

A SIMPLE ENCOMPASSING TEST FOR THE DETERMINISTIC AND BILINEAR UNIT ROOT MODELS

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ABSTRACT

A new parameters' encompassing test is proposed for deciding between the deterministic unit root processes with a structural break and the bilinear unit root model without such break. The test consists in testing three sets of hypotheses regarding parameters in a simple regression model. The test uses the t -ratio and F -statistics, of non-trivial distributions under the null. The finite sample distributions for the relevant statistics are tabulated and the asymptotic distribution of the F -test is derived. The test has been applied for the daily stock price indices for 66 countries, for the period 1992-2001. The results support the conjecture that the bilinear model dominates the structural break model more often than the other way around. Also, it is likely that in practical applications the bilinear unit root process might be mistaken for the deterministic unit root process with a structural break.

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1. THE MODEL

The paper considers two particular stochastic processes embedded within the following general process:

$$u_t = d_t + y_t \quad , \quad y_t = \rho_t y_{t-1} + v_t \quad , \quad (1)$$

where d_t is a deterministic part, $E(y_0^p)^2 < \infty$, $v_t = \delta(L)\varepsilon_t = \sum_{j=0}^{\infty} \delta_j \varepsilon_{t-j}$ and $\sum_{j=0}^{\infty} j |\delta_j| < \infty$ (see Ng and Perron, 2001), ρ_t is series of (possibly degenerated) random variables, $\varepsilon_t \sim IID(0, \sigma^2)$ and $t = 1, 2, \dots, T$. The deterministic part in (1) is given as:

$$d_t = const. + \gamma BREAK_t \quad , \quad (2)$$

where $BREAK_t$ is a variable indicating a nonlinear change in the deterministic part of the process. The most commonly used form of the break is either the 'slope':

$$BREAK_t = \begin{cases} (t - T_B), & \text{for } t > T_B \\ 0, & \text{otherwise} \end{cases}, \quad (3)$$

or the ‘step’:

$$BREAK_t = \begin{cases} 0, & \text{for } t < T_B \\ 1, & \text{otherwise} \end{cases} \quad (4)$$

and T_B indicates the position of the beginning of the structural break in the series. A number of other, more complex, specifications of $BREAK_t$ is also possible (see Vogelsang and Perron, 1998, and Harvey and Mills, 2002).

As far as the unit root part in (1) is concerned, the following terminology is introduced:

- (a) The unit root process has a *deterministic* unit root and is *linear* if in (1) $\rho_t = 1$ and in (2) $\gamma = 0$.
- (b) The unit root process is *deterministic* unit root and is *nonlinear* if $\rho_t = 1$ and $\gamma \neq 0$.
- (c) The unit root process has a *stochastic* unit root if ρ_t is a non-degenerated random variable with the expected value of one. In this paper we only consider the stochastic unit root process without a deterministic part, that is where $\gamma = 0$.

The first two processes, (a) and (b), constitute a family of the deterministic unit root (*URD*) processes. The literature on testing and evaluation of the *URD* processes is well developed. In particular, the nonlinear *URD* models (and such models are considered in this paper) are analysed in Kim, Leybourne and Newbold (2000), Ng and Perron (2001), Perron, (1989), (1990), (1997), Vogelsang, and Perron (1998), Zivot, and Andrews (1992). Among the stochastic unit root processes, this paper focuses on the bilinear unit root (*URB*) process of the first order (see Charemza, Lifshits and Makarova, 2005), defined as:

$$\rho_t = 1 + b\varepsilon_{t-1}, \quad b \neq 0, \quad \text{where } \text{cov}(\rho_t, \varepsilon_t) = 0 \quad .$$

For other specifications of the stochastic unit root models see e.g. Granger and Swanson (1997), Leybourne, McCabe and Tremayne (1996), McCabe and Tremayne (1995), Sollis, Leybourne and Newbold (2000). We consider a simple case of (1) where $d_t = 0$ and $v_t = \varepsilon_t$, which gives:

$$y_t = (1 + b\varepsilon_{t-1})y_{t-1} + \varepsilon_t. \quad (5)$$

For $b = 0$, (5) becomes a simple *URD* process, without any deterministic part (a random walk). The process (5) is nonstationary, since $1 + b^2\sigma^2 \geq 1$ (see Granger and Andersen, 1978). In Charemza, Lifshits and Makarova (2005) it is shown that, under the null that $b=0$ the Student t statistic for \hat{b} in the regression equation:

$$\Delta y_t = \hat{b}y_{t-1}\Delta y_{t-1} + e_t, \quad (6)$$

where e_t are the regression residuals, has an asymptotic standard normal distribution. It is also shown that the similar test statistics can be formulated for a regression containing an intercept and for demeaned series of first differences of y_t . If the relationship between v_t and

ε_t is more complex, that is, where v_t is described by a fully defined moving average process, it is possible to add augmentations to (6), as in the Augmented Dickey-Fuller test.

