

The impact of tax progression on employment, hours and wages

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Abstract

I study the effects of tax progression on working hours, wage rates and unemployment in an equilibrium job search model with fixed and indexed unemployment compensation. In the microeconomic analysis, a lower tax progression is always harmful to the firm. The macroeconomic analysis explains that the elasticity of the labor supply, the initial tax wedge progressivity and the unemployment compensation regime are critical variables for the results. In a numerical simulation we show that it is very unlikely to diminish the unemployment rate of low paid workers as the tax progressivity on their wages decreases.

Keywords: Progressive Taxation; Wage Bargaining; Search Models; Endogenous working time; Unemployment

JEL code: E24; H24; J22; J41; J65

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I Introduction

Should the tax wedge progressivity for low paid workers decrease or increase in Europe? The question is of a great concern though tax progressivity may affect differently the unemployment rate, the economic activity measured by the total hours worked and the earnings. In France, the Ducamin (1995) and De La Martinière's reports recommended to lower tax progressivity on low paid workers by increasing the tax base and reducing the marginal taxes. In 2000, the French government adopted the "prime pour l'emploi", a kind of negative income tax for low paid workers. It had in mind to increase the labor supply of the low paid workers so as to diminish their unemployment rate and to increase their earnings. Theoretically, Pissarides (1998) answered to the question by stating that "a reform of the employment tax structure from regressive to progressive can be one of the very few free lunches that one encounters in the analysis of the economic policy" (p177).

In this paper, we investigate the effects of tax progressivity on employment, wages and hours in a theoretical and numerical set up applied to low paid workers. The literature put forward two main dimensions to answer to our question. First, taking into account exogenous or endogenous working hours may lead to different conclusions. Second, the unemployment compensation regime seems to influence dramatically the results.

In the first dimension, the perfectly competitive labor market framework showed that high marginal taxes had a strong disincentive effect on the labor supply. In this type of model, the individual is supposed to be able to vary his working hours freely. The fall of income tax progressivity allows an increase of the labor supply for given wage rates and average taxes through a *substitution effect*. As marginal taxes decrease, the benefit of working an additional hour rises as the individual equalises marginal revenue and marginal cost of working an extra hour. With lower marginal taxes, the marginal revenue increases and assuming that the marginal cost (foregone leisure) doesn't change, the individual works more hours than before. We will call this effect the *labor supply effect*. This effect is also supposed to enhance economic activity and to allow a drop in unemployment.

However, the impact of a modification in income tax progression has also been studied in imperfect labor market frameworks. They lead to radically different results by taking into account the influence of the tax system on the fixation of the wage rate. In bargaining models, where bargaining constitutes the only market distortion source, involuntary unemployment emerges as the wage rate negotiated is superior to the perfectly competitive hourly wage equilibrium. In this type of models, a lower tax progression rises the incentives of the worker to claim higher wages because the worker gets more from a hourly wage increase. We will call this effect the *wage bargaining effect*¹. Lockwood and Manning (1993)

¹In fact, the wage bargaining effect may be considered as a substitution effect on the wage rate. An

and Koskela and Vilmunen (1996) show in models with exogenous working hours that an increase in income tax progression may enhance employment. In these models, the union is faced to a trade-off between employment and higher net wages. As the income marginal tax rate raises, it becomes less costly for the union to "buy" more jobs through wage moderation, since the employee loses a larger fraction of an increase in the pre-tax wage as before. It makes also more attractive for the union to restrict the wage pressure and to favour employment in the negotiation with the firm. Thus, a higher marginal tax rate imposes a joint loss to the pair composed of the firm and the union (the price of a wage increase will increase) that can be reduced by keeping wages low. Most of the empirical studies put forward this effect, whatever the country, the period or intermission chosen (see Malcomson and Sartor (1987), Holmlund and Kolm (1995)...). However, Sorensen (1997) explains that "the magnitudes of the estimated tax rate elasticities also differ dramatically across studies and countries, so at the present stage there is great uncertainty regarding the quantitative impact of taxation on the wage formation" (p230).

It seemed also interesting to integrate the two polar approaches incarnated by the *labor supply* and *union effect* in the same framework by taking the working hours endogenous. Some researchs were conducted to fill this emptiness. Holmlund and Kolm (1995), Hansen (1999) and Hansen et alii (2000) investigate how the progressive tax system affects wage formation. Hansen's conclusions are astonishing because they do not differ from the previous works. Reducing tax progression results in unambiguous increases in hourly wages, individual working hours and unemployment. The interesting aspect of his paper comes from it's simple framework that is very close to the Pissarides (2000, chap7 and 9) ones. However, a problem subsists in his article. His conclusions rely on the assumption of a recruiting cost depending on the average yearly productivity of an employee. It seems difficult to justify such a representation of the hiring cost in the reality² and it is more reasonable to suppose that the firm posts a vacant job being informed of the cost of it. In our article we therefore prefer to remove this hypothesis and use the more standard assumption of a recruiting cost that does not hinge on working hours. Hansen et alii (2000), with a fixed unemployment benefit, showed that the effect of a decrease in the marginal tax rate is indeterminate. A decrease in tax progression enhances employment if the initial tax system is sufficiently progressive, the union has a low bargaining power and/or the elasticity of labor supply with respect to the hourly wage is high.

The second dimension, put forward by Pissarides (1998) and Holmlund and Kolm (1995) focuses on the impact of unemployment compensation. According to Pissarides (1998), the incidence of taxes on wages and employment hinges on the impact of income

income effect would be represented by a modification of the average tax rate.

²Except for idiosyncratic segments of population

taxes on the replacement rate. He shows for instance that if unemployment benefits are indexed to wages, real wages are flexible and they absorb tax changes without much impact on employment. For a fixed unemployment benefit regime, there are substantial employment effects. As the progressivity increases, the employee's rent decreases because the fixed unemployment compensation being not taxed is not influenced by a progressivity modification.

In this paper, I first analyse the impact of fixed unemployment benefits before studying the effects of an indexed unemployment compensation (a fixed replacement ratio)³. I mix also the models of Pissarides (1998) and Hansen (1999). I take a sufficient canonical model so as to put forward the impact of the *wage bargaining effect* and the *labor supply effect* and range the intensity of each one.

Our analysis proceeds as follows. In the next section, we expose the theoretical framework. The microeconomic analysis of the third section shows that a lower tax progression is always harmful for the firm whatever the unemployment compensation regime. The macroeconomic analysis of the fourth section shows that the elasticity of the labor supply and the initial tax wedge progressivity are critical variables for a *labor supply effect* dominating the *wage bargaining effect*. Indeed, we show that the global effect of a lower tax progression on unemployment might be indetermined in each unemployment compensation regime. This leads us to proceed to a calibration and a simulation with french data to put forward the most important effect in the fifth section. We show that while the working hours increase by dropping the tax progressivity, the unemployment compensation regime is crucial for the results. In fact, it seems impossible to diminish the unemployment rate as the replacement ratio is fixed. However, my main conclusion is consistent with the main literature stream on the question: it is very unlikely to diminish the unemployment rate of low paid workers as the tax progressivity decreases. I also put forward a tradeoff between economic activity measured by total working hours and the unemployment rate.

II The framework

II.1 General assumptions

³One can also put forward the fact that in reality, the unemployment compensation are never fixed in the very long run though the unemployed would protest (as it happened in France in 1997). However, the unemployment compensation is decreasing with the time staid in unemployment and indexed on the past wages so that they are not stricly indexed on wages. Reality lies also between the two polar cases studied.

For the sake of simplicity, we confine our analysis to steady-state values and as in Pissarides (2000) the state budget constraint is omitted⁴. All the earnings are consumed at the present period so that no investment and capital are taken into account.

Our analysis is carried out in a matching framework *a la* Pissarides (2000).

We assume the firm and the job seeker to bargain over the wage rate and working hours. However, we study a right-to-manage model⁵. It allows us to take into account the empirical fact that bargaining agreements often stipulate the two variables at the same period, notably in France for the overtime work or since working time reduction is imposed by law.

To get a better tractability, we will consider only the case of tax progression lowering the marginal tax rate for given average tax rates. We focus our analysis on particular segments and consider certain classes of earnings which are very different so that we don't allow the possibility of a transition of a worker of a certain class to another class⁶. Implicitly, we break the labor market in various segments⁷, hypothesis allowing to consider exogenous marginal tax rates⁸.

II.2 The model

Transaction costs imply that vacant jobs and unemployed workers are matched together in pairs through an imperfect matching process. The matching function gives the rate at which good matches are formed in the market. We take the simplest form of the matching function, say it depends on the number of vacant jobs v and the number of job seekers u ⁹. We suppose that the matching function $M(v, u)$ is increasing in each argument, is continuously differentiable, homogenous of degree one and yields no hiring if v or u is nil. The transition rate for vacancies is also written as

$$\frac{M(v, u)}{v} = q(\theta)$$

⁴This assumption is not crucial since an increase in the unemployment compensation or a decrease in the tax progression for the segment of population studied would be financed by another segment of population.

⁵As Hansen (1999), argues, we are still in the right to manage model here because the firm determines solely the employment and the worker is tempted to reduce the working hours after the agreement. We will assume in the paper that such a behaviour is impossible because each protagonist is engaged in the agreement so that he can't deviate.

⁶The limit of this assumption is that we suppose that a single persons will stay single his whole life he will never get married. We have to make this assumption for the sake of simplicity.

⁷Obviously, we assume that each segment having the same average product will have the same earnings so that we elude asymmetric informations problems on effort. However, taking endogeneous hours of work resumes in a certain way as the analysis on the effort at work.

⁸As in most of the studies, the marginal tax rate is given. This hypothesis does not seem very crucial for the analytical results in comparison to the complexity it would integrate. Numerically, Holmlund and Kolm (1995) show that this assumption does not change dramatically the results.

⁹For simplicity, we assume that the unemployed and firms search intensities are fixed.

where $\theta \equiv \frac{v}{u}$ is the labor market tightness and $q'(\theta) \leq 0$. Similarly, the job finding rate is given by

$$\frac{M(v, u)}{u} = \theta q(\theta)$$

where $\theta q(\theta)$ is increasing in θ .

