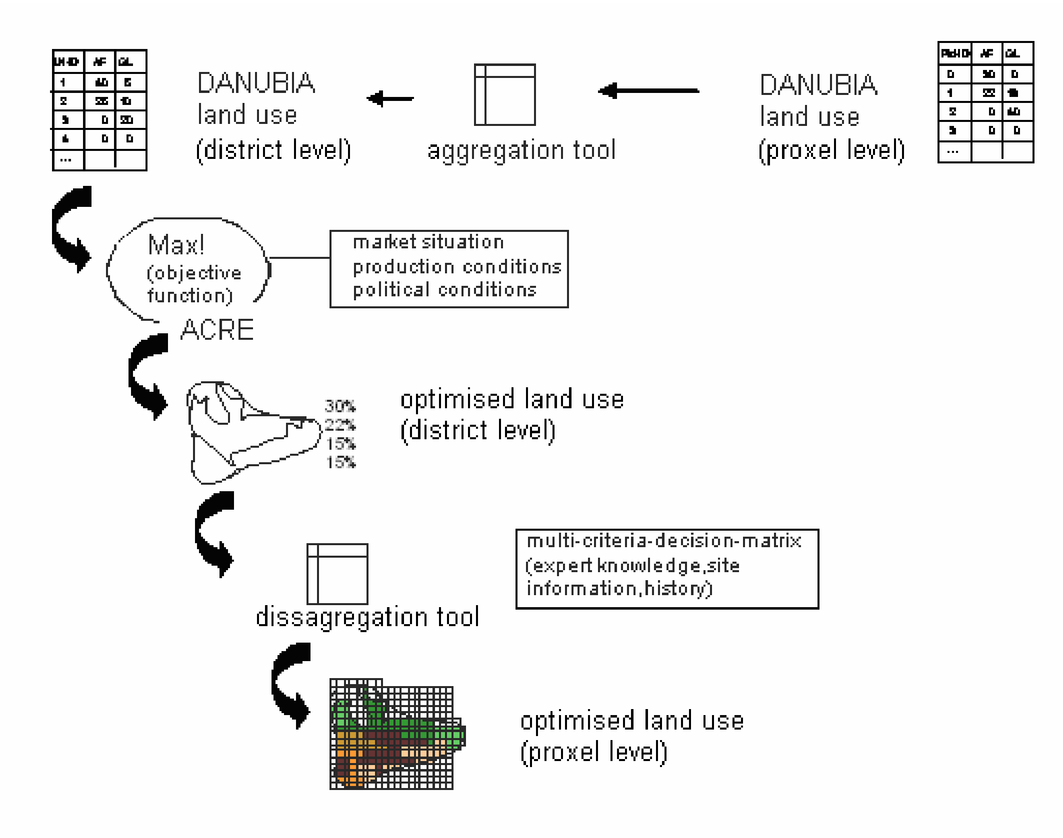

Figure 4. Methodological concept of the farming model in DANUBIA. (Source: adopted from Krimly et al., 2003.)

5.2 Application in RIVERTWIN

The research project RIVERTWIN (A Regional Model for Integrated Water Management in Twinned River Basins) aims at providing a management tool for the implementation of the EU Water Framework Directive. In support of the EU Water Initiative an integrated regional model for the strategic planning of water resources management in twinned river basins will initially be developed and tested in the German Neckar Basin and later on transferred to the river catchments in the partner regions of Uzbekistan and Bénin. The model which is based on a Geographic Information System (GIS) copes with impacts of economic and technological development, demographic trends and effects of global climate and land use changes on the availability and quality of water bodies (Gaiser and Dukhovny, 2004).

The German Neckar catchment covers an area of 13,000 km² enclosing highly populated industrial metropolitan areas such as the “Middle Neckar” and major areas of intensive agricultural use. Important water management issues are frequent floods due to sealing of floodplain areas, poor annual water supply and the deterioration of riverine habitats due to industrial emissions and changes of riverbed morphology.

Major problems for drinking water quality can be attributed to nitrate, phosphate and pesticide leaching from agriculture.

The socio-economic analysis and evaluation of land use and agricultural drinking water demand and its impact on water quality will be conducted by using the regional economic optimization model ACRE. In close cooperation and exchange with regional and local stakeholders in the Neckar basin sustainability indicators and integrated scenarios of land use and climate change will be formulated as a basis for future river basin management plans.

6. Conclusions

ACRE is a regional model that simulates agricultural production. This model is developed to calculate scenarios for global changes as well as political or economic scenarios. For these issues, the optimisation method of PMP is used to maximise the regional total gross margin in the ACRE model.

The PMP-method was originally developed to optimise activities which are not differentiated in levels of flexibility. In an extended version, called the variant-activity PMP, a differentiation is made between total-activities and variant-activities. These activities are characterised by different degrees of flexibility to substitute each other in the optimization process. This development allows the different production intensities in agricultural production to be modelled. Thus, the new approach is used successfully to evaluate agro-environmental programs as well as political instruments.

Based on regional statistical data, ACRE was tested and validated with an ex-post analysis. The calculated forecasting error was insignificant so the ACRE model is regarded as suitable for making forecasts. However, the informational value of prognosis is restricted. On a spatial scale ACRE’s calculation ability is confined to district-level and on a time scale to the time period of one year. Thus, without further data transformation, simulation results can only be interpreted on these scales. Nevertheless, this resolution for regional models is sufficient for the purposes of the projects described.

Currently ACRE is being tested in two large interdisciplinary projects dealing with global and regional change scenarios in two different river basins. Within these projects the technical possibilities (e.g. the possibility of JAVA™-interfaces for data exchange with other models) allow for the implementation of ACRE as a sub model of a farming decision tool. The technical features as well as the quality of forecast results are currently being tested in two interdisciplinary projects.

Even though ACRE can be regarded as a sophisticated regional model, there is room for further development. Further analysis is necessary to exclude inaccuracies in aggregation. Therefore, the model should be rerun with data based on the municipality level. Additional data and parameters could make it possible for ACRE to simulate scenarios with respect to special questions of interest (e.g. consequences of new reforms and laws). Another improvement underway is to allow ACRE to interact with other models such as external production models on a farming scale.

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