2. THE TEST

In order to decide whether a *URD* (possibly nonlinear, with a deterministic break) or a *URB* (without a deterministic break) model describes the series in a more accurate way, a simple encompassing exercise has been performed. It is based on the following regression:

$$\Delta y_t = \hat{c} + \hat{b}y_{t-1}\Delta y_{t-1} + \hat{\gamma}BREAK_t + \sum_{i=1}^k c_i \Delta y_{t-i} + e_t, \quad (7)$$

so that \hat{c} , \hat{b} and $\hat{\gamma}$ are the estimates for c , b and γ . There are three sets of the hypotheses of interest to be considered:

(A) $H_0: b = 0$ and $\gamma = 0$ and $H_1: b \neq 0$ and/or $\gamma \neq 0$

(B) $H_0: \gamma = 0$ and $H_1: \gamma \neq 0$

(C) $H_0: b = 0$ and $H_1: b \neq 0$

The rejection of H_0 in (A) confirms that the unit root process is either subject to bilinearity, or a structural break. Further on, the rejection of H_0 in (B) confirms the existence of the deterministic structural break without much evidence of the *URB* process. If, however, H_0 is rejected in (A) and (C) and is not rejected in (B), this would implicitly confirm the existence of the *URB* and not the *URD* with break process.

For testing these sets of hypotheses we use Student t -statistics and, additionally in (A), the F statistic for testing the joint significance of b and γ on the basis of their estimators \hat{b} and $\hat{\gamma}$. In (A), under the null, the asymptotic distribution of the t -statistic for $\hat{\gamma}$, denoted as $t(\hat{\gamma})$ is standard normal. However, since the finite sample distribution of $t(\hat{b})$ is non-standard, the distribution of the F -statistic is unknown and has to be approximated numerically. The limit distribution of F -statistic in case of ‘step’ is given by the following theorem:

Theorem. *Let the series y_t be generated by*

$$y_t = y_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim IID(0, \sigma^2), \quad y_0 = 0, \quad t = 1, 2, \dots, T. \quad (8)$$

For the regression model (7) with $BREAK_t$ defined by (4), under the null of $b = \gamma = 0$, as $T \rightarrow \infty$ the F -statistic has a limit distribution of the form:

$$F \Rightarrow \frac{1}{2} \left[\frac{\left(\int_0^1 W_1(t) dW_2(t) \right)^2}{\int_0^1 W_1^2(t) dt} + \frac{(1 - \sqrt{\lambda})(4 - \lambda)}{\lambda(1 + \sqrt{\lambda})} W_1^2(1) \right] \sim \frac{1}{2} \left[\chi_1^2(1) + \frac{(1 - \sqrt{\lambda})(4 - \lambda)}{\lambda(1 + \sqrt{\lambda})} \chi_2^2(1) \right], \quad (9)$$

where \Rightarrow denotes weak convergence when $T \rightarrow \infty$ W_1, W_2 are two independent Wiener processes on $[0,1]$, λ is the ratio of time of break T_B to the sample size T , that is

$$\frac{T_B}{T} = \lambda = \text{cons.} \quad (10)$$

and is assumed to be constant, and $\chi_1^2(1), \chi_2^2(1)$, are two independent chi-squared distributions with one degree of freedom¹.