II.2.1 Workers

There are two goods in the economy. First we have the labor, which is the sole input. It is characterized by a continuum of infinite lived individuals¹⁰, which size is normalised at one. Each worker negotiates h hours per unit of time and the cost wage rate w with the firm, and produces p units of product per hour when he is employed¹¹. The household production and search-on-the-job are omitted.

The expected present value W_i of the stream of incomes of a worker i who is employed in the firm i at the stationary equilibrium is:

$$rW_i = I(w_i, h_i) + \lambda(U - W_i) \quad (1)$$

where λ denotes the job destruction rate per unit of time. We also assume that negative idiosyncratic shocks arrive at constant rate λ ¹². These shocks lead to the destruction of the job and the entry of the worker into unemployment.

r is the discount rate

U is the average expected value of the stream of income of an unemployed worker

$I(w_i, h_i)$ denotes the instantaneous flow of utility. It takes the form

$$I(w_i, h_i) = w_i h_i - T(w_i h_i) - a \frac{h_i^\alpha}{\alpha}$$

where

$T(w_i h_i)$, represents the tax wedge on a job i ¹³.

¹⁰Through this article we use the terms "household", "worker" and "individual" interchangeably. Labor supply may however involve a complex joint decision within the households, but, the analysis concentrates on single individuals.

¹¹We abstract from possible effects of working hours on labor productivity. When the worker does not work an extremely high number of hours (which is particular case for low paid workers), it seems consistent to suppose that the profile of his productivity is not really affected by the time spent at work.

¹²Pissarides (2000) argues that the assumption of a constant destruction rate is reasonable in the long run equilibrium setting, as in our paper, but it is less appropriate for the cyclical analysis of the job flows.

¹³In reality, the whole tax wedge is not supported by the employer. However, in our model, the wage rate is flexible so that economically, the tax burden is always carried out by the employee. Nickell and Bell (1995) on 13 countries of the ODCE show that in the long run, the payroll taxes paid by firms are broadly absorbed by the net income levels. Indeed, the equilibrium long run wage cost does not depend on the composition of the tax wedge.

$a \frac{h_i^\alpha}{\alpha}$, indicates a disutility from working and α , the elasticity of marginal disutility with respect to work, is supposed to be bigger than one. a is a ladder factor.

For the sake of simplicity we suppose that agents are not risk averse and the utility function is quasi-linear. Thus, we do not take into account the income effect. We choose such an utility function to set our work in the literature stream and to compare our results to the existing studies. Several reasons support to believe that it does not seem very embarrassing to comparative static results. First, empirical studies find small income effects relative to labor supply effects (substitution effects) (see, Blundell and MaCurdy (1999) or Lockwood and Manning (1993)¹⁴). Second, the income effect is important to study the case of an extensive margin (ie the participation to the labor market) but not for an intensive one (the number of hours worked), now we focus on the intensive margin.

The expected value of the stream of income of an unemployed worker satisfies:

$$rU_i = z + \theta q(\theta)(W - U_i) \quad (2)$$

where W is the average expected present value of a job

z denotes the unemployment compensation

Two kinds of unemployment compensation are taken into account. In the first case, we will consider fixed unemployment compensation and in the second indexed ones.

We have for indexed unemployment compensation:

$$z = \rho(\bar{w}\bar{h} - T(\bar{w}\bar{h})) \quad (3)$$

where ρ is the net replacement rate ratio and

$$\bar{j} = \frac{1}{n} \sum_1^n j_i di \text{ with } j = w, h$$

where n represents the number of worker employed. In this case, the unemployment compensation will be influenced by the progressivity.

For a fixed unemployment compensation, we have,

$$z = \bar{z} \quad (4)$$

¹⁴They argue :”In the long run, it may well be the case that the non-proportional aspects of the tax system have a larger effect on the natural rate of unemployment than the average level of taxation” (see p2). Many studies show for instance that the non compensated elasticities of labor supply are much more higher than the income elasticities, in particular for women.

II.2.2 Firms

An endogeneously sized continuum of competitive firms produce a numeraire good with labor. Each firm produces the same good which is sold on a competitive market so as the firm can not influence the price of the good. We normalize this price at one. The firm is supposed to have one job that can be either filled and producing or vacant and searching.

The asset value J_i of a filled job for the firm i is:

$$rJ_i = h_i(p - w_i) + \lambda(V - J_i) \quad (5)$$

The asset value of a vacancy, denoted by V is:

$$rV = -c + q(\theta)(J_i - V) \quad (6)$$

where J represents the average expected present value of profits from an occupied job c is the cost of a vacant job per unit of time. We suppose that c is a fixed cost. It might depend on the productivity of the employee but it does not change the qualitative results¹⁵. c is composed for instance by the volume of advertisement for capturing an employee on a vacant job.

III Partial equilibrium relations¹⁶

III.1 The negotiations¹⁷ between the firm and the worker

Job seekers and firms with vacant jobs go through a costly and time-consuming search process before they can be matched. They choose the wage rate and the hours of work by taking into account the fact that if they do not agree to form the job, their best alternative is to search for alternative partners at the following period. At the equilibrium, they also prefer to come to an agreement with each other at the present period.

The bargaining outcome solves the asymmetric Nash-bargaining problem which is reduced to the maximisation of the Nash criterium ie the surplus of each protagonist-firm and a worker- weighted by their bargaining power. The bargaining carries on the wage cost

¹⁵Pissarides (2000) takes a hiring cost proportional to productivity, assumption, which seems "natural to make in the long run equilibrium, since the costs of a firm have to rise along with productivity to ensure the existence of a steady state"(p10). Now, the productivity of a worker is constant in our paper and we analyse only certain classes of earnings who approximately have the same productivity so that the assumption of a fixed cost of hiring seems natural to take.

¹⁶We won't distinguish the two cases in the microeconomic analysis since the atomicity of an individual worker does not able him to influence the level of the unemployment compensation.

¹⁷We consider solely individual negotiations

rate and the hours of work¹⁸. The bargaining is decentralised so that the maximisation takes place with the aggregate variables taken as given ie the disagreement point (U, V) .

From this bargaining it can be shown (see appendix VIII.1) that the wage setting equation is:

$$w = p - \frac{(1 - \beta)(r + \lambda)}{\beta(1 - T_M)v h} (W_i - U) \quad (7)$$

where v is the total progression coefficient put forward by Musgrave and Musgrave(1989), which we call the progression index¹⁹. We have :

$$v = \frac{1 - T_m}{1 - T_M}$$

which is the ratio of the tax progression index supported by the firm and the worker.

T_M is the average tax wedge and T_m , the marginal tax wedge. We impose $T_m \geq 0$, with T_m the constant marginal tax rate.

If $v < 1$, an increase of 1% of the hourly net wage corresponds to a raise of less than 1 % of the hourly cost wage. In this case the marginal tax rate on income is higher than the average tax rate so that we have a progressive tax system. By considering only values of $v \leq 1$ ie cases of tax progressivity, we stick to the reality because direct taxes on labor are progressive in most of the industrialised countries.

We have as well from the maximisation, the equation giving the choice of hours worked:

$$h^{\alpha-1} = \frac{p}{a} v (1 - T_M) \quad (8)$$

(see appendix VIII.1). Because we concentrate our study on the efficient bargaining, the working hours are set where marginal disutility of work equals the marginal gain to the two parties resulting in a Pareto-optimal outcome. Here, the number of working hours is chosen by comparing the joint marginal costs and gains of one more hour to the firm and to the worker. The difference lies in the marginal cost which includes the product from one more hour and not only the wage rate (see Pissarides, 2000). The variation in progressivity removes the contract curve in the sense that the hours worked increase with the progression index. The parameter α determines the responsiveness of hours with

¹⁸It is indifferent to bargain over the wage cost rate or the tax net wage rate since there is a monotonic relation between the two in our model (the output prices are fixed).

¹⁹The progression index represents the elasticity of the net income with respect to the gross income or here the cost for the firm. The use of this index is identical to a linear tax schedule of the form $T(wh) = awh + b$, where a is the marginal tax rate and b is an endogenous negative allowance. We suppose that this elasticity is constant with the wage, assumption that does not seem crucial because the progression index is a local measure of the progressivity ie carries on a particular wage segment of the population.

respect to changes in progressivity. The larger α ie the less elastic the labor supply is, the less responsive hours are to changes in the progression index and the less the labor supply effect is important. We can as well put forward the fact that there is no direct effect of the wage rate on the working hours (since T_M is assumed to be constant).

III.2 The influence of a lower tax progression

Now, I turn to the impact of an exogenous change in the marginal tax rate through the progression index ν on the wage rate, the working hours, the utility of the employee and the profit of the firm.

In this section we will see that:

Proposition 1 *At a partial equilibrium level, a lower tax progression implies larger working hours, a higher wage rate, has a positive influence on the employee's rent and a negative impact on the profit per worker of the firm.*

Proof. See appendix VIII.2 ■

We obtain (see appendix VIII.2):

$$\xi_{h,\nu} = \frac{1}{\alpha - 1}, \text{ which sign is always positive.}$$

$\xi_{w,\nu} = \frac{p-w}{w} \left(\frac{1}{\alpha - 1} + \beta \right)$, which sign is always positive. The economic intuition is as follows. A lower marginal tax rate on labor income reduces the cost to the employer of providing the worker with some given increase in the after-tax wage rate. At the same time, it raises the cost of the worker of conceding more profit to the employer. The job seeker asks also for a higher wage rate. A lower tax progressivity will thus lower the tax on his rent and increase at the margin his implicit market power.

To put forward the impact on the worker's utility, we look at the evolution of it's rent. We obtain (see appendix VIII.2) :

$$\frac{d(W - U)}{d\nu} = \frac{\beta(p - w)}{r + \lambda} h(1 - T_M)$$

which is always positive.

This means that the employee's rent increases as tax progressivity declines. It is logical because the utility of an unemployed being constant, it is treated as it were not taxed. A decrease in progressivity raises the wage rate and the attractivity of work with respect to leisure.

Finally, we have the elasticity of the intertemporal profit per job with respect to the index of tax progression:

$$\xi_{J_i,\nu} = -\beta$$

This elasticity is always negative. The profit of the firm will also always diminish as the tax progression decreases, except when the bargaining power of the employee is nil.