Proof of the Theorem is based on the following Lemma:

Lemma. Let the series y_t be generated by (8). For the regression model (7) with $BREAK_t$ defined by (4) under the null of $b = \gamma = 0$, as $T \rightarrow \infty$:

$$\begin{aligned} 1) \quad \sqrt{T}\hat{c} &\Rightarrow \frac{\sigma}{\sqrt{\lambda}}W_1(1), \\ 2) \quad T\hat{b} &\Rightarrow \frac{\int_0^1 W_1(t)dW_2(t)}{\sigma \int_0^1 W_1^2(t)}, \\ 3) \quad \sqrt{T}\hat{\gamma} &\Rightarrow \frac{\sigma}{\sqrt{\lambda}(1+\sqrt{\lambda})}W_1(1), \end{aligned}$$

where \Rightarrow denotes weak convergence, W_1, W_2 are two independent Wiener processes on $[0,1]$ and λ is defined by (10).

The proofs of Lemma and the Theorem above are given in Appendix A.

It can also be shown that in (B), under the null, the asymptotic distribution of $t(\hat{\gamma})$ is not normal and it depends on b . Moreover for $t(\hat{b})$ in (C), under the null, that is for the Student- t statistic for the parameter b , the asymptotic normality is no longer the case, although the limit distribution remains well-defined as long as $0 < b \leq 1/\sqrt{T}$ (see Lifshits, 2002).

In order to facilitate testing, critical values for particular tests (assuming additionally the normality of ε_t) have been computed in a series of Monte Carlo experiments. Applying (7) to data generated by (5) with $b = 0$ gives approximate distributions of relevant statistics under the null in (A). Similarly, applying (7) to data generated by (5) with non-zero values of b gives distributions of the statistics in (B) under the null. In our experiments the

¹ We are grateful to Mikhail Lifshits for additional comment on the independence of the chi squared variables in this theorem.

parameter b has been chosen in such a way that the largest one corresponds to $b = 1/\sqrt{T}$ and the subsequent b 's are proportionally smaller; b is defined as $b = d/\sqrt{T}$, where $d = 1, 0.5, 0.25$ and 0 . For $d = 0$, the data generating process correspond to the null hypothesis in (A), otherwise it is that of (B). Value of T_B , that is, time indicator of a break, see (3) and/or (4), has been set at $0.85T$. This corresponds to the empirical example used in this paper (see further in Section 3). For each set of parameters and each sample size ($T = 1,000, 1,500, 2,500$), 100,000 replications were made. It turned out that sample size do not affect the results markedly.

Table 1 presents the relevant percentiles obtained in each experiment. The way of using this table is the following. To test the set of (A) hypotheses use values given in column headed $d = 0$. For testing the set of (B) hypotheses, it is necessary to compute $\delta = \hat{b}\sqrt{T}$. Next, choose a column headed by d to which δ could be rounded and use its percentiles as the critical values.

Table 1. Critical values for the encompassing test

Test	percent.	$d = 0$	$d = 0.25$	$d = 0.50$	$d = 1$
$t(b)$	1%	-3.45	-3.04	-2.85	-2.56
	5%	-2.75	-2.38	-2.12	-1.61
	10%	-2.40	-1.98	-1.69	-1.05
	90%	2.40	3.11	3.96	5.96
	95%	2.75	3.75	5.41	9.09
	99%	3.45	5.62	10.37	16.19
$t(\gamma)$	1%	as in $N(0,1)$	-2.95	-3.38	-3.87
	5%	as in $N(,1)$	-2.04	-2.11	-2.93
	10%	as in $N(0.1)$	-1.55	-1.60	-2.25
	90%	as in $N(0,1)$	1.83	2.19	2.76
	95%	as in $N(0,1)$	2.40	2.82	3.57
	99%	as in $N(0,1)$	3.54	4.04	4.96
$F(b,\gamma)$	90%	4.47	N/A	N/A	N/A
	95%	5.42	N/A	N/A	N/A
	99%	7.53	N/A	N/A	N/A

3. EMPIRICAL RESULTS

In the empirical analysis we use daily (session-to session) data on prices and returns on 66 stock markets worldwide for the period from the 23rd of March 1992 until the 6th of December 2001. The length of the series varies from 1100 to 2534. It is assumed here that the possible breakpoint in the series, that is T_B in (3) and/or (4), is unknown. As the criteria for selection of the break date we use Zivot and Andrews (1992) criterion, according to which the break is in the period where the linear unit root t -statistic is the smallest. Other criteria considered by Vogelsang and Perron (1998) were also used, giving similar results.

It can be concluded that the *URB* process without a break encompasses the *URD* process with a break if for (7) the F -statistic is significant, b is significant under (A) and (B) and γ is not significant under (B). Analogously, the *URD* process with a deterministic break

encompasses the *URB* process without a break if, in addition to the *F*-statistic being significant in (A), γ is significant under (B) and b is not significant under (C).

The detailed results of the encompassing exercise are presented at Table B1 at the Appendix B. They indicate that the *URB*-no break test shows, on the balance, some advantage over the *URD*-break approach. In 26 cases out of 66 the result is such that the *t*-statistic for \hat{b} under the set of hypotheses (C) remains significant while the corresponding statistic for $\hat{\gamma}$ loses its significance under the set of hypotheses (B). Since in all these cases the *URD* model, while tested separately, appears to be significant, it might be concluded that a true bilinear unit root process might frequently be mistaken for the deterministic unit root process with a structural break. In 6 cases, however, the *URD* model seems to encompass the *URB* model. This includes the important USA market, for which the deterministic unit root model with a structural break appears to be superior. Among 6 countries for which the *URD* model encompasses *URB*, there is only one, Jordania, which is not within the *G25* group of the 25 richest (in terms of per capita real *GDP*) countries. Additionally we have found that only 6 countries belonging to the *G25* group are among the 26 for which the *URB* model encompasses *URD*. Hence, it seems that unit root bilinearity is more typical for the emerging and developing markets rather than for the mature markets.

Under (B), a relatively large number of insignificant results suggest that the problem of identification of underlying stochastic patterns for financial time series is far from being solved. True dynamics of these series is presumably more complicated and involves a mixture of a stochastic unit root process with that of a deterministic break. This, however, requires further investigation.

4. CONCLUSIONS

Stochastic unit root modelling and, in particular, the bilinear unit root approach presented here, offers an attractive alternative to the traditional (deterministic) unit root analysis. The concept of the bilinear unit roots can substantially enrich the analysis traditionally conducted within the deterministic unit root framework. More specifically, a possible speculative bubble interpretation of the *URB* model and the computationally simple nature of its tests create an interesting tool for the analysis of ups and downs on financial markets. So far, speculative bubbles have been regarded as difficult to treat on the grounds of unit root econometrics (see e.g. Evans, 1991). Results presented here also reveal that a substantial number of the empirical financial time series exhibits unit root bilinearity. Testing for this particular form of the stochastic unit root is feasible and can be done without the need for developing specialised software. Finally the *URB* process, being lowly parameterised, does not require specific assumptions or additional tests regarding the nature or timing of the structural breaks.

APPENDIX A. PROOFS OF LEMMA AND THEOREM

Proof of the Lemma. Consider the data generating process (*DGP*) given by equation:

$$y_t = y_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim IID(0, \sigma^2), \quad y_0 = 0, \quad t = 1, 2, \dots, T. \quad (11)$$

For the parameters of the equation of interest (7) the usual *OLS* estimator is given as:

$$\left[\hat{c}, \hat{b}, \hat{\gamma} \right]' = (X'X)^{-1} X'Y, \quad (12)$$

where:

$$X = \begin{bmatrix} 1 & y_1 \Delta y_1 & 0 \\ 1 & y_1 \Delta y_1 & 0 \\ \dots & \dots & \dots \\ 1 & y_{T_B-2} \Delta y_{T_B-2} & 0 \\ 1 & y_{T_B-1} \Delta y_{T_B-1} & 1 \\ \dots & \dots & \dots \\ 1 & y_{T-1} \Delta y_{T-1} & 1 \end{bmatrix}, \quad \text{and} \quad Y = \begin{bmatrix} \Delta y_2 \\ \Delta y_3 \\ \dots \\ \Delta y_T \end{bmatrix}, \quad (13)$$

and T_B is a moment of a possible break. To obtain the asymptotic of matrices $X'X$ and $X'Y$ (with X and Y defined by (13)) under the *DGP* of (11) and the null of $b = \gamma = 0$, let us apply the Donsker 's theorem and some results of Charemza, Lifshits and Makarova (2005), namely of the Statement, section 1 and Lemma C, sections (3) and (4), which give:

$$\begin{aligned} T^{-1}(XX')_{11} &\Rightarrow 1, \quad T^{-1}(XX')_{12} \Rightarrow \sigma^2 \left(\int_0^1 W_1(t) dW_1(t) + 1 \right), \quad T^{-1}(XX')_{13} \Rightarrow 1 - \lambda, \\ T^{-2}(XX')_{22} &\Rightarrow \sigma^4 \int_0^1 W_1^2(t) dt, \quad T^{-1}(XX')_{23} \Rightarrow (1 - \lambda) \sigma^2 \left(\int_0^1 W_1(t) dW_1(t) + 1 \right), \quad T^{-1}(XX')_{31} \Rightarrow 1 - \lambda, \end{aligned} \quad (14)$$

and

$$T^{-1/2}(X'Y)_{11} \Rightarrow \sigma W(1), \quad T^{-1}(X'Y)_{21} \Rightarrow \sigma^3 \int_0^1 W_1(t) dW_2(t), \quad T^{-1/2}(X'Y)_{31} \Rightarrow (1 - \sqrt{\lambda}) \sigma W(1), \quad (15)$$

where $(XX')_{ij}$ and $(XY)_{i1}$ ($i, j = 1, 2, 3$) are corresponding elements of matrices XX' and XY , \Rightarrow denotes weak convergence, W_1, W_2 are two independent Wiener processes on $[0, 1]$ and λ is defined by (10).

Combination of (12), (14) and (15) after some usual algebra complete the proof of the Lemma.

Proof of the Theorem. The test statistics for $b = \gamma = 0$ in (7) has a form of the F -test:

$$F = \frac{(RSS_{(R)} - RSS_{(UR)})/2}{RSS_{(UR)}/(T-4)}, \quad (16)$$

where $RSS_{(R)}$ is sum of squared OLS -residuals from the restricted regression, that is:

$$\Delta y_t = c + w_t, \quad (t = 2, 3, \dots, T), \quad (17)$$

and $RSS_{(UR)}$ is sum of squared OLS -residuals from the unrestricted regression, that is from (7). For the restricted model (17) sum of squared OLS -residuals are:

$$RSS_{(R)} = \sum_{t=2}^T \hat{w}_t^2 = \sum_{t=2}^T (\Delta y_t)^2 - T \left(\frac{1}{T} \sum_{t=2}^T \Delta y_t \right)^2, \quad (18)$$

and, under the DGP of (11) and the null of $b = \gamma = 0$, we get:

$$T \sum_{t=2}^T (\Delta y_t)^2 = T \sum_{t=2}^T \varepsilon_t^2 \Rightarrow \sigma^2, \quad (19)$$

and:

$$T \left(\frac{1}{T} \sum_{t=2}^T \Delta y_t \right)^2 = \left(\frac{1}{\sqrt{T}} \sum_{t=2}^T \varepsilon_t \right)^2 \Rightarrow [\sigma W_1(1)]^2. \quad (20)$$

Sum of squared OLS -residuals from the unrestricted regression of (7) with the use of (12) and (13) may be decomposed as:

$$\begin{aligned} RSS_{(UR)} = \sum_{t=2}^T e_t^2 &= \sum_{t=2}^T (\Delta y_t)^2 + T\hat{c} + \hat{b}^2 \sum_{t=2}^T (y_{t-1} \Delta y_{t-1})^2 - 2\hat{c} \sum_{t=2}^T \Delta y_{t-1} \\ &\quad - 2\hat{b} \sum_{t=2}^T y_{t-1} \Delta y_{t-1} \Delta y_t + 2\hat{b}\hat{c} \sum_{t=2}^T y_{t-1} \Delta y_{t-1} - 2\hat{\gamma} \sum_{t=T_B}^T \Delta y_{t-1} \\ &\quad + 2\hat{\gamma}\hat{c}(T - T_B) + 2\hat{\gamma}\hat{b} \sum_{t=T_B}^T y_{t-1} \Delta y_{t-1} + \hat{\gamma}^2 (T - T_B). \end{aligned} \quad (21)$$

From, (18), (19) and (20) we immediately obtain that, under the null and as $T \rightarrow \infty$, the denominator in (16) will converge to σ^2 , that is:

$$(T-4)^{-1}RSS_{(UR)} = (T-4)^{-1} \left(\sum_{t=2}^T (\varepsilon_t)^2 - T \left(\frac{1}{T} \sum_{t=2}^T \varepsilon_t \right)^2 \right) \Rightarrow \sigma^2. \quad (22)$$

For the nominator in (16), applying the Lemma for decomposition (21), we obtain:

$$RSS_{(R)} - RSS_{(UR)} \Rightarrow \sigma^2 \frac{\left(\int_0^1 W_1(t) dW_2(t) \right)^2}{\int_0^1 W_1^2(t) dt} + \sigma^2 \frac{(1-\sqrt{\lambda})(4-\lambda)}{\lambda(1+\sqrt{\lambda})} W_1^2(1). \quad (23)$$

Combination of (16), (22) and (23) complete the proof of Theorem.