To understand some mechanism more intuitively, we intend to study the impact of the negotiation power of the worker. It can be noticed that for a worker's negotiation power of 1, the elasticity of the wage rate with respect to the index, is nil and for a very weak negotiation power of the worker ($\beta \rightarrow 0$), we obtain $\xi_{w,v} = \frac{(\frac{p}{w} - 1)\alpha}{\alpha - 1}$, which is positive. This means that the wage rate always increases when the initial intertemporal profit of the firm is different of zero. As the negotiation power of the worker is nil, the wage rate increases for a lower tax progressivity because the worker's rent would decrease too much if the increased working hours were not offset by an increase of the wage rate. The firm accepts also to compensate him so as the worker's rent does not change²⁰. In effect, we can remark that for $\beta = 0$, we have $\xi_{J,v} = 0$ and for $\beta = 1$, we have $\xi_{J,v} = -1$. In the first case, even with an incentive to ask for a higher hourly wage as the tax progressivity decreases, the worker is only able to ask for an increase corresponding to the supplement of hours he works. In this case, the profit of the firm is not influenced because the increase of the wage rate is exactly offset by the increase of the hours worked. The pie does also not increase because the jump of the hours worked is offset by the increase of the disutility of the worker. The second case corresponds to the inverse situation. The profit per hour is nil at the initial equilibrium. The increase of the wage rate will thus decrease the profit and the increase of working hours worsen this tendency.

All these results on the profit are *a priori* astonishing. In effect, we saw in appendix VIII.2 that:

$$\xi_{J,v} = \xi_{h,v} + \xi_{w,v} \varepsilon_{J,w}$$

This equation explains that the increase in the hourly wage has to be offset by an increase in the hours worked so that the decrease of the profit per hour is compensated and the profit per worker becomes positive. Other said it means that the labor supply *effect* dominates the *wage bargaining effect*. However, at the microeconomic level, this case does never happen because the worker is not concerned with the influence of his own negotiations on the labor market tightness.

In fact, the result is relied to the specification of the instantaneous utility function of the worker. He values more an increase of one percent of the hourly wage than an increase of the working hours because hours and wage rates act the same way in the net wage but the working hours are decreasing the utility of the worker.

²⁰This effects hinges clearly on the specification of the utility function of the worker.

IV The general equilibrium model

The macroeconomic model allows to put forward the spillovers between firms and employees. Because the matching function is increasing in both arguments, the unemployed workers and vacant jobs are complements. When a new firm enters the market and offers a vacant job, the labor market tightness increases and an unemployed has more chances to find a job. There are also positive external effects of the entry of a new firm on the workers, effect that is called positive intergroup external effect. On the same side of the market, there is a congestion effect (called negative intragroup effect) coming from the fact that a new entrant firm renders more difficult the other firms to fill their vacancies.

By closing the model and thus endogenizing these two probabilities, we implicitly give an employment concern to the workers because now they have the power to influence by their bargaining the probabilities of matches to succeed and the expected present value of the stream of income of an unemployed. Thus, contrary to the microeconomic level, the welfare of an unemployed is endogenous.

The labor market tightness is a critical variable because it transmits the effects of parameter changes to all the endogenous variables. η , the elasticity in the matching function, captures the intergroups and intragroups externalities effects. We will take it as exogenous assuming that the matching function is of a Cobb-Douglas form.

The Beveridge curve defining a negative relation between vacancies and unemployment shows the inadequate unemployment by equalizing flows in with flows out of unemployment. Assuming unchanged labour force we also have a steady state equilibrium unemployment rate equal to:

$$u = \frac{s}{s + \theta q(\theta)}$$

IV.1 The price setting relation

The free entry condition reads as $V=0$ and implies from the equation (6),

$$J = \frac{c}{q(\theta)} \tag{9}$$

which is the average recruitment cost per worker. The equation (9) means that firms enter the market as long as the expected return from one more vacant job is driven to zero, or other said as the intertemporal profit of a filled job is compensated by the actualised cost of a vacant job. Contrary to many studies on the influence of tax progressivity, it is implicitly assumed that employed individuals and working hours are not perfect

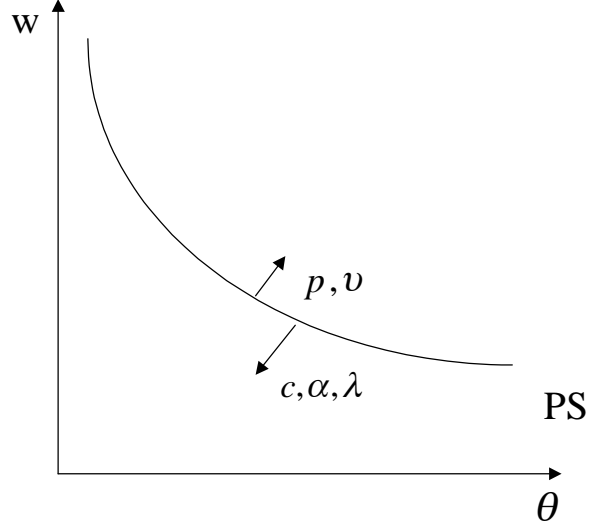


Figure 1: The price setting curve

substitutes. It is straightforward to see that the firm intends to fix the highest working hours as possible to carry "scale economies" on the vacant job cost.

In the symmetric equilibrium, we have $w_i = w$ and $h_i = h$ so that $J_i = J$ for all i . By substituting the equations (9) and (8) in the equation (5) and taking into account that $V=0$, we obtain a price setting equation (or job creation curve):

$$w = p - \frac{c(r + \lambda)}{\left(\frac{p}{a}v(1 - T_M)\right)^{\alpha - 1} q(\theta)} \quad (10)$$

This equation defines the labor demand schedule in the plane (θ, w) and shows that the firm enters until the hourly wage cost equals the net gain equal to the average product minus the expected capitalized value of hiring costs. This relation is decreasing in θ . When the wage rate falls, the expected profits on a job rise so that there are more entry of firms and more job openings. As usual, the wage rate increases with the productivity of the worker. It increases with the hours worked because of the scale economies on the hiring cost and with the probability of a vacant job to be filled because it diminishes the time of the job to stay vacant. For the same reason, it decreases with the separation rate. The wage rate is also decreasing to the extent of the hiring cost because the higher it is, the less will be the intertemporal profit on a job.

IV.2 The wage setting relation

IV.2.1 For indexed unemployment compensation

From the Nash bargaining, at the symmetric equilibrium, we obtain (see appendix VIII.3) the wage setting equation:

$$w = v \frac{p(\beta + \frac{(1-\beta)}{\alpha} \frac{r+\lambda}{r+\lambda+\theta q(\theta)})}{\beta v + (1-\beta)(1-\rho) \frac{r+\lambda}{r+\lambda+\theta q(\theta)}} \quad (11)$$

How does the wage setting curve evolve with the modification of an exogenous variable, the other given? It shifts to the left in the mark (θ, w) with :

- the productivity p of the worker because it increases the pie to share which allows a higher wage rate.

- the bargaining power of the worker β . As usual, the higher the bargaining power of the worker, the higher is the wage rate (see appendix VIII.4).

- the progression index ν . This effect resumes the *wage bargaining effect*. A lower marginal tax may encourage the worker to drive up the wage rate, thereby rising the number of hours "sold" to the employer, since the lower marginal tax rate enhances the individual's net gain from working an extra hour.

- the net replacement ratio ρ . Implicitly, for a higher net replacement ratio, the reservation wage rate increases and the worker asks for a higher wage rate.

It shifts to the right in a mark (θ, w) with α . The higher the elasticity of labor supply, the more the individual reacts to a modification of the marginal tax wedge and accepts to work a long time for a given tax wedge. The firm can afford to negotiate a higher wage rate with the worker because for a lower tax progression, the size of the pie is increased (for θ given).

This relation defines an increasing and concave curve in the (θ, w) plan which might be interpreted by the fact that the worker is interested in the employment situation. In fact, the *wage bargaining effect* is higher when θ is higher ie the employment situation is better. More vacant jobs enhance the wage rate because they increase the possibility of an unemployed to find a job, which implicitly gives him more bargaining power.

IV.2.2 For a fixed unemployment compensation

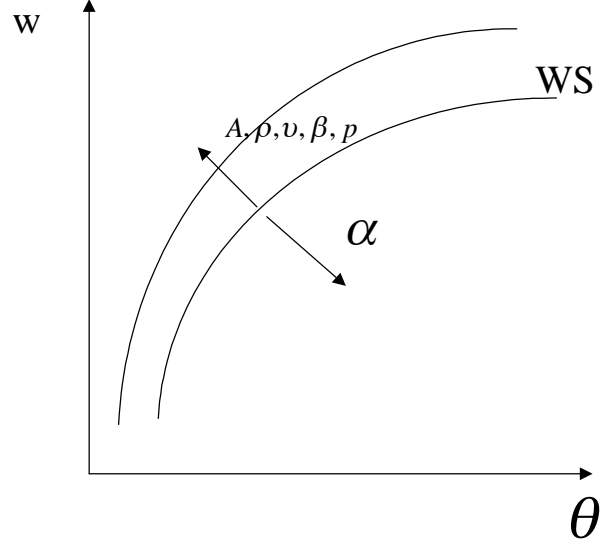


Figure 2: The wage setting curve for indexed unemployment compensation (with ρ) or a fixed unemployment compensation (with A)

From the Nash bargaining, at the symmetric equilibrium, we obtain (see appendix VIII.3) the wage setting equation:

$$w = \frac{pv\beta + (1 - \beta) \frac{r + \lambda}{r + \lambda + \theta q(\theta)} \left(\frac{pv}{\alpha} + \frac{\bar{z}}{1} \right)}{\beta v + (1 - \beta) \frac{r + \lambda}{r + \lambda + \theta q(\theta)}} \frac{\left(\frac{p}{a} v(1 - T_M) \right)^{\alpha - 1} (1 - T_M)}{1} \quad (12)$$

We obtain the same results and economic intuitions as for an indexed unemployment compensation.

IV.3 The impact of a lower tax progression

We intend to find the effect of tax progression on the wage rate, the labor market tightness, the working hours, the profit of the firm and the unemployment rate for each unemployment compensation regime. Not to cumbersome the notations, we won't give analytical conditions on the elasticity of labour supply. Our analysis lies on total elasticities with respect to the progression index. These elasticities are derived from the partial elasticities coming from the wage setting, the price setting and from the equilibrium working hours relations.

IV.3.1 On the working hours

The impact is the same for the two unemployment compensation regimes. We have,

$$\xi_{h,\nu} = \frac{1}{\alpha - 1}$$

, which is positive. Solely the *labor supply effect* coming from the modification of the marginal tax influences the working hours.

IV.3.2 On the wage rate

For an indexed unemployment compensation In appendix VIII.5, we show that,

$$\xi_{w,\nu} = \frac{\eta \varepsilon_{w,\nu}^p (\alpha - 1) + \varepsilon_{w,\theta}^p}{(\alpha - 1) (\eta - \varepsilon_{w,\theta}^p \varepsilon_{J,w})}$$

is positive, calling $\xi_{w,\nu}$ the total elasticity of the wage rate with respect to the progression index ν . The economic intuition is as in the microeconomic model, say a lower tax progression encourages the worker to claim a higher wage rate because the net gain from an increase is higher (the wage setting curve shifts to the left). The price setting curve shifts to the right because a lower tax progressivity increases the working hours, which allows the firm to bear a higher hourly wage.

There are several partial elasticities not seen in the microeconomic analysis to explain.

$\varepsilon_{w,\theta}^p$ is the partial elasticity of the wage rate with respect to the labor market tightness. This elasticity shows the impact of the employment situation on the wage rate bargained. It captures also the employment concern of the pair worker-firm on the employment. The higher the labor market tightness, the higher is the job finding rate for an unemployed. In this case, the worker has less to worry about the employment and claims for a higher wage rate.

One can analyse the special effect of the worker's negotiation power. As it is equal to one, we have $\xi_{w,\nu} = 0$ and we obtain the same result as in the microeconomic analysis. As it is nil, we have $\xi_{w,\nu} = 1$. In this case, the initial wage is so weak that a decrease in the tax progressivity increasing working hours, the firm is constrained to negotiate a high increase of the wage rate to preserve the worker's rent.

For a fixed unemployment compensation In appendix VIII.5, we show that,

$$\xi_{w,\nu} = \frac{\eta \varepsilon_{w,\nu}^p (\alpha - 1) + \varepsilon_{w,\theta}^p + \eta \varepsilon_{w,h}^p}{(\alpha - 1)(\eta - \varepsilon_{w,\theta}^p \varepsilon_{J,w})}$$

Contrary to the case of a fixed replacement ratio, $\xi_{w,\nu}$ is not always positive. The economic intuition remains the same as for the indexed unemployment compensation except for $\varepsilon_{w,h}^p$, which is negative. This effect is *a priori* counterintuitive. In fact, for a decrease of the working hours, the worker has a too small wage to offset the distutility of work. His utility might also be smaller than the utility of an unemployed. To keep working, he asks for a higher wage rate in the negotiation and the firm has an interest to give him this increase since its profit on a filled job is higher than the profit on a vacant job which is nil. Inversely, as the hours of work increase, the rent of a worker increases a lot. His rent is also sufficiently increased, which reduces his pressure on the wage rate to raise it more.

One can analyse the special effect of the worker's negotiation power. As it is equal to one, we have $\xi_{w,\nu} = 0$ and we obtain the same result as for the indexed unemployment compensation. As it is nil, we have $\xi_{w,\nu} = \frac{p\nu}{p\nu + \alpha Y} < 1$. The economic intuition is also the same as for an indexed unemployment compensation. However, this elasticity is lower than in the previous case.

IV.3.3 On the profit of the firm

For an indexed unemployment compensation We have:

$$\xi_{J,\nu} = \frac{1}{\alpha - 1} \frac{\eta}{(\eta - \varepsilon_{w,\theta}^p \varepsilon_{J,w})} ((\alpha - 1) \varepsilon_{J,w} \varepsilon_{w,\nu}^p + 1)$$

which sign is indetermined. The first term is positive because $\varepsilon_{J,w}$ is negative and the other elasticities are positive. The second term has an indetermined sign because its first term is negative and its second term is positive. The higher $\varepsilon_{w,\nu}^p$ ie the more the worker claims a wage increase for a lower progressivity (high bargaining effect), the more chances the second term is negative and the total elasticity is negative. The higher the elasticity of labor supply ie the lower α , the more chances $\xi_{J,\nu}$ is positive.

The first term in the term in bracket is negative. It shows that there is a negative impact of the variation of the progressivity on the wage rate which *ceteris paribus* diminishes the profit to the extent of the elasticity of the labor supply.

Thus this equation explains that the increase in the wage rate might be lower as in the microeconomic model. The latter increase might also be offset by a sufficiently high increase in the hours worked so that the decrease of the profit per hour is compensated and

the profit per worker becomes positive. Other said it means that the *labor supply effect* dominates the *wage bargaining effect*. Our results are also different from the microeconomic level where the profit was always decreasing for a lower tax progression. Intuitively, one can anticipate that for a higher elasticity of labor supply, the *labor supply effect* has more chances to dominate the *wage bargaining effect*.

One can analyse the special effect of the worker's negotiation power. As it is equal to one, we have $\xi_{J,\nu} = 1$ and we obtain the same result as in the microeconomic analysis. The profit jumps on because the working hours increase and the wage rate does not. As it is nil, we have $\xi_{J,\nu} = \frac{-v}{\alpha(1-\rho) - v}$. This elasticity is negative as the initial progressivity is sufficiently low (the initial wage rate is also relatively high and its increase is harmful for the profit), the elasticity of the labor supply is sufficiently low (the wage rate increase is not offset by the increase of the working hours) and the replacement ratio ie in fact the reservation wage is sufficiently high.

For an fixed unemployment compensation We have:

$$\xi_{J,\nu} = \frac{1}{\alpha - 1} \frac{\eta}{(\eta - \varepsilon_{w,\theta}^p \varepsilon_{J,w})} \left((\alpha - 1) \varepsilon_{J,w} \varepsilon_{w,v}^p + 1 + \varepsilon_{w,h}^p \varepsilon_{J,w} \right)$$

which sign is indetermined. We remark again that the probability for the profit to increase for a lower progressivity seems higher as in the case of an indexed unemployment compensation. The last term in the term in bracket is positive. The economic intuition behind this term comes from the effect of the concavity of the wage rate with respect to the working hours. In fact, an increase in the working hours limits the wage rate increase call of the worker and this has a positive effect on the profit. When this effect is sufficiently important, the total effect of a decrease of the tax progressivity on the profit of the firm is positive.

One can analyse the special effect of the worker's negotiation power. As it is equal to one, we have $\xi_{J,\nu} = 1$ and we obtain the same result as in the microeconomic analysis. The profit jumps on because the working hours increase and the wage rate does not. As it is nil, we have $\xi_{J,\nu} = \frac{-p\nu + Y\alpha}{p(\alpha - \nu) - Y}$, which sign is indetermined.

IV.3.4 On the labor market tightness

The impact on the labor market tightness has the same form for each unemployment compensation regime. We have:

$$\xi_{\theta,\nu} = \frac{\xi_{J,\nu}}{\eta}$$

Since $\eta < 1$, the effect of a modification of the progression tax index on the profit of each firm is amplified by the spillovers in the matching function.

IV.3.5 On the unemployment rate

We obtain, for the two unemployment compensation regimes, the elasticity of the unemployment rate with respect to the tax progression index $\xi_{u,\nu}$ (see appendix VIII.5):

$$\xi_{u,\nu} = -\xi_{\theta,\nu} \frac{\theta q(\theta)(1-\eta)}{s + \theta q(\theta)}$$

which sign is indetermined and depends on the sign of $\xi_{\theta,\nu}$. But the effect of the last elasticity is attenuated or amplified by the second member of this relation. The effect on the unemployment is also larger as:

- the exogenous employment destruction rate is small, which signifies that a filled job is hold a sufficient long time to offset the cost of a vacant job. The firms will also tend to open more vacant jobs because the expected value of a filled job increases.

- the job finding rate $\theta q(\theta)$ is high (see appendix VIII.6) initially ie the labor market is very tight, the implicit power of the worker is very high. In this case, there are strong probabilities for $\xi_{\theta,\nu}$ to be negative and a lower tax progression has more chances to increase the unemployment rate.

We can summarise the results put forward in this section by the following proposition:

Proposition 2 *At a macroeconomic level, for indexed and fixed unemployment compensation, a lower tax progression has a positive influence on the wage rate, the working hours, an indetermined effect on the labor market tightness, the profit of the firms and the unemployment rate. The solution of the tradeoff between economic activity and unemployment is indetermined.*

V A numerical simulation

Now, we try a rough calibration of our model to evaluate the relative strength of the counteractive effects we pointed out in the previous subsections. We choose to limit our numerical simulation to the macroeconomic analysis as its analytical results are ambiguous. We first expose the calibration methodology. Then we present the simulations.

V.1 The calibration of the model

There are two kinds of parameters in the model. Parameters whose values are known and parameters whose values are set to reproduce some key facts of the French economy over the current period.

First, we adopt the following constant returns Cobb-Douglas matching function $m(u, v) = do u^\eta v^{1-\eta} = do \theta^{-\eta}$, and we take $\eta = \beta = 0.5$ (see Blanchard and Diamond, 1989). For an analysis depending not solely on the variation of the variables but on their level too, we choose to take into account a ladder factor in the matching function relation.

Second, we get empirical evaluation of the discount factor, of the separation rate, of the average and marginal tax wedge rates. Since wage bargaining generally stands for a year, according to the Lois Auroux, our basis time will be a year. We set also the discount rate around 5%. The rate of job destruction is set at 0.2. It is higher than the rate given by the OECD data (1995) but it suits better with the situation of low wage persons, which generally are unqualified. The labor is assumed to be homogenous in our model. However, we intend to study the tax modification of different segments of the labor market by classes of earnings. We choose to study the classes in reference to the income level of the average production worker. We include into the tax wedge personal income taxes, social security contributions by employers and employees, plus consumption taxes. For the sake of simplicity and to avoid the problems of labor supply of a household, we consider only the single persons. Two classes are given for references²¹. The first earns 67% of the the income level of the average production worker (APW), the second 100%. We are not also interested in the upper class of earnings, which can be justified by the fact that their elasticity of labor supply is very weak.