APPENDIX B. EMPIRICAL RESULTS

Table B1 presents the empirical results of the encompassing test exercise. After the codes identifying particular countries (see Table B2 below), the F -statistics for testing the joint significance of the parameters b and γ under the set of hypotheses (A) in (7) are given. Their significance is indicated in the third column as follows: ‘0’ means no significance, ‘+’, ‘++’ and ‘+++’ means that the statistic is significant respectively at 0.10, 0.05 and 0.01 level of significance. Column 4 gives the estimates of the t -statistics for b . The next two columns show their significance in (A) and (B). The last three columns show the values of Student- t statistics for γ in (A) and (B) respectively. The cases where the t -statistic for b under the set of hypotheses (B) remains significant while the corresponding statistic for γ loses its significance are underlined. The codes for countries which belong to the richest in terms of per capita real GDP (so called the $G25$ group) are marked by *. The 6 cases, for which the URD model seems to encompass the URB model are marked by boldfacing.

Table B1: Results of encompassing test

Code	Bilinearity stat.			Break statistic				
	$F(b, \gamma)$	Snfce (A)	$t(b)$	Snfce (A)	(B)	$t(\gamma)$	Snfce (A)	(B)
ARG	33.59	+++	5.62	+++	0	-1.43	0	0
AUS1*	7.67	+++	-2.72	+++	+	-0.54	0	0
AUS2*	5.45	++	-2.32	+++	0	-0.23	0	0
BANG	15.57	+++	3.89	+++	0	0.65	0	0
BEL*	102.76	+++	10.01	+++	++	-1.63	0	0
BRA	21.36	+++	4.60	+++	0	-0.40	0	0
CAN*	8.44	+++	-2.60	+++	+	-1.30	+	0
CHIL	155.72	+++	12.48	+++	++	-0.13	0	0
CHIN	2.58	0	1.23	+++	0	-1.04	0	0
COL	6.39	++	-2.53	+++	+	0.11	0	0
CROA	4.33	0	2.04	+++	0	0.42	0	0
CYPR*	34.76	+++	5.83	+++	0	-0.85	0	0
CZE	43.18	+++	6.57	+++	+	-0.06	0	0