For the sake of simplicity, in our model, the negotiations carry on the cost rate and the working hours. We have also to proceed to transformations to obtain the average and marginal taxes englobing the taxes paid by the employee and the firm. In appendix VIII.7 and VIII.8, we explain how we proceeded to these transformations and give the data on taxes available.

Table 1: Tax rates on labour income corresponding to our definition in France, 1999 (% of the gross labor wage), Source: The author, from OECD(2000),

Wage level (% of APW)	67	100
Progression index (v)	0.59	0.90

²¹The higher progressivity (higher marginal tax rates and lower average tax rates) appears for people getting out of a precarious situation ie people working in part time work. However, the data on these population are not available yet for us. Our data apply also to an average level.

Finally, we take several elasticities of the marginal disutility of work α ²² to encounter different *labor supply effects*. The empirical studies give controversial results on the elasticity of the labor supply with respect to working hours lying on different specifications. Bourguignon and Magnac (1990) proceeding to a study for France, found a range of 0.05 to 1. Following the works of Blundell and MaCurdy (1999), it seems reasonable to take non compensated elasticities ε in the range of 0.1 to 1. This last elasticity seems extremely high but will allow us to encounter the effects of an important *labor supply effect*. One can also assume that some people of the 67% APW segment might have a very strong elasticity of labor supply²³. We choose also to overvalue these elasticities to see if the labor supply effect might dominate the wage bargaining effect. Some examples of the elasticities used are given in the next table.

elasticity of the labor supply with respect to working hours ε	0.1	0.5	1
elasticity of the marginal disutility of the work α	11	3	2

The second group of parameters is set to reproduce the French economic situation in 1999 without a modification of the progressivity. We set the productivity parameter which has no influence on the steady state as a numeraire. This assumption is not crucial because we are interested in the modification of the parameters. The last parameter we need are the replacement ratio ρ (or the fixed unemployment benefit, which is computed the same way), the cost of a vacant job c , the ladder factors a and d_0 . They result from the calibration and will be different for each group.

Two arguments favor an endogenousing of the unemployment compensation. First, it allows to get a negative relation between the unemployment compensation and the initial unemployment rate²⁴. If we had taken ρ as given, we would have artificially obtained a higher initial wage rate, which would have increased at a lower rate as for a low replacement rate. The firm making its decision on the variation of the wage rate would then have a slower decrease of the profit as the replacement rate and the initial wage rate is higher, which biases the analysis counterintuitively in favour of the labour supply effect. Second, it allows to compare two different segment of the population and different elasticities of labor supply. This way, we use the relation between the initial unemployment rate, the wage rate, the replacement rate and the positive relation between the macro derivative of the unemployment rate with respect to the replacement rate (see Appendix VIII.9). The

²²We assume this elasticity to be constant. We obtain the values of the elasticity of the marginal disutility of the work α , defined by $\alpha = \frac{1 + \varepsilon}{\varepsilon}$.

²³A majority of these people has an important elasticity because their choice is to participate or not to the labor market. The values of the elasticities are also very much overvalued.

²⁴Implicitly, we assume that solely the upper class of earnings contribute to the financing of a higher replacement ratio.

macroeconometrics evaluations of this derivative are quite uncertain. They conclude to a level from 0.1 to 0.15 (see Scarpetta,1996 or Holmlund,1998). We compute the initial wage rate and the replacement ratio in order to get a steady state value of unemployment rate corresponding to the French level of unemployment of each wage segment, their average and marginal tax rates and by taking an average macro derivative of 0.12. We will also match a person with no qualification with a wage rate of 67% of the APW and etc... Until now, we do not have data on these rates. We will also roughly approximate the wage segment by the qualification level.

Table 2: source Enquêtes emploi,2000

	Unemployment rate
No qualification(0.67APW)	15
Low qualification(APW)	10

This approximation is bad and opens to criticism, notably because we assume that a worker stays his whole life at the same qualification level. However, the initial unemployment rate does not seem to be a crucial variable on the strength of the counteractive effects we study.

The remaining parameters, c and do are computed by using the wage setting, the price setting and the matching function relations. We compute these values in order to get a steady state vacant job finding rate corresponding to the French level. A job is approximaty vacant for 5 weeks in France until it is filled. The vacant job duration in a year is also about 10.

The parameter a is calibrated for the initial progression index and allows us to set the initial hours worked at one.²⁵

V.2 The simulation results

V.2.1 General report

First, the hourly wage always increases as the total progressivity diminishes. This result is also consistent with the study of Lockwood and Manning (1993), Paddoa-Schioppa (1990) and Hansen and alii (2000).

Second, the only case for an unemployment rate to diminish with the total progressivity happens as the unemployment compensation is fixed (see 3 and 4). Even with a biased

²⁵For the sake of simplicity, we choose not to give the values of the calibrated parmeters here because as we shall see next, they are different for each case studied.

case in favour of the labour supply effect, the unemployment rate does never diminish for a lower progressivity for a fixed replacement ratio. The intuition is straightforward. For a lower progressivity, in the indexed unemployment compensation regime, the worker's rent does not increase as much as in the fixed unemployment benefits regime. It encourages the worker to ask always for higher wage increases because the cost of being unemployed is less important (the reservation wage increases). The worker does also take less into account the impact of the negotiation on the unemployment rate. The profit per worker has also more chances to diminish for a lower progressivity.

Thirdly, in the case of a fixed unemployment benefit, it is possible to put forward for credible elasticities of labor supply, cases where the unemployment rate decreases for a lower progressivity (see 5). However, one needs a quite high elasticity of labor supply for an unemployment rate being highly affected by the tax progressivity index.

The impact on the unemployment hinges on two critical parameters. The elasticity of labor supply influences the shape of the profit and unemployment curves. For appropriate values of this parameter, the shapes of the previous curves become convex. The initial tax wedge progressivity is important for the political recommendation because it allows to know in which part of the curves the total tax progressivity system of the segment is located.

The unique case where a lower tax progression enhances employment is as the elasticity of labor supply and initial tax progression are sufficiently high (see 5 and 6). The economic intuition is as follows. A lower tax progression on labor income diminishes the cost to the employer of providing the worker an increase in the wage rate and increases the cost to the worker of conceding more profit to the employer. In this case, a wage rate increase is too costly for the pair, which decides to increase the hours of work. It is also very likely that the *labor supply effect* dominates the *wage bargaining effect*. However this case is not relevant to a weak initial progressivity. The explanation lies in the initial individual hours, which are implicitly much more higher²⁶. In this case, the increase in the individual hours is not sufficiently high to offset the hourly wage's increase and the profit per worker drops. Less vacant jobs are also opened in the economy and the unemployment rises.

Finally, the general result is that the most convincing conclusion is a *wage bargaining effect* dominating the *labor supply effect*.

Table summing up the general report:

²⁶In our simulations, we take the same initial working hours but our parameter α captures in a certain kind the responsiveness of the working hours. It can be seen that for progression index increasing by 1%, the hours react by $1.71 \frac{1}{1-\alpha}$ in the case of an 0.67 APW and by $1.2 \frac{1}{1-\alpha}$ in the case of an APW. Implicitly, it means that the total disutility of work is lower or other said that the initial hours worked are lower.

For a decrease of the tax progression:
Hourly wage always increases
Unemployment rate decreases only fixed unemployment compensation
critical parameters for unemployment : elas of labor supply, initial tax prog

V.2.2 The tradeoff interest issue

When there is no tradeoff between the economic activity and unemployment²⁷

At first, this case never happens for a fixed replacement ratio. We will also analyse solely the case of fixed unemployment benefits.

This case happens solely²⁸ as the elasticity of labor supply is quite high (more than 0.5) and the initial progressivity is high. In the biased case for the *labor supply effect*, (see 4), the initial hours are quite low . For a progression index rising from 0.6 to 0.9 , the increase of the cost rate does not weight to much on the profit of the firm. The decrease of the profit per hour is also largely offset by the individual hours of work and finally the profit per job will enhance.

When the tradeoff profiles The tradeoff between the economic activity represented by the total working hours in the economy and unemployment arises as the initial tax wedge progressivity is weak even when the elasticity of labor supply is high. In this case, for a lower marginal tax wedge, the cost rate is so high that even a weak increase in the cost rate can never be offset by hours of work (which are initially very high too).

Two different cases hinging on the unemployment benefit arise.

For a fixed unemployment compensation This case happens for elasticities lower than 0.4. Two different cases depending on the labor supply elasticity and the initial progressivity arise.

First, for a high initial progressivity (case of an 0.67 APW). As the elasticity of labor supply is very low²⁹ (see 7), by lowering the tax progression index from 0.59 to 0.5 ie rising the marginal tax rate from 49% to 56%, the cost rate diminishes of 0.5% and the total hours of work about 0.5%. As they were low initially, their very small decrease do not hinge too much on the profit per worker all the more since the decrease of the cost rate enhances sharply the profit per hour. Finally, the profit per worker is improved by more than 10%.

²⁷It is straightforward to see that the conflict does not arise for a high initial tax progression. But, this case is not interesting since a higher tax progression would increase the unemployment rate and decrease the total working hours.

²⁸We disregard the case where the progression index is higher than one.

²⁹We are thus in the case of no response of hours of work which was put forward by the aforementioned papers in the introduction.

More firms open jobs on the labor market, which by a positive intergroup externality diminishes the job spell duration of the unemployed and diminishes the unemployment (about 1%).

As the elasticity of labor supply is a little bit higher (see 8), the profit will always drop because the initial hours worked are higher and the sharp decrease of the individual hours do not offset the decrease of the hourly wage.

Second, for a low initial progressivity (case of an APW), the results are in an opposite direction : it seems better to lower the progressivity. In fact, the unemployment rate will increase a little bit and the working hours much more.

For an indexed unemployment compensation The initial progressivity does not really apply since it is impossible to diminish the unemployment rate by lowering the progressivity. We also restrict our analysis to an 0.67 APW. An increase in the tax progressivity index from 0.6 to 0.5 decreases the hours about 3% and the hourly wage about 0.7%. Because the hours of work are initially low and the wage rate is initially high, its drop will offset the decrease of the working hours and the profit per worker increases about 10% and the unemployment rate might decrease more than 1% (see 9). The total hours worked decrease about 1.5% but this might not be very important because they were initially low (the decrease is not to high in absolute value term).

Table summing up the tradeoff issue with examples:

	Elas lab sup	Ini tax prog	Var margi tax	Var unem	Var tot hours
No tradeoff					
Fix comp	high(>0.5)	High	High(-)	Low<-0.5%	+
Tradeoff					
Fix comp	0.1	High	High (+7%)	-1%	-0.5%
Ind comp	0.2	Not apply	High (+7%)	-1%	-1.5%

V.2.3 Should the tax wedge progression index be reduced in France?

All the problem lies on the extent to which the economic activity gains from a lower tax progression, coming from a higher efficiency of the job matching process, are set against the welfare loss from a higher involuntary unemployment. There may also be an optimal amount of tax progressivity without any appeal to equity considerations.

The decision has also to be based on the elasticity of the labor supply. A political recommendation will also be completely different for each segment of the population. For the single male workers, for instance, the elasticity seems quite lower than for single women

having children. But the problem is even not settled yet because one has to put forward a welfare analysis.

We will restrict our analysis to indexed unemployment benefit³⁰. As we have seen, it seems better in this case to increase the progressivity. From a welfare perspective, it might be a good answer to lower inequalities between the employed and the involuntary unemployed because a decrease of the progression index from 0.6 to 0.5 would decrease the inequality about 15% (see 9).

Finally, a trade-off between equity and economic activity arises and a modification of the tax wedge progression does not seem to be a free-lunch as was announced by Pissarides (1998).

VI Conclusion

In a job search equilibrium framework, we showed that at a microeconomic level, a lower tax progression is always bad for employment through a too high wage increase. At the macroeconomic level, we demonstrated that the unemployment compensation regime, the elasticity of the labor supply and the initial progressivity were critical parameters for political recommendations. Our numerical results suggest that a lower tax progression increases the wage pressure so that the profit per worker of the firm is reduced. This effect is hardly offset by an increase in working hours so that the involuntary unemployment increases and the economic activity measured by the total hours of work enhances for certain segments of the population. We learnt about the tradeoff issue between economic activity and unemployment that there are few cases when the tradeoff does not arise. The next step of a research on this topic would be to have a political recommendation approach.

However, there are a number of reasons to exercise caution in policy prescriptions underlined in our paper.

Our results hinges strongly on our representation of the utility of a worker. By taking into account in their function the substitution and income effects, Cahuc and Zylberberg (2001) show analytically for the two unemployment compensation regimes an unambiguously negative relation between the progressivity and the unemployment rate.

The results obtained are without any concern about the participation rate. It might be interesting to study this case since the elasticity of non active persons are generally higher than working people.

³⁰The analysis is straightforward for fixed unemployment benefits because the welfare gap between the employees and the unemployed increases as the tax progression is lower

A crucial element to take into account is the state budget equilibrium. As the economic activity measured by the total hours worked is affected by a tax reform, the government's budget is affected, as well. This kind of influence would allow to study simultaneous modifications of the marginal and average tax rates. As is shown in Rasmussen (1999), the combination of a long-run equilibrium analysis and a balanced-budget tax reform can account for adverse effects on employment. Finally, the incentives to make an effort at work underlined in Andersen and Rasmussen (1997) do not have to be neglected. A shift of the tax burden away from the low paid labor to the high earnings might enhance the incentives of a certain class to work more but be harmful for the effort of the more withdrawalled categories of earnings. In the short run, this might decrease the economic activity. In the long run, one might as well anticipate the adverse effects on the human capital investment.

All these issues are left for future researchs.

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VIII APPENDIX

VIII.1 Determination of wages and hours in the microeconomic analysis

The Nash Bargaining problem writes as

$$\underset{w_i, h_i}{Max} \Omega(w_i, h_i) = [W_i(w_i, h_i) - U]^\beta [J_i(w_i, h_i) - V]^{1-\beta}$$

where $\beta \in (0, 1)$ represents the bargaining power of the worker.

The first order conditions are:

with respect to w : $\frac{\beta}{W_i - U} * \frac{\partial(W_i - U)}{\partial w_i} + \frac{1 - \beta}{J_i - V} * \frac{\partial(J_i - V)}{\partial w_i} = 0$
Using this expression, it follows that:

$$W_i - U = \frac{\beta v(1 - T_M)}{1 - \beta} (J_i - V)$$

We obtain using the expression of $J_i - V$ coming from the relations (5) and (6), the expression of the wage rate:

$$w_i = p - \frac{(1 - \beta)(r + \lambda)}{\beta(1 - T_M)v h_i} (W_i - U) \quad (13)$$

with respect to h : $\frac{\beta}{W_i - U} * \frac{\partial(W_i - U)}{\partial h_i} + \frac{1 - \beta}{J - V} * \frac{\partial(J - V)}{\partial h_i} = 0$

This expression compared to the first order condition on the wage rate, gives the optimal working hours:

$$h^{\alpha-1} = \frac{p}{a} v(1 - T_M) \quad (14)$$

VIII.2 Total elasticities in the microeconomic analysis

To find the total elasticities of the wage rate, we differentiate the relation(13).

We obtain, at the symmetric equilibrium,

$$dw = -\frac{(1 - \beta)(r + \lambda)}{\beta(1 - T_M)} \left[\frac{d(W - U)}{v h} + (W - U) \left(-\frac{dv}{v^2 h} - \frac{dh}{h^2 v} \right) \right] \quad (15)$$

with $d(W - U) = \frac{v(1 - T_M)}{r + \lambda} [h dw + dh(w - p)]$

Replacing the expressions $(W - U)$ and $d(W - U)$, from (15) we have after a few calculations:

$$\frac{dw}{dv} = (p - w) \left(\frac{dh}{dv} \frac{1}{h} + \frac{\beta}{v} \right) \quad (16)$$

We find easily from (14) that $\xi_{h,v} = \frac{1}{\alpha - 1} > 0$

From (16), $\xi_{w,v} = \frac{p-w}{w} \left(\frac{1}{\alpha-1} + \beta \right) > 0$

To derive the elasticity of the employee's rent with respect to the tax progression index, we differentiate the relation (13), we have:

$$d(W - U) = \frac{v(1 - T_M)}{r + \lambda}(hdw + wdh - pdh), \text{ which after some calculations gives:}$$

$$\frac{d(W - U)}{dv} = \frac{\beta(p - w)}{r + \lambda}h(1 - T_M)$$

Differentiating the intertemporal profit relation $J = (h(\eta - w))/(r + \lambda)$ and taking V as given, we obtain

$$\frac{dJ_i}{J_i} = -\frac{w}{p - w} \frac{dw}{w} + \frac{dh}{h}. \text{ By dividing every term of this relation by } \frac{dv}{v}, \text{ we have}$$

$$\xi_{J,v} = \xi_{h,v} + \xi_{w,v} \varepsilon_{J,w} \text{ with } \varepsilon_{J,w} = -\frac{w}{p - w}.$$

We also have:

$$\xi_{J,v} = \frac{\frac{dJ}{J}}{\frac{dv}{v}} = -\beta < 0$$

VIII.3 Determination of wages and hours in the macroeconomic analysis

At the macroeconomic level, the first order conditions write as in the microeconomic case. However, the expected income of an unemployed is endogenous.

Using the relations (1),(2) and the free condition entry, the first order condition on the wage rate at the symmetric equilibrium,

$$\beta(1 - T_m)(p - w)h = (1 - \beta)(I(w, h) - rU)$$

becomes :

$$\beta(1 - T_m)(p - w)h = (1 - \beta)I(w, h) \frac{r + \lambda}{r + \lambda + \theta q(\theta)}$$

For an indexed unemployment compensation, by introducing the progression index, taking the (14) and (3) relations and isolating the wage rate, we obtain:

$$w = v \frac{p\beta + (1 - \beta) \frac{p}{\alpha} \frac{r + \lambda}{r + \lambda + \theta q(\theta)}}{\beta v + (1 - \beta)(1 - \rho) \frac{r + \lambda}{r + \lambda + \theta q(\theta)}} \quad (17)$$

For a fixed unemployment compensation, by introducing the progression index, taking the (14) and (4) relations and isolating the wage rate, we obtain:

$$w = \frac{p\beta v + (1 - \beta) \frac{r + \lambda}{r + \lambda + \theta q(\theta)} \left(\frac{pv}{\alpha} + Y \right)}{\beta v + (1 - \beta) \frac{r + \lambda}{r + \lambda + \theta q(\theta)}} \quad (18)$$

by calling

$$Y = \frac{\bar{z}}{\left(\frac{p}{a} v (1 - T_M) \right)^{\alpha - 1} (1 - T_M)}$$

VIII.4 Impact of the negotiation power

The impact of the negotiation power on the wage rate is derived from the wage setting relation. For an indexed unemployment compensation, we have:

$$\frac{\partial w}{\partial \beta} = \alpha \frac{r + \lambda}{r + \lambda + \theta q(\theta)} (\alpha(1 - \rho) - v)$$

But, it can be shown that :

$$p - w = p \frac{r + \lambda}{r + \lambda + \theta q(\theta)} (1 - \beta) \left(1 - \rho - \frac{v}{\alpha} \right)$$

We always have $p > w$ because otherwise the intertemporal profit on an occupied job would be lower than the intertemporal profit on a vacant job. The last condition implies also that $\alpha(1 - \rho) > v$, which means that $\frac{\partial w}{\partial \beta} > 0$

VIII.5 Determination of the partial and total elasticities in the macroeconomic analysis

We choose to give the resolution method for an indexed unemployment compensation. The same method is applicable to a fixed unemployment compensation.