Table B1 (continued)
Bilinearity stat. **Break statistic**

DEN*	8.83	+++	-2.21	+++	0	-1.99	++	++
EGY	10.71	+++	2.87	++	0	-1.57	0	0
EST	1.33	0	-1.13	+++	0	0.23	0	0
FIN*	13.88	+++	-3.46	+++	+++	-1.38	+	0
FRA*	10.34	+++	2.67	+	0	-1.79	0	++
GER*	10.19	+++	2.82	++	0	-1.50	0	0
GRE	59.42	+++	7.62	+++	+	-1.17	0	0
HK*	6.96	++	2.48	+	0	-0.91	0	0
HUN	18.39	+++	-4.25	+++	+++	-0.58	0	0
ICE*	13.85	+++	3.58	+++	0	-1.01	0	0
INDIA	44.84	+++	6.68	+++	+	-0.43	0	0
INDO	46.91	+++	6.79	+++	+	-0.90	0	0
IRE*	7.48	++	-2.62	+++	+	-0.79	0	0
ISR*	3.73	0	1.19	+++	0	-1.52	0	0
ITA*	5.58	++	-1.76	+++	0	-1.57	+	0
JAP*	5.50	++	2.09	+++	0	-1.06	0	0
JORD	15.81	+++	3.47	+++	0	1.93	0	++
KOR	7.86	+++	2.76	++	0	0.49	0	0
LAT	17.39	+++	4.17	+++	0	0.18	0	0
LEBAN	46.57	+++	6.78	+++	+	-0.74	0	0
LIT	45.72	+++	6.76	+++	+	-0.17	0	0
LUX*	63.35	+++	7.42	+++	+	-2.87	+	+++
MAL	21.76	+++	4.66	+++	0	-0.22	0	0
MAU	11.40	+++	3.05	++	0	-1.44	0	0
MEX	72.29	+++	8.48	+++	+	-0.59	0	0
MOR	68.73	+++	8.24	+++	+	-0.90	0	0
NETH*	10.18	+++	2.01	+++	0	-2.48	+	+++
NIG	0.27	0	-0.52	+++	0	0.07	0	0
NOR*	5.91	++	-2.00	+++	0	-1.37	+	0
NZEL*	0.03	0	0.17	+++	0	-0.02	0	0
PAK	18.89	+++	4.34	+++	0	-0.27	0	0
PER	85.86	+++	9.26	+++	++	-0.45	0	0
PHI	90.71	+++	9.48	+++	++	-0.88	0	0
POL	5.33	+	-2.30	+++	0	-0.20	0	0
POR	9.12	+++	-2.61	+++	+	-1.51	+	0
ROM	2.65	0	-1.50	+++	0	0.64	0	0
RUS	3.32	0	-1.82	+++	0	0.01	0	0
SAFR	49.80	+++	7.06	+++	+	0.00	0	0
SING*	22.89	+++	4.76	+++	0	-0.44	0	0
SLOVA	9.14	+++	-2.83	+++	++	1.08	0	0
SLOVE	234.24	+++	15.29	+++	++	0.71	0	0
SPA*	6.06	++	-2.26	+++	0	-0.97	0	0
SRIL	273.56	+++	16.50	+++	+++	1.14	0	0
SUE*	16.14	+++	3.45	+++	0	-2.05	0	++
SWE*	18.65	+++	-4.15	+++	+++	-1.18	0	0
TAI	0.75	0	0.81	+++	0	-0.32	0	0
THA	39.07	+++	6.25	+++	+	0.00	0	0
TUR	20.93	+++	-4.51	+++	+++	-0.74	0	0
UK*	18.65	+++	4.02	+++	0	-1.58	0	0
US*	5.46	++	1.43	+++	0	-1.85	+	++
VEN	10.84	+++	3.14	++	0	-1.00	0	0
ZIMB	283.14	+++	16.82	+++	+++	0.60	0	0

Table B2. Country codes and abbreviations

(codes for countries which belong to the *G25* countries are marked by *)

ARGENTINA	ARG
AUSTRALIA	AUS1*
AUSTRIA	AUS2*
BANGLADESH	BANG
BELGIUM	BEL*
BRAZIL	BRA
CANADA	CAN*
CHILE	CHIL
CHINA	CHIN
COLUMBIA	COL
CROATIA	CROA
CYPR	CYPR*
CZECH REPUBLIC	CZE
DENMARK	DEN*
EGYPT	EGY
ESTONIA	EST
FINLAND	FIN*
FRANCE	FRA*
GERMANY	GER*
GREECE	GRE
HONG KONG	HK*
HUNGARY	HUN
ICELAND	ICE*
INDIA	INDIA
INDONESIA	INDO
IRELAND	IRE*
ISRAEL	ISR*
ITALY	ITA*
JAPAN	JAP*
JORDANIA	JORD
KOREA, REPUBLIC OF	KOR
LATVIA	LAT
LEBANON	LEBAN
LITHUANIA	LIT
LUXEMBOURG	LUX*
MALAYSIA	MAL
MAURITIUS	MAU
MEXICO	MEX
MOROCCO	MOR
NETHERLANDS	NETH*
NIGERIA	NIG

Table B2 (continued)

NORWAY	NOR*
NEW ZEALAND	NZEL*
OMAN	OMAN
PAKISTAN	PAK
PERU	PER
PHILIPPINES	PHI
POLAND	POL
PORTUGAL	POR
ROMANIA	ROM
RUSSIA	RUS
SOUTH AFRICA	SAFR
SINGAPORE	SING*
SLOVAKIA	SLOVA
SLOVENIA	SLOVE
SPAIN	SPA*
SRI LANKA	SRIL
SWITZERLAND	SUE*
SWEDEN	SWE*
TAIWAN, PROVINCE OF CHINA	TAI
THAILAND	THA
TURKEY	TUR
UNITED KINGDOM	UK*
UNITED STATES	US*
VENEZUELA	VEN
ZIMBABWE	ZIMB

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