From the relation (17), we have:

$$\frac{dw}{w} = \varepsilon_{w,v}^p * \frac{dv}{v} + \varepsilon_{w,\theta}^p * \frac{d\theta}{\theta} \quad (19)$$

with $\varepsilon_{i,j}^p$, the partial elasticity of i with respect to j , the other endogenous variables taken as given. It is easy to demonstrate that:

$$\varepsilon_{w,v}^p = \frac{(1-\beta)(r+\lambda)(1-\rho)}{(1-\beta)(1-\rho)(r+\lambda) + \beta v(r+\lambda + \theta q(\theta))} \text{ with } 0 < \varepsilon_{w,v}^p < 1$$

$$\varepsilon_{w,\theta}^p = \frac{(1-\beta)(r+\lambda)(\theta q(\theta))\beta(1-\rho - \frac{v}{\alpha})\theta}{[\beta v(r+\lambda + \theta q(\theta)) + (1-\beta)(1-\rho)(r+\lambda)] \left[\beta(r+\lambda + \theta q(\theta)) + \frac{1}{\alpha}(1-\beta)(r+\lambda) \right]}$$

with $0 < \varepsilon_{w,\theta}^p < 1$

From the price setting relation, we have:

$$\frac{1}{\eta}(-\varepsilon_{J,w} * \frac{dw}{w} - \frac{dh}{h}) + \frac{d\theta}{\theta} = 0 \quad (20)$$

where $\eta = -\frac{q(\theta)\theta}{q(\theta)}$

From the relation determining the working hours, we have:

$$(\alpha - 1) * \frac{dh}{h} = \frac{dv}{v} \quad (21)$$

On the matrix form, we have from the relations (19), (20) and (21):

$$\begin{pmatrix} 1 & -\varepsilon_{w,\theta}^p & 0 \\ -\frac{\varepsilon_{\pi,w}}{\eta} & 1 & -\frac{1}{\eta} \\ 0 & 0 & \alpha - 1 \end{pmatrix} \begin{pmatrix} \frac{dw}{w} \\ \frac{d\theta}{\theta} \\ \frac{dh}{h} \end{pmatrix} = \begin{pmatrix} \varepsilon_{w,v}^p \\ 0 \\ 1 \end{pmatrix} * \frac{dv}{v}$$

We also find the total elasticities of the endogenous variables with respect to the progression index. Calling $\xi_{i,v} = \frac{\frac{di}{i}}{\frac{dv}{v}}$, the total elasticity of the variable i with respect to the index progression, we obtain:

$$\xi_{w,v} = \frac{\eta \varepsilon_{w,v}^p + \frac{\varepsilon_{w,\theta}^p}{\alpha - 1}}{\eta - \varepsilon_{w,\theta}^p \varepsilon_{J,w}} > 0 \text{ because } 1 > \eta > 0, \varepsilon_{w,v}^p > 0, \varepsilon_{w,\theta}^p > 0, \alpha > 1 \text{ and } \varepsilon_{J,w} < 0$$

$$\xi_{\theta,v} = \frac{1}{(\alpha - 1)(\eta - \varepsilon_{w,\theta}^p \varepsilon_{J,w})} ((\alpha - 1) \varepsilon_{\pi,w} \varepsilon_{w,v}^p + 1), \text{ which sign}$$

is indetermined because the first term is negative and the second is positive

$$\xi_{h,v} = \frac{1}{\alpha - 1} > 0$$

To the end, from the differentiation of the Beveridge relation ($u = \frac{s}{s + \theta q(\theta)}$), we obtain:

$$\xi_{u,v} = -\xi_{\theta,v} \frac{\theta q(\theta)(1-\eta)}{s + \theta q(\theta)}, \text{ which sign is indetermined too}$$

For a fixed unemployment compensation, it can be shown, calling $X = \frac{r + \lambda}{r + \lambda + \theta q(\theta)}$, that :

$$\begin{aligned}\varepsilon_{w,v}^p &= \frac{(1 - \beta)Xv(p\beta + \frac{(1 - \beta)Xp}{\alpha} - Y\beta)}{((1 - \beta)X + \beta v)(pv\beta + (1 - \beta)X(\frac{pv}{\alpha} + Y))} \text{ with } 0 < \varepsilon_{w,v}^p < 1 \\ \varepsilon_{w,\theta}^p &= \frac{(1 - \beta)v\theta\beta(\frac{pv}{\alpha} + Y - p)\frac{\partial X}{\partial \theta}}{((1 - \beta)X + \beta v)(pv\beta + (1 - \beta)X(\frac{pv}{\alpha} + Y))} \text{ with } 0 < \varepsilon_{w,\theta}^p < 1 \text{ because } \frac{\partial X}{\partial \theta} < 0 \\ \text{and } \frac{pv}{\alpha} + Y - p &< 0 \\ \varepsilon_{w,h}^p &= -\frac{(1 - \beta)XY}{pv\beta + (1 - \beta)X(\frac{pv}{\alpha} + Y)} < 0\end{aligned}$$

VIII.6 Evolution of the elasticity of the unemployment rate with respect to the job finding rate

We have the derivative:

$$\frac{\partial \left(\frac{\theta q(\theta)(1 - \eta)}{s + \theta q(\theta)} \right)}{\partial (\theta q(\theta))} = \frac{(1 - \eta)s}{(s + \theta q(\theta))^2} > 0$$

VIII.7 Transformation of taxes

For the sake of simplicity, in our model, the negotiations carry on the cost rate and the working hours. We have also to proceed to transformations to obtain the average and marginal taxes englobing the taxes paid by the employee and the firm. Our progression index (called coefficient of residual income) will not have the same expression as in Musgrave (1976)

First, we have:

$$wh = w_g h + S(w_g h) \quad (22)$$

where w is the cost rate for the firm

w_g denotes the gross wage rate

$S(w_g h)$ represents payroll taxes

$$w_E h = w_g h - \tau(w_g h) \quad (23)$$

where w_E indicates the net wage rate perceived by the employee

$\tau(w_g h)$ represents all the taxes paid by the employee

So we obtain:

$$wh = w_g h(1 + S_M(w_g h)) \quad (24)$$

with $S_M(w_g h)$, the average payroll taxes, which are supposed to be constant

and

$$w_E h = w_g h(1 - \tau_M(w_g h)) \quad (25)$$

with, $\tau_M(w_g h)$, the average taxes paid by the employee, which are supposed to be constant

We obtain also:

$$w_g = \frac{w}{1 + S_M(w_g h)} \quad (26)$$

and

$$w_E = w \frac{(1 - \tau_M(w_g h))}{1 + S_M(w_g h)} \quad (27)$$

In our model, we have also assumed that:

$$\frac{(1 - \tau_M(w_g h))}{1 + S_M(w_g h)} = 1 - T_M(wh) \quad (28)$$

where $T_M(wh)$, represents the average total tax paid by the employer and the employee

Now, we search the expression of the total marginal tax rate $T_m(wh)$ as a function of the marginal tax rate paid by the employee $S_m(w_g h)$ and marginal tax rate paid by the firm $\tau_m(w_g h)$. These two marginal tax rates will be our public economic variables.

By differentiating the relation (28), we obtain:

$$(-dw_g * h \tau_m - dh * w_g \tau_m)(1 + S_M(w_g h))^{-1} - (1 - \tau_M(w_g h))(dw_g * h S_m)(1 + S_M(w_g h))^{-2} = -T_m(wh)(dw * h + dh * w)$$

After simplifications, this expressions becomes:

$$\frac{\tau_m(dw_g * h + dh * w_g)}{1 + S_M(w_g h)} + \frac{(1 - \tau_M(w_g h))(dw_g * h + dh * w_g)S_m}{(1 + S_M(w_g h))^2} = T_m(wh)(dw * h + dh * w) \quad (29)$$

Now, we have to find the expression $(dw * h + dh * w)$.

From the differentiation of the relation (22), we have:

$$(dw * h + dh * w) = (dw_g * h + dh * w_g)(1 + S_m)$$

Thus, the relations(29) and (22) give the marginal tax rate :

$$T_m(wh) = \frac{1}{(1 + S_M(w_g h))(1 + S_m)} \left(\tau_m + \left(1 - \frac{(1 - \tau_M(w_g h))}{1 + S_M(w_g h)} \right) S_m \right)$$

Our coefficient residual income becomes also:

$$v = \frac{v_e}{v_f} = \frac{1 - T_m(wh)}{1 - T_M(wh)} = \frac{(1 + S_M(w_g h))}{(1 - \tau_M(w_g h))} - \frac{\tau_m}{(1 - \tau_M(w_g h))(1 + S_m)} - \frac{S_m}{(1 + S_M(w_g h))(1 + S_m)}$$

Our data form the OECD(2001), give us:

-the marginal rate of income tax plus employee and employer contributions, by wage level as percentage of the labor costs, which writes with our expressions as:

$$\frac{(\tau_m + S_m)w_g h}{wh} = \frac{\tau_m + S_m}{1 + S_M}$$

- the marginal rate of income tax plus employee contributions, by wage level as percentage of the gross wage, which is our τ_m .

-the income tax plus employee contributions, by wage level as percentage of the gross wage, which corresponds to our $\tau_M(w_g h)$

-the annual labor costs (which is wh) and net income (which is $.w_E h$) by wage level.

The ratio of these two quantities gives:

$$\frac{wh}{w_E h} = \frac{1 + S_M}{1 - \tau_M}$$

Finally, we have all the data to calculate the total average tax rate $T_M(wh)$, which we assume to be constant over the wage level studied and the coefficient residual income v , which is our economic policy variable.

VIII.8 Taxes data

For each class we have the following average, marginal tax wedge rate.

Table 3: Tax rates on labour income supported by single workers in France, 1999 (% of the gross labor wage), Source: The author, from OECD(2000),p44 and 47

Wage level (% of APW)	67	100
Average tax	0.235	0.277
Marginal tax	0.486	0.348
Progression index (v_e)	0.67	0.90

Table 4: Tax rates on labour income supported by the firm for single workers in France, 1999 (% of the gross labor wage), Source: the author, from OECD(2000), p46 and 48

Wage level (% of APW)	67	100
Average tax	0.281	0.393
Marginal tax	0.456	0.393
Progression index (v_f)	1.136	1

Table 5: Total progression index for single workers in France, 1999 (% of the gross labor wage), Source: the author, from OECD(2000)

Wage level (% of APW)	67	100
Progression index (v)	0.59	0.90

VIII.9 Calibration: the macroderivative of the unemployment rate with respect to the replacement rate

To compute the initial replacement rate and the hourly wage, we use the expression of the wage setting relation and isolate the $\theta q(\theta)$. We reintegrate this term in the relation of the unemployment rate (beveridge curve) and obtain the unemployment rate as a function of the wage rate and the replacement ratio. In fact, we obtain:

$$u = \frac{\lambda}{\lambda + (r + \lambda) \frac{\frac{pv}{1 + T_M^f} (\beta + \frac{1 - \beta}{\alpha}) - w(\beta v + (1 - \beta)(1 - \rho))}{\beta v (w - \frac{p}{1 + T_M^f})}} \quad (30)$$

which is increasing in the replacement ratio and the wage rate.

Derivating this relation with respect to the replacement ratio, we obtain the macroderivative of the unemployment rate with respect to the replacement rate:

$$\begin{aligned}
\text{DERIV} = \frac{\partial u}{\partial \rho} = & \frac{\frac{sw(1-\beta)(1-\rho)}{\beta v(\frac{p}{1+T_M^f} - w)}}{(\lambda + (r + \lambda) \frac{\frac{pv}{1+T_M^f}(\beta + \frac{1-\beta}{\alpha}) - w(\beta v + (1-\beta)(1-\rho))}{\beta v(w - \frac{p}{1+T_M^f})})^2} \quad (31)
\end{aligned}$$

By rearranging the relations (31) and (30), we obtain the initial wage rate as a function of the macroderivative and the initial unemployment of the segment of population considered in France. Using the (30), we obtain the French replacement ratio linked to our model.

To compute the last parameter c , do and the initial tightness, we have a three relations and three unknowns system given by the wage setting and price setting relation and the vacant job finding rate.

IX Graphics

The graphics show the deviation in percentage from the initial value except for the unemployment rate.

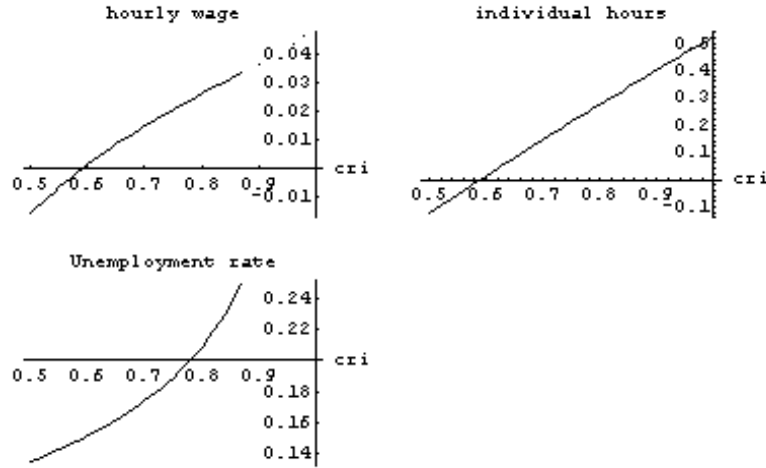


Figure 3: Case biased in favour of the labour supply effect. Case of an 0.67APW, with $\text{elas} = 1$, $w_0 = 0.74$ and $\rho = 0.54$

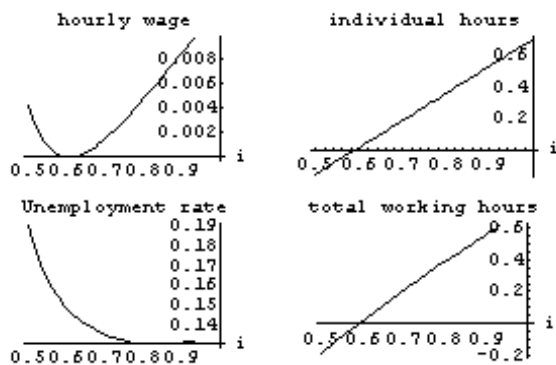


Figure 4: Biased case in favour of a labour supply effect. Case of an 0.67 APW, with $\text{elas} = 1$, $w_0 = 0.76$ and $A = 0.33$

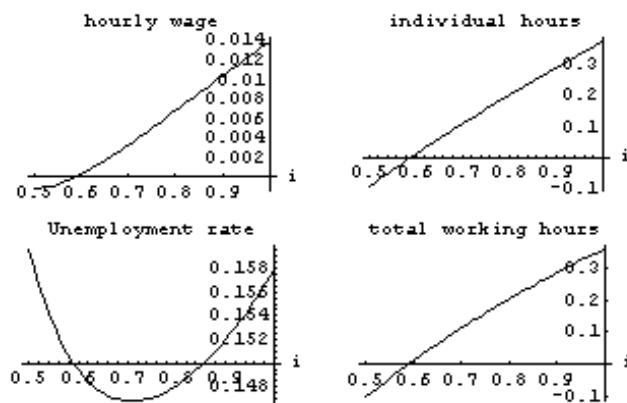


Figure 5: Case of an 0.67APW, with $\text{elas} = 0.6$, $w_0 = 0.75$, $A = 0.35$ and $a_0 = 0.35$

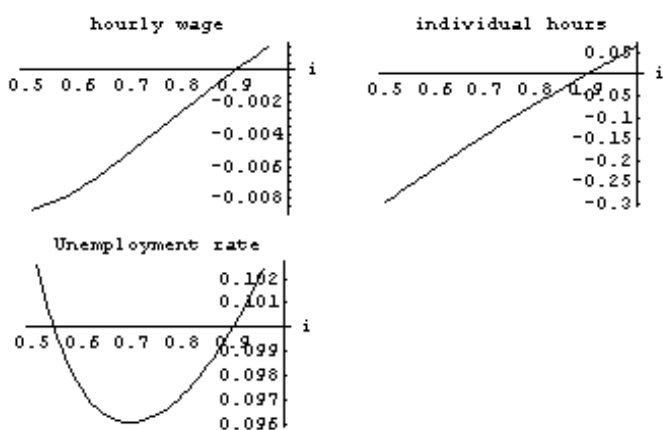


Figure 6: Case of an APW, with $\text{elas} = 0.6$, $w_0 = 0.7$, $A = 0.24$ and $a_0 = 0.47$

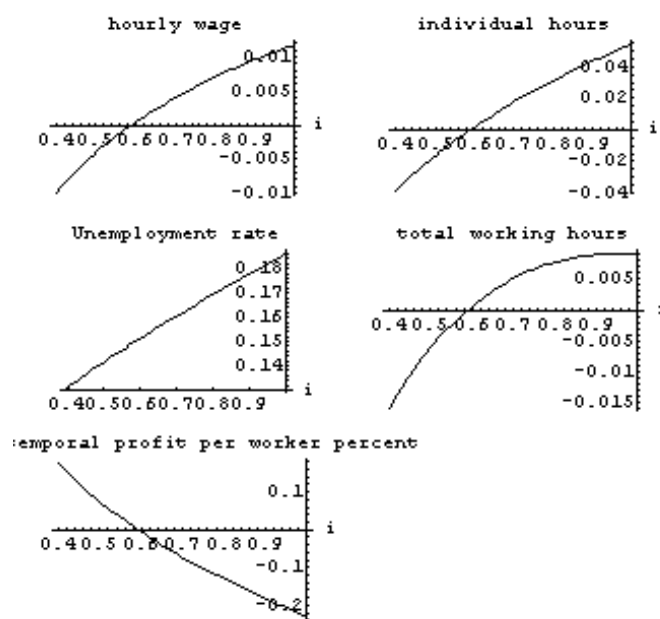


Figure 7: Case of an 0.67APW, with $elas = 0.1$, $wo=0.75$, $A=0.46$ and $ao=0.35$

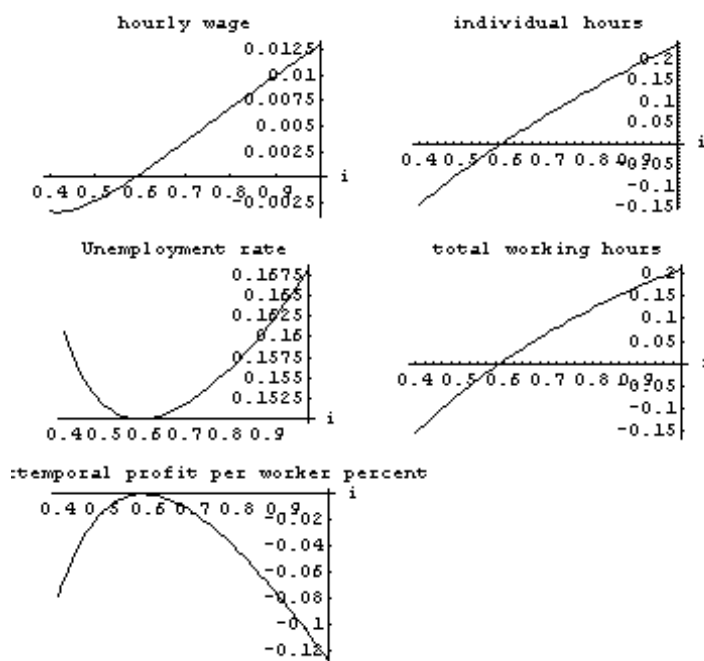


Figure 8: Case of an 0.67APW, with $elas = 0.4$, $wo=0.75$, $A=0.39$ and $ao=0.35$

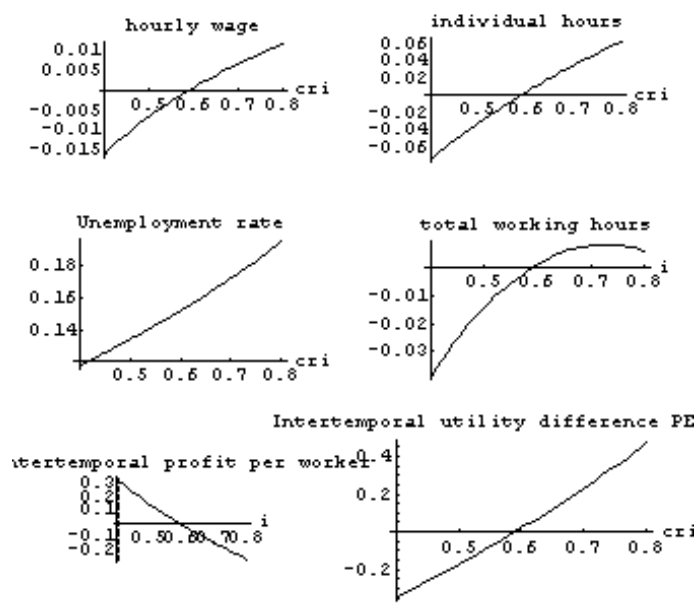


Figure 9: Case of an 0.67APW, with $\text{elas} = 0.2$, $\text{wo} = 0.755$ and $\rho = 0.78$